# Bottom-Up Parsing
# LR Parsing. Parser Generators.

Lecture 6

# Bottom-Up Parsing

- Bottom-up parsing is more general than top-down parsing
  - And just as efficient
  - Builds on ideas in top-down parsing
  - Preferred method in practice

- Also called LR parsing
  - L means that tokens are read left to right
  - R means that it constructs a rightmost derivation !

# An Introductory Example

- LR parsers don't need left-factored grammars and can also handle left-recursive grammars

- Consider the following grammar:

$$E \rightarrow E + ( E ) \mid \text{int}$$

  - Why is this not LL(1)?

- Consider the string: int + ( int ) + ( int )

# The Idea

- LR parsing *reduces* a string to the start symbol by inverting productions:

str = input string of terminals

repeat

- Identify $\beta$ in str such that $A \rightarrow \beta$ is a production (i.e., str = $\alpha\ \beta\ \gamma$)
- Replace $\beta$ by $A$ in str (i.e., str becomes $\alpha\ A\ \gamma$)

until str = $S$

# A Bottom-up Parse in Detail (1)

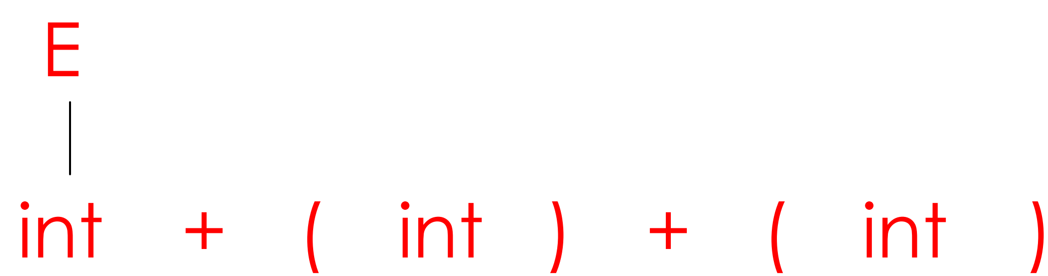
int + (int) + (int)

int + ( int ) + ( int )

# A Bottom-up Parse in Detail (2)

int + (int) + (int)

E + (int) + (int)

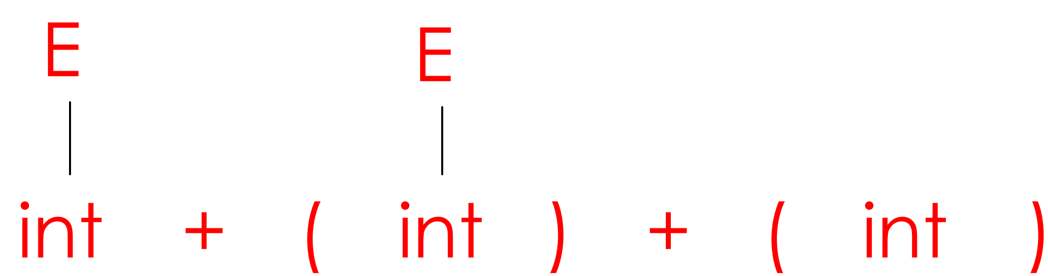


# A Bottom-up Parse in Detail (3)

int + (int) + (int)

E + (int) + (int)

E + (E) + (int)



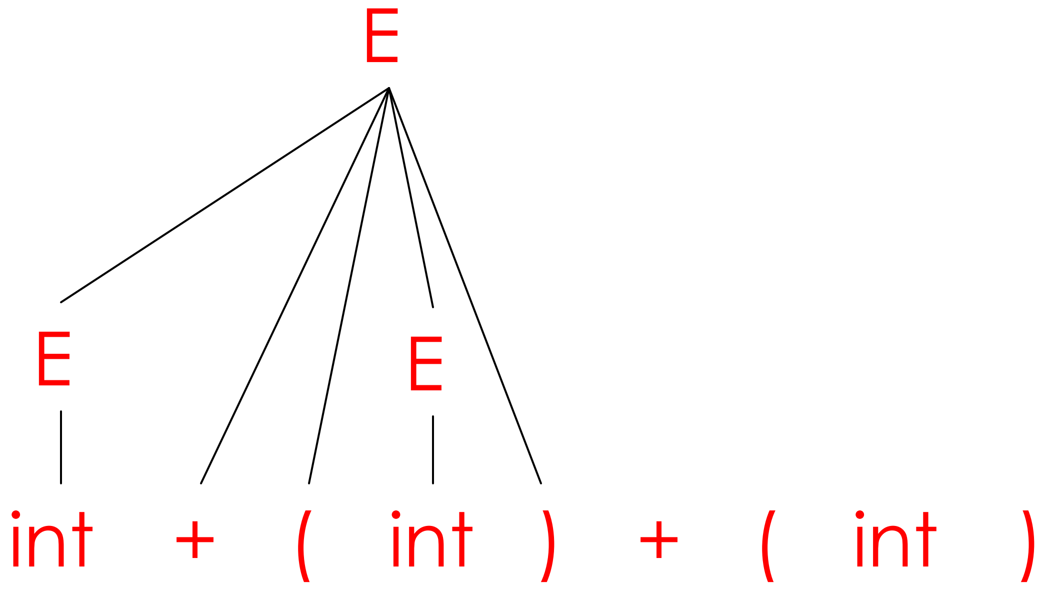
# A Bottom-up Parse in Detail (4)

int + (int) + (int)

E + (int) + (int)

E + (E) + (int)

E + (int)



# A Bottom-up Parse in Detail (5)

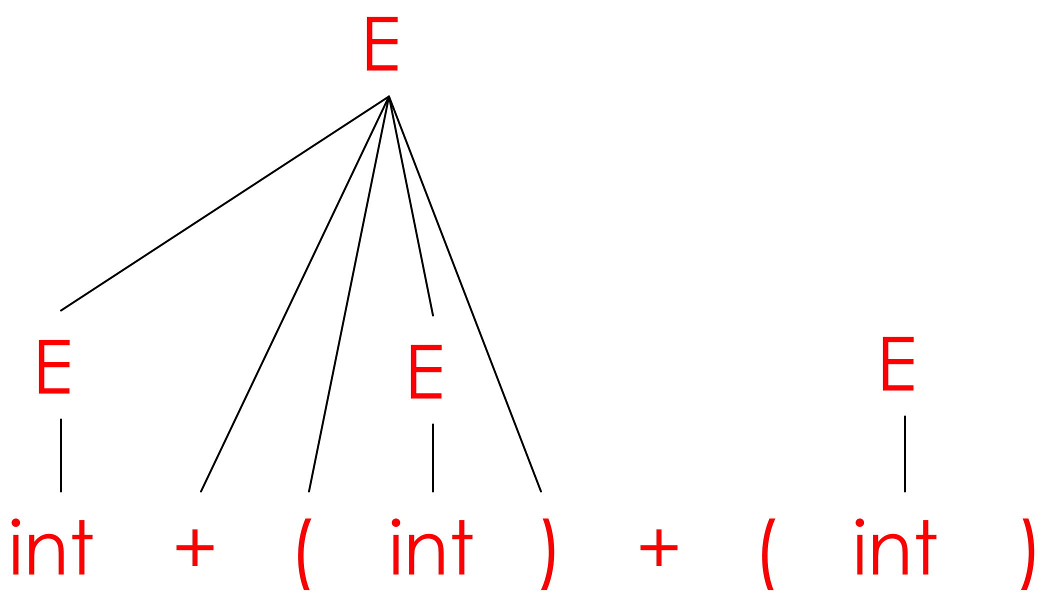
int + (int) + (int)

E + (int) + (int)

E + (E) + (int)

E + (int)

E + (E)



# A Bottom-up Parse in Detail (6)

int + (int) + (int)

E + (int) + (int)

