Chapter 1 Answers

1.1. Converting from polar to Cartesian coordinates:

$\frac{1}{2}e^{j\pi} = \frac{1}{2}\cos\pi = -\frac{1}{2}$, $\quad \frac{1}{2}e^{-j\pi} = \frac{1}{2}\cos(-\pi) = -\frac{1}{2}$

$e^{j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = j$, $\quad e^{-j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$

$e^{j5\frac{\pi}{2}} = e^{j\frac{\pi}{2}} = j$, $\quad \sqrt{2}e^{j\frac{\pi}{4}} = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right) = 1 + j$

$\sqrt{2}e^{\frac{9j\pi}{4}} = \sqrt{2}e^{\frac{j\pi}{4}} = 1 + j$, $\quad \sqrt{2}e^{\frac{-9j\pi}{4}} = \sqrt{2}e^{\frac{-j\pi}{4}} = 1 - j$

$\sqrt{2}e^{\frac{-j\pi}{4}} = 1 - j$

1.2. Converting from Cartesian to polar coordinates:

$5 = 5e^{j0}$, $\quad -2 = 2e^{j\pi}$, $\quad -3j = 3e^{-j\frac{\pi}{2}}$

$\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j\frac{\pi}{3}}$, $\quad 1 + j = \sqrt{2}e^{j\frac{\pi}{4}}$, $\quad (1-j)^2 = 2e^{-j\frac{\pi}{2}}$

$j(1-j) = e^{j\frac{\pi}{4}}$, $\quad \frac{1+j}{1-j} = e^{j\frac{\pi}{2}}$, $\quad \frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = e^{-j\frac{\pi}{12}}$

1.3. **(a)** $E_\infty = \int_0^\infty e^{-4t}dt = \frac{1}{4}$, $P_\infty = 0$, because $E_\infty < \infty$

(b) $x_2(t) = e^{j(2t+\frac{\pi}{4})}$, $|x_2(t)| = 1$. Therefore, $E_\infty = \int_{-\infty}^{\infty}|x_2(t)|^2dt = \int_{-\infty}^{\infty}dt = \infty$, $P_\infty =$

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x_2(t)|^2dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt = \lim_{T\to\infty}1 = 1$$

(c) $x_3(t) = \cos(t)$. Therefore, $E_\infty = \int_{-\infty}^{\infty}|x_3(t)|^2dt = \int_{-\infty}^{\infty}\cos^2(t)dt = \infty$,

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos^2(t)dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left(\frac{1+\cos(2t)}{2}\right)dt = \frac{1}{2}$$

(d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$, $|x_1[n]|^2 = \left(\frac{1}{4}\right)^n u[n]$. Therefore, $E_\infty = \sum_{n=-\infty}^{\infty}|x_1[n]|^2 = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{4}{3}$,

$P_\infty = 0$, because $E_\infty < \infty$.

(e) $x_2[n] = e^{j(\frac{\pi n}{2}+\frac{\pi}{8})}$, $|x_2[n]|^2 = 1$. Therefore, $E_\infty = \sum_{n=-\infty}^{\infty}|x_2[n]|^2 = \infty$,

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x_2[n]|^2 = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}1 = 1.$$

(f) $x_3[n] = \cos(\frac{\pi}{4}n)$. Therefore, $E_\infty = \sum_{n=-\infty}^{\infty}|x_3[n]|^2 = \sum_{n=-\infty}^{\infty}\cos^2(\frac{\pi}{4}n) = \infty$,

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\cos^2(\frac{\pi}{4}n) = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\left(\frac{1+\cos(\frac{\pi}{2}n)}{2}\right) = \frac{1}{2}$$

1.4. **(a)** The signal $x[n]$ is shifted by 3 to the right. The shifted signal will be zero for $n < 1$ and $n > 7$.

(b) The signal $x[n]$ is shifted by 4 to the left. The shifted signal will be zero for $n < -6$ and $n > 0$.

(c) The signal $x[n]$ is flipped. The flipped signal will be zero for $n < -4$ and $n > 2$.

(d) The signal $x[n]$ is flipped and the flipped signal is shifted by 2 to the right. This new signal will be zero for $n < -2$ and $n > 4$.

(e) The signal $x[n]$ is flipped and the flipped signal is shifted by 2 to the left. This new signal will be zero for $n < -6$ and $n > 0$.

1.5. (a) $x(1-t)$ is obtained by flipping $x(t)$ and shifting the flipped signal by 1 to the right. Therefore, $x(1-t)$ will be zero for $t > -2$.

(b) From (a), we know that $x(1-t)$ is zero for $t > -2$. Similarly, $x(2-t)$ is zero for $t > -1$. Therefore, $x(1-t) + x(2-t)$ will be zero for $t > -2$.

(c) $x(3t)$ is obtained by linearly compressing $x(t)$ by a factor of 3. Therefore, $x(3t)$ will be zero for $t < 1$.

(d) $x(t/3)$ is obtained by linearly stretching $x(t)$ by a factor of 3. Therefore, $x(t/3)$ will be zero for $t < 9$.

1.6. (a) $x_1(t)$ is not periodic because it is zero for $t < 0$.

(b) $x_2[n] = 1$ for all n. Therefore, it is periodic with a fundamental period of 1.

(c) $x_3[n]$ is as shown in the Figure S1.6.

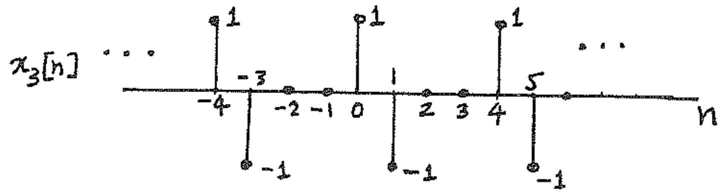

Figure S1.6

Therefore, it is periodic with a fundamental period of 4.

1.7. (a)

$$\mathcal{E}v\{x_1[n]\} = \frac{1}{2}(x_1[n] + x_1[-n]) = \frac{1}{2}(u[n] - u[n-4] + u[-n] - u[-n-4])$$

Therefore, $\mathcal{E}v\{x_1[n]\}$ is zero for $|n| > 3$.

(b) Since $x_2(t)$ is an odd signal, $\mathcal{E}v\{x_2(t)\}$ is zero for all values of t.

(c)

$$\mathcal{E}v\{x_3[n]\} = \frac{1}{2}(x_1[n] + x_1[-n]) = \frac{1}{2}[(\frac{1}{2})^n u[n-3] - (\frac{1}{2})^{-n} u[-n-3]]$$

Therefore, $\mathcal{E}v\{x_3[n]\}$ is zero when $|n| < 3$ and when $|n| \to \infty$.

(d)

$$\mathcal{E}v\{x_4(t)\} = \frac{1}{2}(x_4(t) + x_4(-t)) = \frac{1}{2}[e^{-5t}u(t+2) - e^{5t}u(-t+2)$$

Therefore, $\mathcal{E}v\{x_4(t)\}$ is zero only when $|t| \to \infty$.

1.8. (a) $\mathcal{R}e\{x_1(t)\} = -2 = 2e^{0t}\cos(0t+\pi)$

(b) $\mathcal{R}e\{x_2(t)\} = \sqrt{2}\cos(\frac{\pi}{4})\cos(3t+2\pi) = \cos(3t) = e^{0t}\cos(3t+0)$

(c) $\mathcal{R}e\{x_3(t)\} = e^{-t}\sin(3t+\pi) = e^{-t}\cos(3t+\frac{\pi}{2})$

(d) $\mathcal{R}e\{x_4(t)\} = -e^{-2t}\sin(100t) = e^{-2t}\sin(100t+\pi) = e^{-2t}\cos(100t+\frac{\pi}{2})$

1.9. (a) $x_1(t)$ is a periodic complex exponential.

$$x_1(t) = je^{j10t} = e^{j(10t+\frac{\pi}{2})}$$

The fundamental period of $x_1(t)$ is $\frac{2\pi}{10} = \frac{\pi}{5}$.

(b) $x_2(t)$ is a complex exponential multiplied by a decaying exponential. Therefore, $x_2(t)$ is not periodic.

(c) $x_3[n]$ is a periodic signal.

$$x_3[n] = e^{j7\pi n} = e^{j\pi n}$$

$x_3[n]$ is a complex exponential with a fundamental period of $\frac{2\pi}{\pi} = 2$.

(d) $x_4[n]$ is a periodic signal. The fundamental period is given by $N = m(\frac{2\pi}{3\pi/5}) = m(\frac{10}{3})$. By choosing $m = 3$, we obtain the fundamental period to be 10.

(e) $x_5[n]$ is not periodic. $x_5[n]$ is a complex exponential with $\omega_0 = 3/5$. We cannot find any integer m such that $m(\frac{2\pi}{\omega_0})$ is also an integer. Therefore, $x_5[n]$ is not periodic.

1.10.

$$x(t) = 2\cos(10t+1) - \sin(4t-1)$$

Period of first term in RHS $= \frac{2\pi}{10} = \frac{\pi}{5}$
Period of second term in RHS $= \frac{2\pi}{4} = \frac{\pi}{2}$
Therefore, the overall signal is periodic with a period which is the least common multiple of the periods of the first and second terms. This is equal to π.

1.11.

$$x[n] = 1 + e^{j\frac{4\pi}{7}n} - e^{j\frac{2\pi}{5}n}$$

Period of the first term in the RHS = 1
Period of the second term in the RHS $= m(\frac{2\pi}{4\pi/7}) = 7$ (when $m = 2$)
Period of the third term in the RHS $= m(\frac{2\pi}{2\pi/5}) = 5$ (when $m = 1$)
Therefore, the overall signal $x[n]$ is periodic with a period which is the least common multiple of the periods of the three terms in $x[n]$. This is equal to 35.