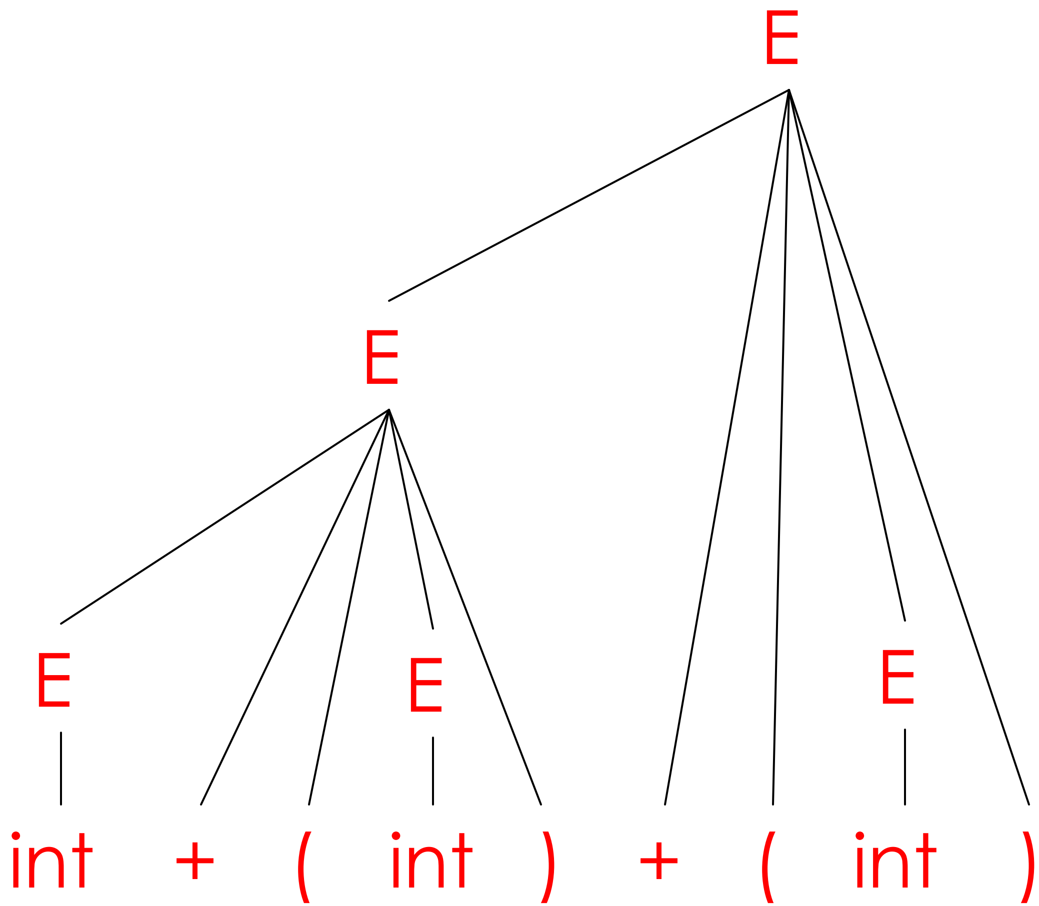
E + (E) + (int)

E + (int)

E + (E)

E

A rightmost derivation in reverse



# Important Fact #1

Important Fact #1 about bottom-up parsing:

*An LR parser traces a rightmost derivation in reverse*

# Where Do Reductions Happen

Important Fact #1 has an interesting consequence:

- Let $\alpha\beta\gamma$ be a step of a bottom-up parse
- Assume the next reduction is by $A \rightarrow \beta$
- Then $\gamma$ is a string of terminals !

Why? Because $\alpha A\gamma \rightarrow \alpha\beta\gamma$ is a step in a right-most derivation

# Notation

- Idea: Split string into two substrings
  - Right substring (a string of terminals) is as yet unexamined by parser
  - Left substring has terminals and non-terminals
- The dividing point is marked by a I
  - The I is not part of the string
- Initially, all input is unexamined: $I x_1 x_2 \ldots x_n$

# Shift-Reduce Parsing

- Bottom-up parsing uses only two kinds of actions:

*Shift*

*Reduce*

# Shift

*Shift:* Move I one place to the right

- Shifts a terminal to the left string

$$E + (\mid \text{int}\ ) \Rightarrow E + (\text{int} \mid )$$

# Reduce

*Reduce:* Apply an inverse production at the right end of the left string

- If $E \to E + ( E )$ is a production, then

$$E + (\underline{E + ( E )} \mid ) \Rightarrow E + (\underline{E} \mid )$$

# Shift-Reduce Example

| int + (int) + (int)\$   shift

int + ( int ) + ( int )

# Shift-Reduce Example

| int + (int) + (int)$   shift

int | + (int) + (int)$   red. E → int

int   +   (   int   ) +   (   int   )

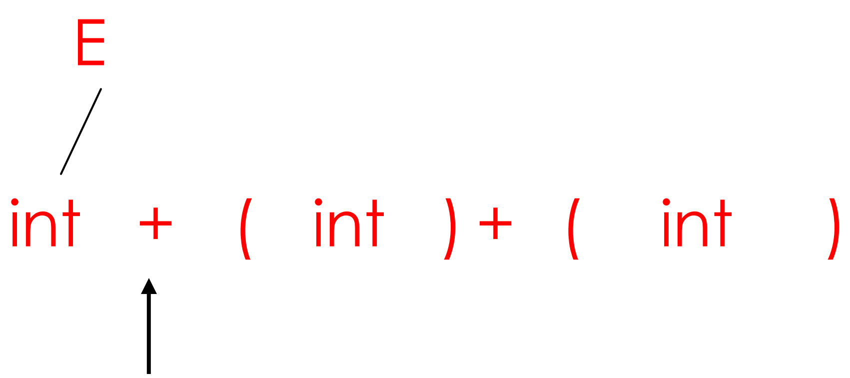
↑

# Shift-Reduce Example

| int + (int) + (int)$    shift

int | + (int) + (int)$    red. E → int

E | + (int) + (int)$    shift 3 times



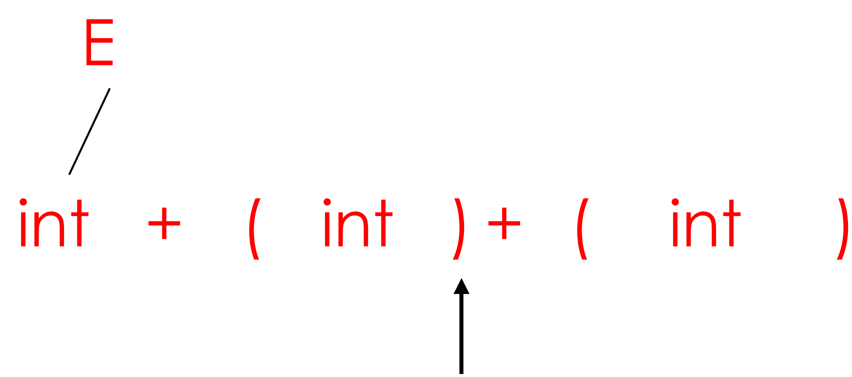
# Shift-Reduce Example

| int + (int) + (int)$   shift

int | + (int) + (int)$   red. E → int

E | + (int) + (int)$   shift 3 times

E + (int | ) + (int)$   red. E → int



# Shift-Reduce Example

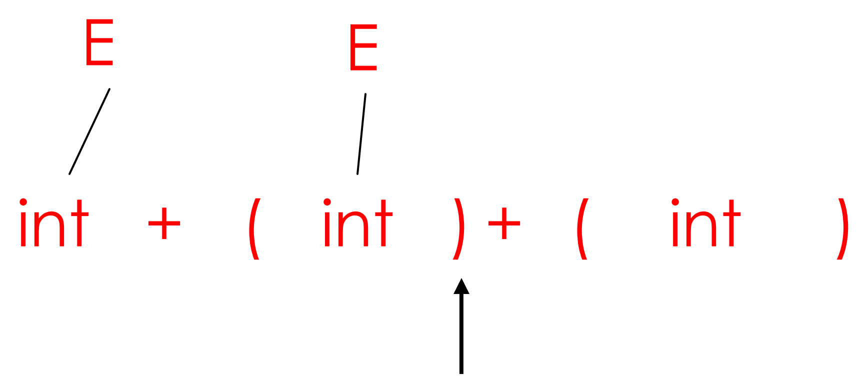
| int + (int) + (int)$ shift

int | + (int) + (int)$ red. E $\rightarrow$ int

E | + (int) + (int)$ shift 3 times

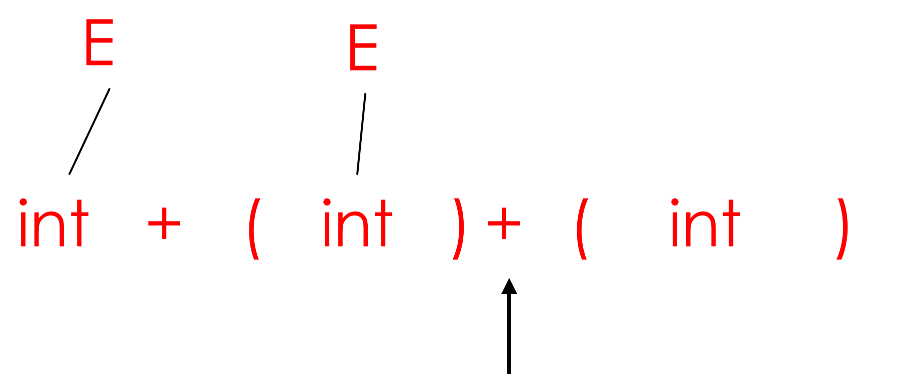
E + (int | ) + (int)$ red. E $\rightarrow$ int

E + (E | ) + (int)$ shift



# Shift-Reduce Example

| | |
|---|---|
| \| int + (int) + (int)$ | shift |
| int \| + (int) + (int)$ | red. E → int |
| E \| + (int) + (int)$ | shift 3 times |
| E + (int \| ) + (int)$ | red. E → int |
| E + (E \| ) + (int)$ | shift |
| E + (E) \| + (int)$ | red. E → E + (E) |



# Shift-Reduce Example

| int + (int) + (int)$ shift

int | + (int) + (int)$ red. E → int

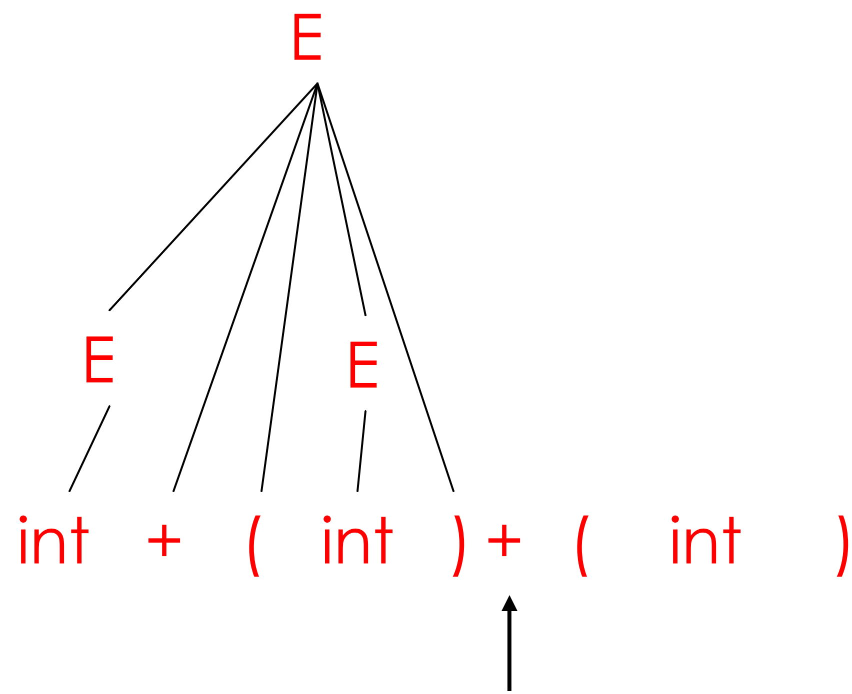
E | + (int) + (int)$ shift 3 times

E + (int | ) + (int)$ red. E → int

E + (E | ) + (int)$ shift

E + (E) | + (int)$ red. E → E + (E)

E | + (int)$ shift 3



# Shift-Reduce Example

| int + (int) + (int)$    shift

int | + (int) + (int)$    red. E → int

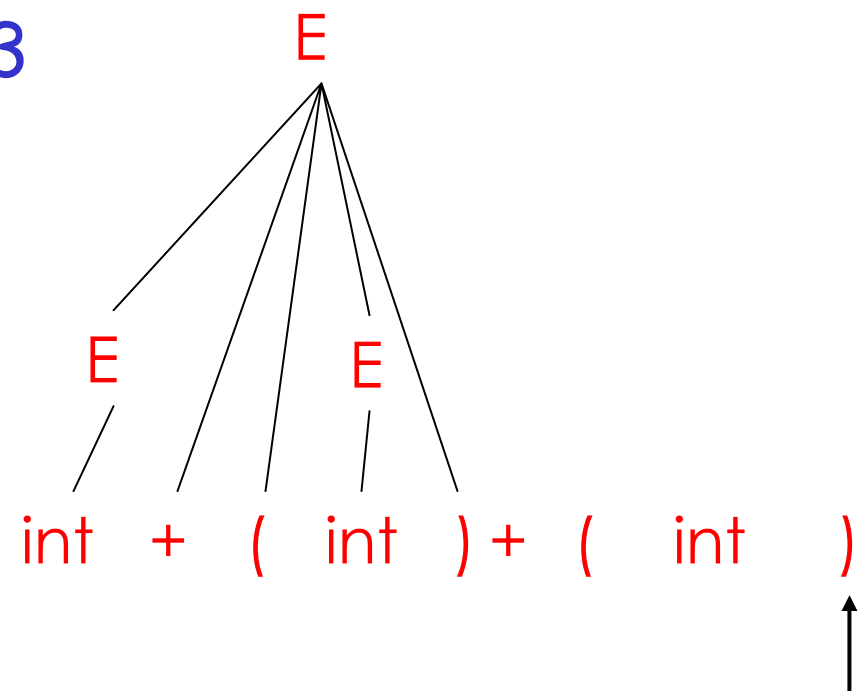
E | + (int) + (int)$    shift 3 times

E + (int | ) + (int)$    red. E → int

E + (E | ) + (int)$    shift

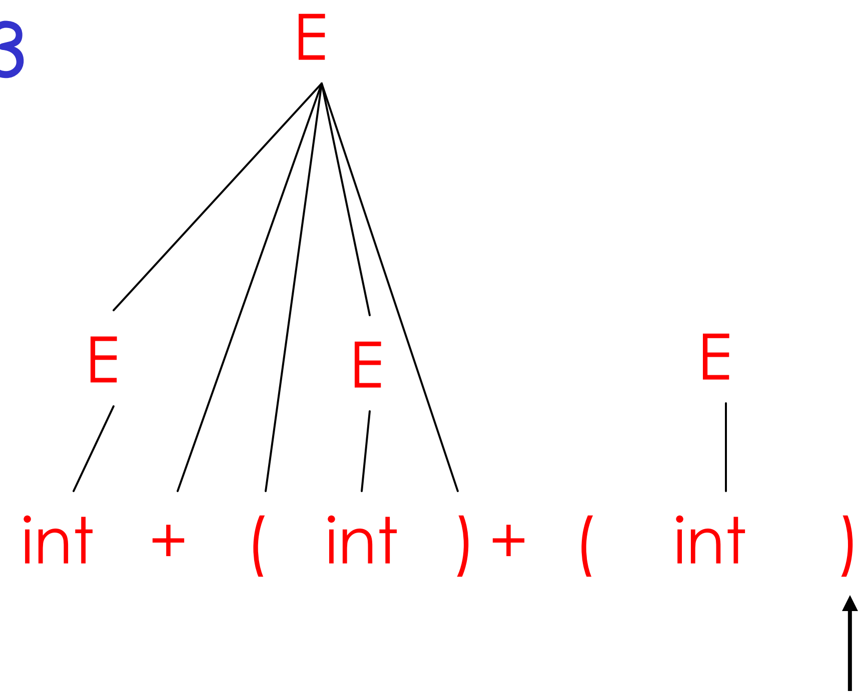
E + (E) | + (int)$    red. E → E + (E)

E | + (int)$    shift 3



# Shift-Reduce Example

| | |
|---|---|
| \| int + (int) + (int)\$ | shift |
| int \| + (int) + (int)\$ | red. E → int |
| E \| + (int) + (int)\$ | shift 3 times |
| E + (int \| ) + (int)\$ | red. E → int |
| E + (E \| ) + (int)\$ | shift |
| E + (E) \| + (int)\$ | red. E → E + (E) |
| E \| + (int)\$ | shift 3 |



# Shift-Reduce Example

| int + (int) + (int)$ shift

int | + (int) + (int)$ red. E → int

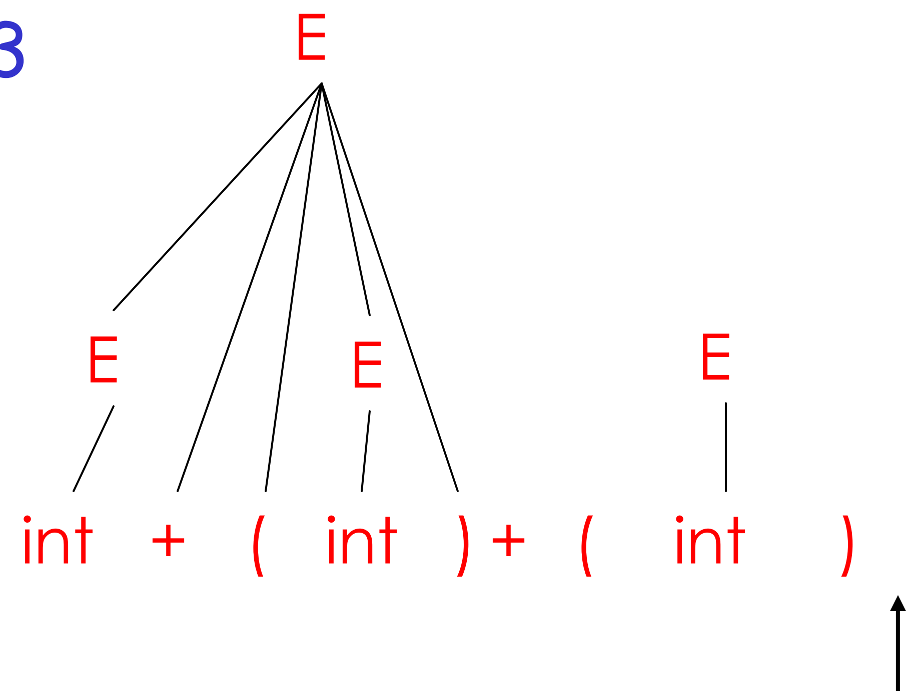
E | + (int) + (int)$ shift 3 times

E + (int | ) + (int)$ red. E → int

E + (E | ) + (int)$ shift

E + (E) | + (int)$ red. E → E + (E)

E | + (int)$ shift 3



# Shift-Reduce Example

| int + (int) + (int)$ shift

int | + (int) + (int)$ red. E → int

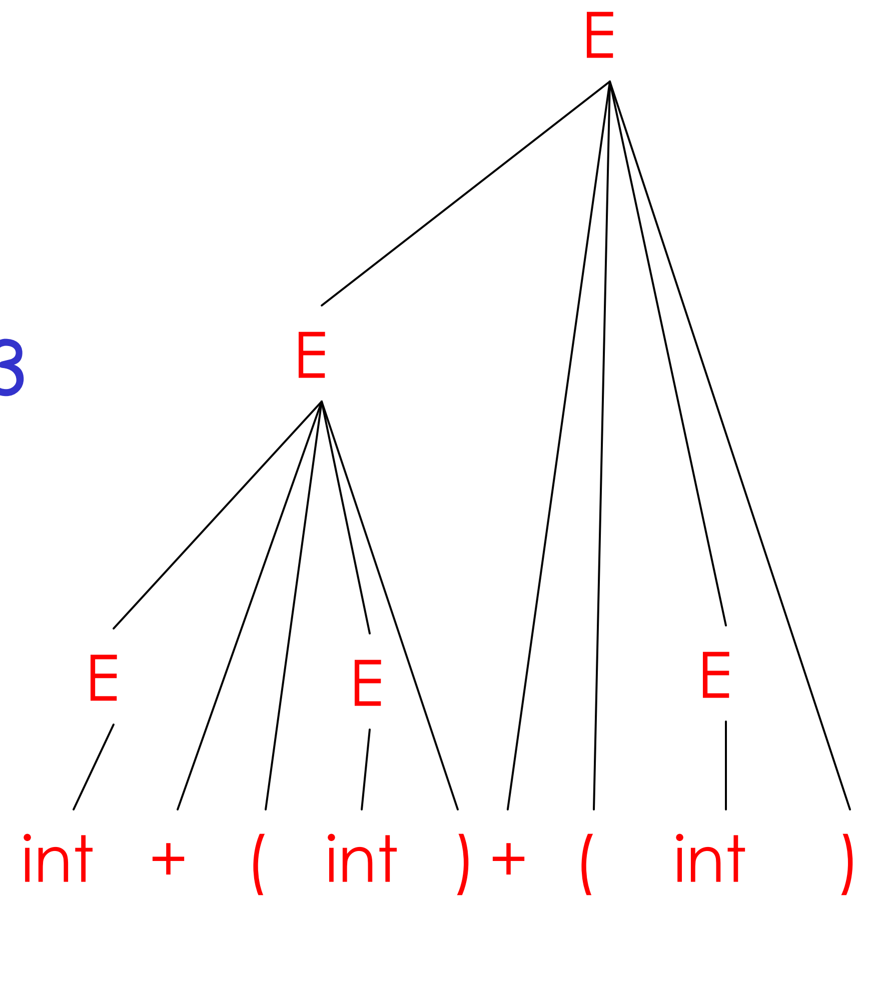
E | + (int) + (int)$ shift 3 times

E + (int | ) + (int)$ red. E → int

E + (E | ) + (int)$ shift

E + (E) | + (int)$ red. E → E + (E)

E | + (int)$ shift 3



# The Stack

- Left string can be implemented by a stack
  - Top of the stack is the |

- Shift pushes a terminal on the stack

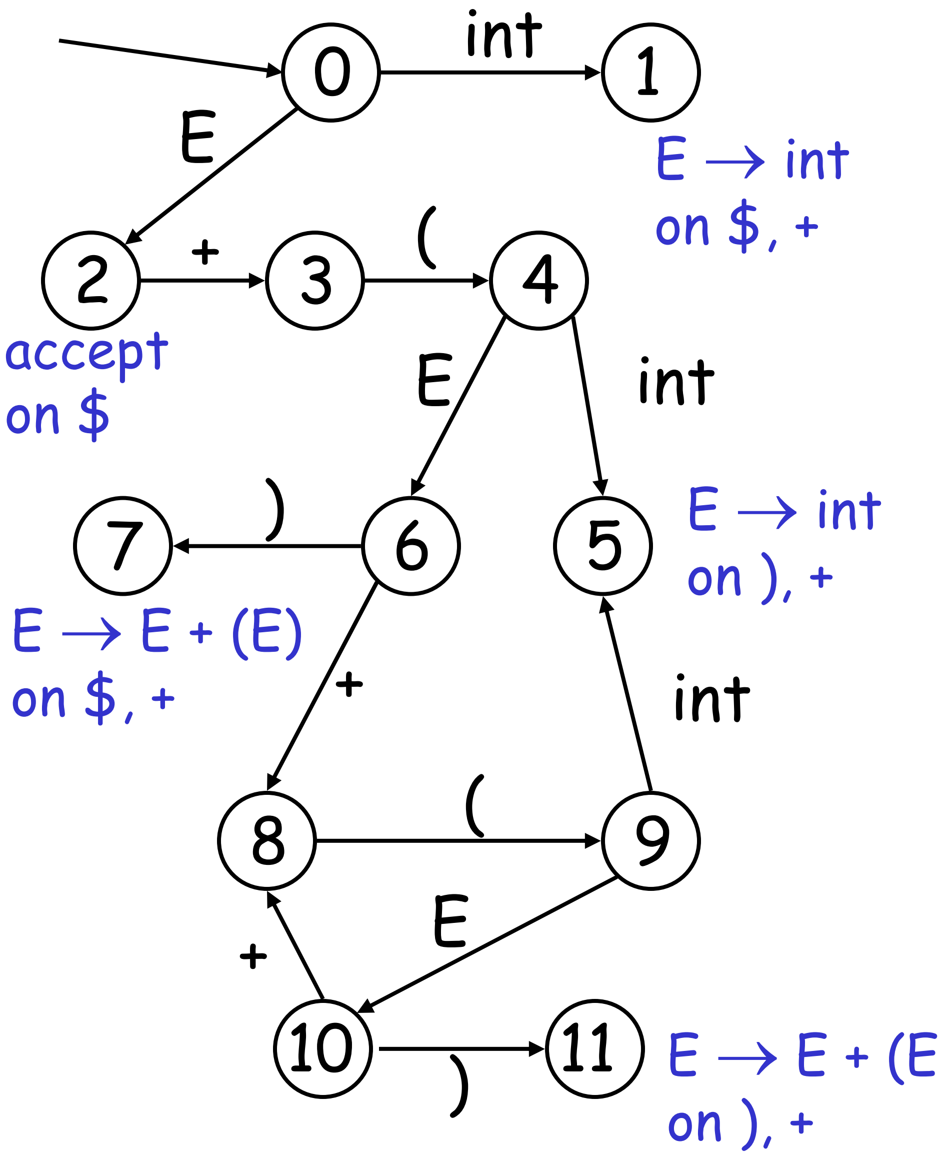
- Reduce pops 0 or more symbols off of the stack (production rhs) and pushes a non-terminal on the stack (production lhs)

# Key Issue: When to Shift or Reduce?

- Decide based on the left string (the stack)
- Idea: use a finite automaton (DFA) to decide when to shift or reduce
  - The DFA input is the stack
  - The language consists of terminals and non-terminals

- We run the DFA on the stack and we examine the resulting state X and the token tok after |
  - If X has a transition labeled tok then shift
  - If X is labeled with "$A \to \beta$ on tok" then reduce

# LR(1) Parsing. An Example



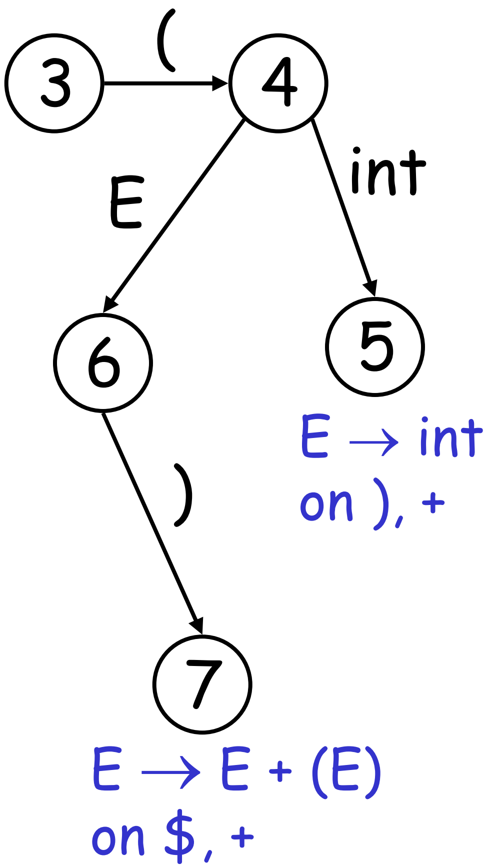

| | |
|---|---|
| I int + (int) + (int)$ | shift |
| int I + (int) + (int)$ | E → int |
| E I + (int) + (int)$ | shift(x3) |
| E + (int I ) + (int)$ | E → int |
| E + (E I ) + (int)$ | shift |
| E + (E) I + (int)$ | E → E+(E) |
| E I + (int)$ | shift (x3) |

# Representing the DFA

- Parsers represent the DFA as a 2D table
  - Recall table-driven lexical analysis
- Lines correspond to DFA states
- Columns correspond to terminals and non-terminals
- Typically columns are split into:
  - Those for terminals: action table
  - Those for non-terminals: goto table

# Representing the DFA. Example

- The table for a fragment of our DFA:




| | int | + | ( | ) | $ | E |
|---|---|---|---|---|---|---|
| … | | | | | | |
| 3 | | | s4 | | | |
| 4 | s5 | | | | | g6 |
| 5 | | $r_{E \to int}$ | | $r_{E \to int}$ | | |
| 6 | s8 | | s7 | | | |
| 7 | | $r_{E \to E+(E)}$ | | | $r_{E \to E+(E)}$ | |
| … | | | | | | |

# The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
  - This is wasteful, since most of the work is repeated

- Remember for each stack element on which state it brings the DFA

- LR parser maintains a stack

$$\langle\, sym_1, state_1 \,\rangle \; . \; . \; . \; \langle\, sym_n, state_n \,\rangle$$

$state_k$ is the final state of the DFA on $sym_1 \ldots sym_k$

# The LR Parsing Algorithm

Let I = w$ be initial input
Let j = 0
Let DFA state 0 be the start state
Let stack = $\langle$ dummy, 0 $\rangle$
    repeat
        case action[top_state(stack), I[j]] of
            shift k:  push $\langle$ I[j++], k $\rangle$
            reduce X $\rightarrow \alpha$:
                pop $|\alpha|$ pairs,
                push $\langle$X, Goto[top_state(stack), X]$\rangle$
            accept: halt normally
            error: halt and report error

# LR Parsing Notes

- Can be used to parse more grammars than LL
- Most programming languages grammars are LR
- Can be described as a simple table
- There are tools for building the table
- How is the table constructed?

# Key Issue: How is the DFA Constructed?

- The stack describes the context of the parse
  - What non-terminal we are looking for
  - What production rhs we are looking for
  - What we have seen so far from the rhs

- Each DFA state describes several such contexts
  - E.g., when we are looking for non-terminal E, we might be looking either for an int or a E + (E) rhs

# LR(1) Items

- An <u>LR(1) item</u> is a pair:

$$X \to \alpha.\beta, a$$

  - $X \to \alpha.\beta$ is a production
  - $a$ is a terminal (the lookahead terminal)
  - LR(1) means 1 lookahead terminal

- $[X \to \alpha.\beta, a]$ describes a context of the parser
  - We are trying to find an $X$ followed by an $a$, and
  - We have $\alpha$ already on top of the stack
  - Thus we need to see next a prefix derived from $\beta a$

# Note

- The symbol I was used before to separate the stack from the rest of input
  - $\alpha$ I $\gamma$, where $\alpha$ is the stack and $\gamma$ is the remaining string of terminals
- In items . is used to mark a prefix of a production rhs:

$$X \to \alpha.\beta, a$$

  - Here $\beta$ might contain non-terminals as well
- In both case the stack is on the left

# Convention

- We add to our grammar a fresh new start symbol $S$ and a production $S \to E$
  - Where $E$ is the old start symbol
- The initial parsing context contains:

$$S \to .E, \$$$

  - Trying to find an $S$ as a string derived from $E\$$
  - The stack is empty

# LR(1) Items (Cont.)

- In context containing

  $$E \rightarrow E + .\,(\,E\,),\ +$$

  - If ( follows then we can perform a shift to context containing

    $$E \rightarrow E + (.\,E\,),\ +$$

- In context containing

  $$E \rightarrow E + (\,E\,)\,.,\ +$$

  - We can perform a reduction with $E \rightarrow E + (\,E\,)$
  - But only if a + follows

# LR(1) Items (Cont.)

- Consider the item

$$E \rightarrow E + (.\ E\ ),\ +$$

- We expect a string derived from $E\ )\ +$
- There are two productions for E

$$E \rightarrow \text{int} \quad \text{and} \quad E \rightarrow E + (\ E)$$

- We describe this by extending the context with two more items:

$$E \rightarrow .\text{int},\ )$$

$$E \rightarrow .E + (\ E\ ),\ )$$

# The Closure Operation

- The operation of extending the context with items is called the closure operation

Closure(Items) =
  repeat
    for each $[X \to \alpha.Y\beta, a]$ in Items
      for each production $Y \to \gamma$
        for each $b \in First(\beta a)$
          add $[Y \to .\gamma, b]$ to Items
  until Items is unchanged

# Constructing the Parsing DFA (1)

- Construct the start context: Closure({$S \rightarrow .E, \$$})

$$
\begin{aligned}
S &\rightarrow .E, \$ \\
E &\rightarrow .E+(E), \$ \\
E &\rightarrow .int, \$ \\
E &\rightarrow .E+(E), + \\
E &\rightarrow .int, +
\end{aligned}
$$

- We abbreviate as:

$$
\begin{aligned}
S &\rightarrow .E, \$ \\
E &\rightarrow .E+(E), \$/+ \\
E &\rightarrow .int, \$/+
\end{aligned}
$$

# Constructing the Parsing DFA (2)

- A DFA state is a **closed** set of LR(1) items
- The start state contains $[S \to .E, \$]$
- A state that contains $[X \to \alpha., b]$ is labeled with "reduce with $X \to \alpha$ on b"
- And now the transitions …

# The DFA Transitions

- A state "State" that contains $[X \to \alpha.y\beta, b]$ has a transition labeled $y$ to a state that contains the items "Transition(State, y)"
  - y can be a terminal or a non-terminal

Transition(State, y)

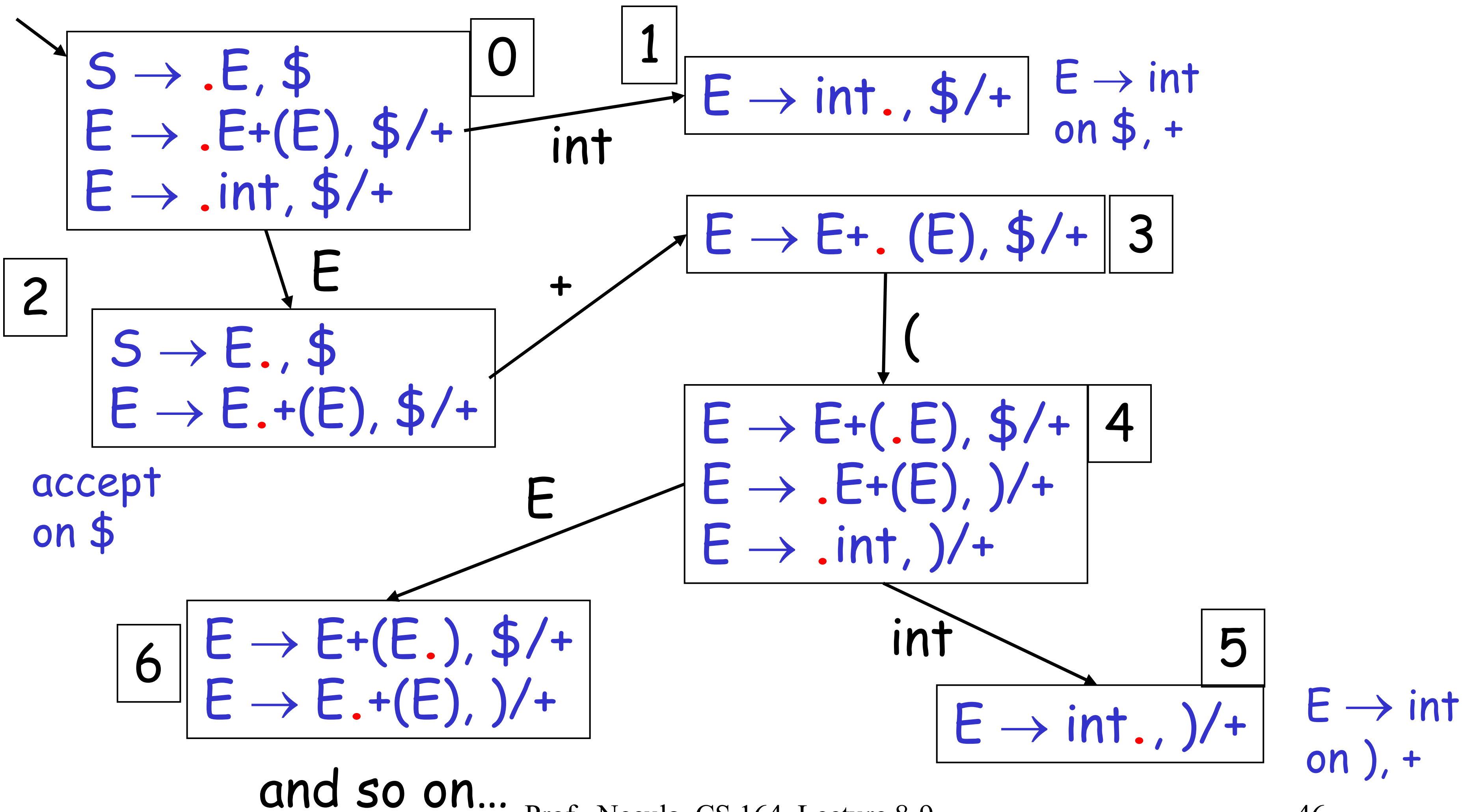
  Items Ã ∅

  for each $[X \to \alpha.y\beta, b]$ 2 State

    add $[X \to \alpha y.\beta, b]$ to Items

  return Closure(Items)

# Constructing the Parsing DFA. Example.



# LR Parsing Tables. Notes

- Parsing tables (i.e. the DFA) can be constructed automatically for a CFG
- But we still need to understand the construction to work with parser generators
  - E.g., they report errors in terms of sets of items
- What kind of errors can we expect?

# Shift/Reduce Conflicts

- If a DFA state contains both

  $[X \to \alpha.a\beta,\ b]$ and $[Y \to \gamma.,\ a]$

- Then on input "a" we could either
  - Shift into state $[X \to \alpha a.\beta,\ b]$, or
  - Reduce with $Y \to \gamma$

- This is called a <u>shift-reduce conflict</u>

# Shift/Reduce Conflicts

- Typically due to ambiguities in the grammar
- Classic example: the dangling else

  $S \rightarrow$ if E then S | if E then S else S | OTHER

- Will have DFA state containing

  [$S \rightarrow$ if E then S., else]

  [$S \rightarrow$ if E then S. else S, x]

- If else follows then we can shift or reduce
- Default (bison, CUP, etc.) is to shift
  - Default behavior is as needed in this case

# More Shift/Reduce Conflicts

- Consider the ambiguous grammar

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

- We will have the states containing

$$[E \rightarrow E * . E, \quad +] \qquad\qquad [E \rightarrow E * E., \quad +]$$
$$[E \rightarrow . E + E, \quad +] \quad \Rightarrow^{E} \quad [E \rightarrow E . + E, \quad +]$$
$$\ldots \qquad\qquad\qquad\qquad \ldots$$

- Again we have a shift/reduce on input +
  - We need to reduce (* binds more tightly than +)
  - Recall solution: declare the precedence of *

# More Shift/Reduce Conflicts

- In bison declare precedence and associativity:

```
%left +
%left *
```

- Precedence of a rule = that of its last terminal
  - See bison manual for ways to override this default

- Resolve shift/reduce conflict with a <u>shift</u> if:
  - no precedence declared for either rule or terminal
  - input terminal has higher precedence than the rule
  - the precedences are the same and right associative

# Using Precedence to Solve S/R Conflicts

- Back to our example:

$$[E \to E * .\ E, +] \qquad [E \to E * E., +]$$
$$[E \to .\ E + E, +] \Rightarrow^{E} [E \to E\ .+ E, +]$$
$$\ldots \qquad \ldots$$

- Will choose reduce because precedence of rule $E \to E * E$ is higher than of terminal $+$

# Using Precedence to Solve S/R Conflicts

- Same grammar as before

$$E \to E + E \mid E * E \mid \text{int}$$

- We will also have the states

$$[E \to E + .\ E, +] \qquad\qquad [E \to E + E., +]$$

$$[E \to .\ E + E, +] \quad \Rightarrow^{E} \quad [E \to E\ .\ + E, +]$$

$$\dots \qquad\qquad \dots$$

- Now we also have a shift/reduce on input +
  - We choose reduce because $E \to E + E$ and + have the same precedence and + is left-

# Using Precedence to Solve S/R Conflicts

- Back to our dangling else example

  $[S \rightarrow \text{if } E \text{ then } S.,\quad \text{else}]$

  $[S \rightarrow \text{if } E \text{ then } S. \text{ else } S,\quad x]$

- Can eliminate conflict by declaring else with higher precedence than then
  - Or just rely on the default shift action
- But this starts to look like "hacking the parser"
- Best to avoid overuse of precedence declarations or you'll end with unexpected parse trees

# Reduce/Reduce Conflicts

- If a DFA state contains both

  $[X \to \alpha., a]$ and $[Y \to \beta., a]$

  - Then on input "a" we don't know which production to reduce

- This is called a reduce/reduce conflict

# Reduce/Reduce Conflicts

- Usually due to gross ambiguity in the grammar
- Example: a sequence of identifiers

$$S \to \varepsilon \mid id \mid id\ S$$

- There are two parse trees for the string id

$$S \to id$$
$$S \to id\ S \to id$$

- How does this confuse the parser?

# More on Reduce/Reduce Conflicts

- Consider the states $[S \to id\,.,\ \$]$
  $[S' \to .\,S,\ \$]$ $[S \to id\,.\,S,\ \$]$
  $[S \to .,\ \$]$ $\Rightarrow^{id}$ $[S \to .,\ \$]$
  $[S \to .\,id,\ \$]$ $[S \to .\,id,\ \$]$
  $[S \to .\,id\,S,\ \$]$ $[S \to .\,id\,S,\ \$]$



- Reduce/reduce conflict on input $\$$

# Using Parser Generators

- Parser generators construct the parsing DFA given a CFG
  - Use precedence declarations and default conventions to resolve conflicts
  - The parser algorithm is the same for all grammars (and is provided as a library function)
- But most parser generators do not construct the DFA as described before
  - Because the LR(1) parsing DFA has 1000s of states even for a simple language

# LR(1) Parsing Tables are Big

- But many states are similar, e.g.

1: $E \rightarrow \text{int}\,\textcolor{red}{.}\,,\ \$/+$ — $E \rightarrow$ int on \$, +

and

5: $E \rightarrow \text{int}\,\textcolor{red}{.}\,,\ )/+$ — $E \rightarrow$ int on ), +

- Idea: merge the DFA states whose items differ only in the lookahead tokens
  - We say that such states have the same core
- We obtain

1': $E \rightarrow \text{int}\,\textcolor{red}{.}\,,\ \$/+/)$ — $E \rightarrow$ int on \$, +, )

# The Core of a Set of LR Items

- Definition: The <u>core</u> of a set of LR items is the set of first components
  - Without the lookahead terminals

- Example: the core of

$$\{ [X \to \alpha.\beta, b], [Y \to \gamma.\delta, d]\}$$

is

$$\{X \to \alpha.\beta,\ Y \to \gamma.\delta\}$$

# LALR States

- Consider for example the LR(1) states

$$\{[X \to \alpha., a], [Y \to \beta., c]\}$$
$$\{[X \to \alpha., b], [Y \to \beta., d]\}$$

- They have the same core and can be merged
- And the merged state contains:

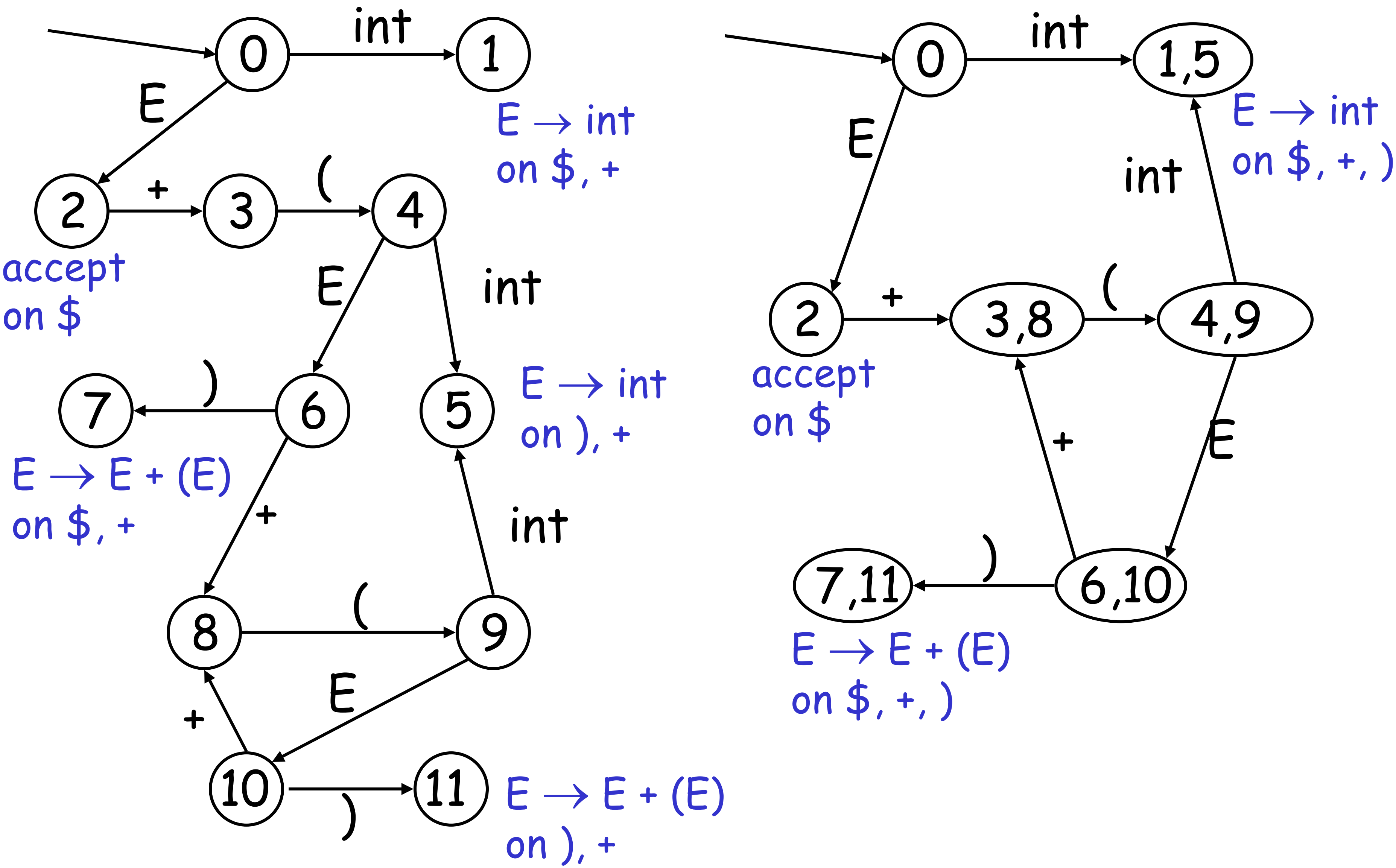
$$\{[X \to \alpha., a/b], [Y \to \beta., c/d]\}$$

- These are called LALR(1) states
  - Stands for LookAhead LR
  - Typically 10 times fewer LALR(1) states than LR(1)

# A LALR(1) DFA

- Repeat until all states have distinct core
  - Choose two distinct states with same core
  - Merge the states by creating a new one with the union of all the items
  - Point edges from predecessors to new state
  - New state points to all the previous successors

# Conversion LR(1) to LALR(1). Example.



# The LALR Parser Can Have Conflicts

- Consider for example the LR(1) states

  $\{[X \to \alpha., a], [Y \to \beta., b]\}$

  $\{[X \to \alpha., b], [Y \to \beta., a]\}$

- And the merged LALR(1) state

  $\{[X \to \alpha., a/b], [Y \to \beta., a/b]\}$

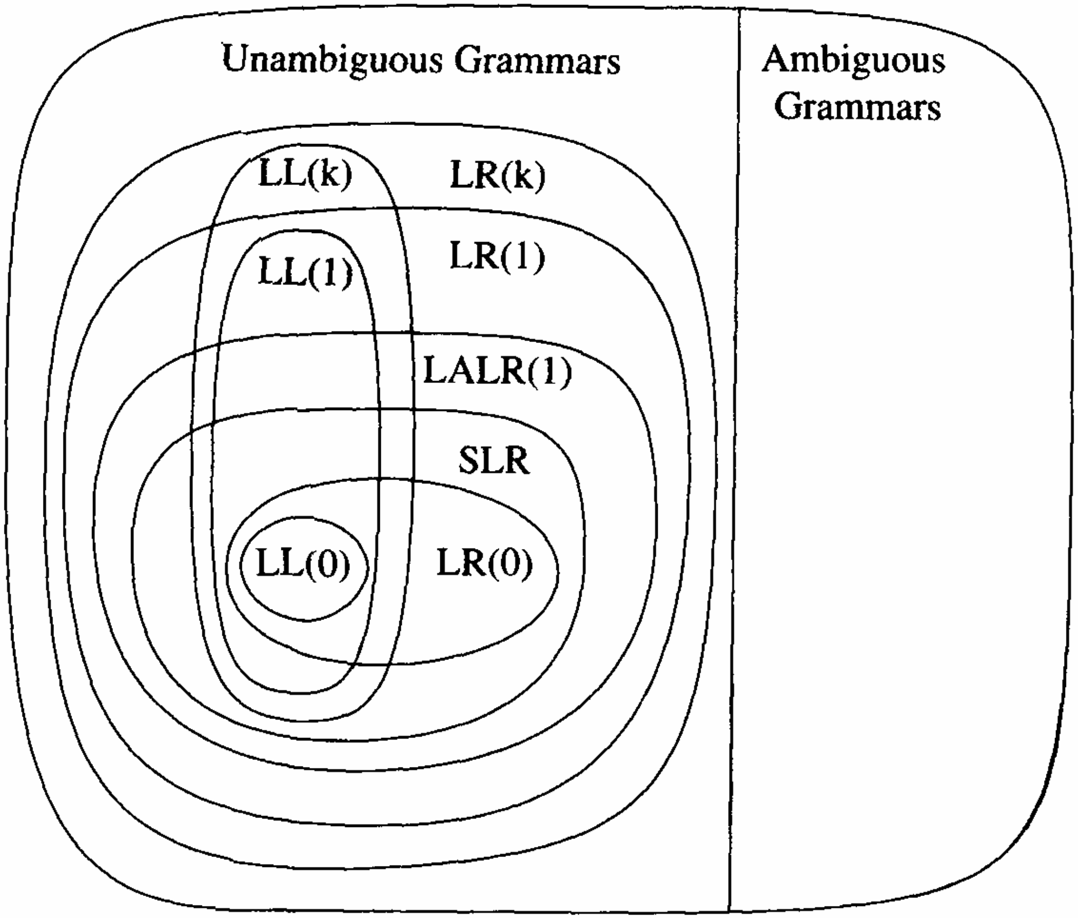
- Has a new reduce-reduce conflict

- In practice such cases are rare

# LALR vs. LR Parsing

- LALR languages are not natural
  - They are an efficiency hack on LR languages
- Any reasonable programming language has a LALR(1) grammar
- LALR(1) has become a standard for programming languages and for parser generators

# A Hierarchy of Grammar Classes




From Andrew Appel, "Modern Compiler Implementation in Java"

# Notes on Parsing

- Parsing
  - A solid foundation: context-free grammars
  - A simple parser: LL(1)
  - A more powerful parser: LR(1)
  - An efficiency hack: LALR(1)
  - LALR(1) parser generators

- Now we move on to semantic analysis

# Supplement to LR Parsing

Strange Reduce/Reduce Conflicts
Due to LALR Conversion
(from the bison manual)

# Strange Reduce/Reduce Conflicts

- Consider the grammar

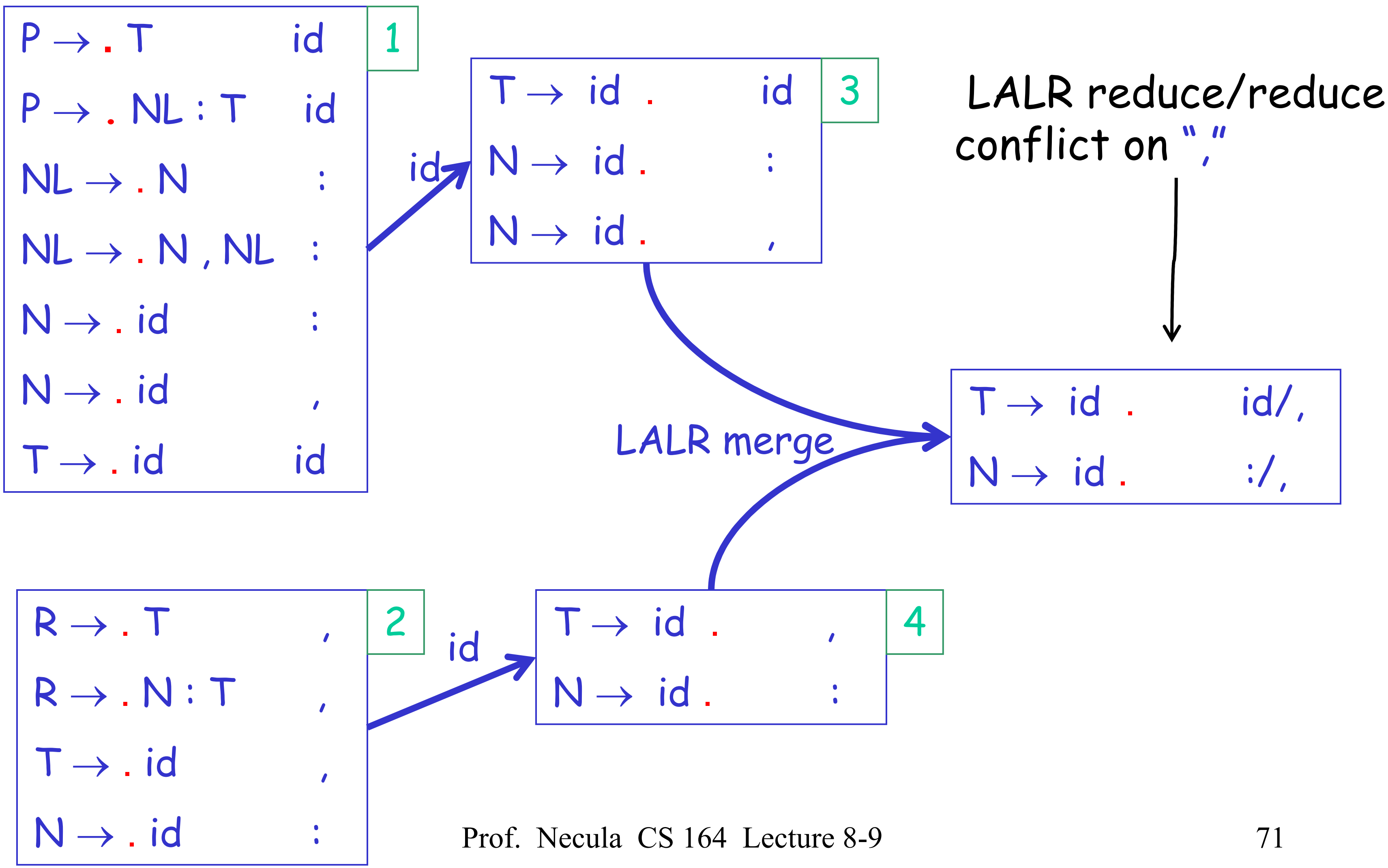
$$S \to P\ R\ ,$$
$$NL \to N \mid N\ ,\ NL$$
$$P \to T \mid NL : T$$
$$R \to T \mid N : T$$
$$N \to \text{id}$$
$$T \to \text{id}$$

- P - parameters specification
- R - result specification
- N - a parameter or result name
- T - a type name
- NL - a list of names

# Strange Reduce/Reduce Conflicts

- In P an id is a
  - N when followed by , or :
  - T when followed by id
- In R an id is a
  - N when followed by :
  - T when followed by ,
- This is an LR(1) grammar.
- But it is not LALR(1). Why?
  - For obscure reasons

# A Few LR(1) States



# What Happened?

- Two distinct states were confused because they have the same core
- Fix: add dummy productions to distinguish the two confused states
- E.g., add

$$R \to \text{id bogus}$$

  - bogus is a terminal not used by the lexer
  - This production will never be used during parsing
  - But it distinguishes R from P

# A Few LR(1) States After Fix

| | | 1 |
|---|---|---|
| $P \to .\ T$ | id | |
| $P \to .\ NL : T$ | id | |
| $NL \to .\ N$ | : | |
| $NL \to .\ N , NL$ | : | |
| $N \to .\ id$ | : | |
| $N \to .\ id$ | , | |
| $T \to .\ id$ | id | |

id $\to$

| | | 3 |
|---|---|---|
| $T \to id\ .$ | id | |
| $N \to id\ .$ | : | |
| $N \to id\ .$ | , | |

Different cores $\Rightarrow$ no LALR merging

| | | 2 |
|---|---|---|
| $R \to .\ T$ | , | |
| $R \to .\ N : T$ | , | |
| $R \to .\ id\ bogus$ | , | |
| $T \to .\ id$ | , | |
| $N \to .\ id$ | : | |

id $\to$

| | | 4 |
|---|---|---|
| $T \to id\ .$ | , | |
| $N \to id\ .$ | : | |
| $R \to id\ .\ bogus$ | , | |