1.12. The signal $x[n]$ is as shown in Figure S1.12. $x[n]$ can be obtained by flipping $u[n]$ and then shifting the flipped signal by 3 to the right. Therefore, $x[n] = u[-n+3]$. This implies that $M = -1$ and $n_0 = -3$.

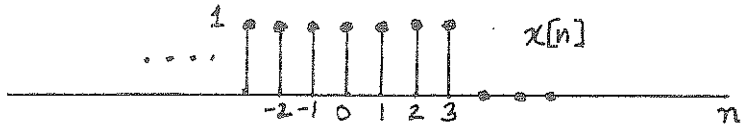

Figure S1.12

1.13.

$$y(t) = \int_{-\infty}^{t} x(\tau)dt = \int_{-\infty}^{t} (\delta(\tau+2) - \delta(\tau-2))dt = \begin{cases} 0, & t < -2 \\ 1, & -2 \le t \le 2 \\ 0, & t > 2 \end{cases}$$

Therefore,

$$E_\infty = \int_{-2}^{2} dt = 4$$

1.14. The signal $x(t)$ and its derivative $g(t)$ are shown in Figure S1.14.

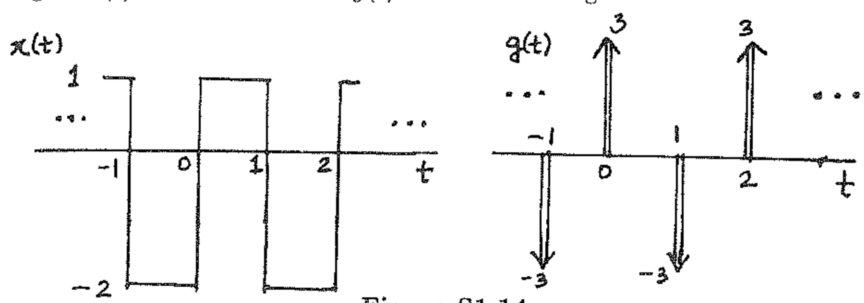

Figure S1.14

Therefore,

$$g(t) = 3\sum_{k=-\infty}^{\infty} \delta(t-2k) - 3\sum_{k=-\infty}^{\infty} \delta(t-2k-1)$$

This implies that $A_1 = 3$, $t_1 = 0$, $A_2 = -3$, and $t_2 = 1$.

1.15. (a) The signal $x_2[n]$, which is the input to S_2, is the same as $y_1[n]$. Therefore,

$$\begin{aligned} y_2[n] &= x_2[n-2] + \frac{1}{2}x_2[n-3] \\ &= y_1[n-2] + \frac{1}{2}y_1[n-3] \\ &= 2x_1[n-2] + 4x_1[n-3] + \frac{1}{2}(2x_1[n-3] + 4x_1[n-4]) \\ &= 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4] \end{aligned}$$

The input-output relationship for S is

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

(b) The input-output relationship does not change if the order in which S_1 and S_2 are connected in series is reversed. We can easily prove this by assuming that S_1 follows S_2. In this case, the signal $x_1[n]$, which is the input to S_1, is the same as $y_2[n]$. Therefore,

$$\begin{aligned} y_1[n] &= 2x_1[n] + 4x_1[n-1] \\ &= 2y_2[n] + 4y_2[n-1] \\ &= 2(x_2[n-2] + \frac{1}{2}x_2[n-3]) + 4(x_2[n-3] + \frac{1}{2}x_2[n-4]) \\ &= 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4] \end{aligned}$$

The input-output relationship for S is once again

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

1.16. (a) The system is not memoryless because $y[n]$ depends on past values of $x[n]$.

(b) The output of the system will be $y[n] = \delta[n]\delta[n-2] = 0$.

(c) From the result of part (b), we may conclude that the system output is always zero for inputs of the form $\delta[n-k]$, $k \in \mathcal{I}$. Therefore, the system is not invertible.

1.17. (a) The system is not causal because the output $y(t)$ at some time may depend on future values of $x(t)$. For instance, $y(-\pi) = x(0)$.

(b) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = x_1(\sin(t))$$

$$x_2(t) \longrightarrow y_2(t) = x_2(\sin(t))$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= x_3(\sin(t)) \\ &= ax_1(\sin(t)) + bx_2(\sin(t)) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Therefore, the system is linear.

1.18. (a) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \longrightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2[n] \longrightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is,

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= \sum_{k=n-n_0}^{n+n_0} x_3[k] \\ &= \sum_{k=n-n_0}^{n+n_0} (ax_1[k] + bx_2[k]) = a \sum_{k=n-n_0}^{n+n_0} x_1[k] + b \sum_{k=n-n_0}^{n+n_0} x_2[k] \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is linear.

(b) Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n - n_1]$$

The output corresponding to this input is

$$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{k=n-n_0}^{n+n_0} x_1[k - n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$$

Also note that

$$y_1[n - n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k].$$

Therefore,

$$y_2[n] = y_1[n - n_1]$$

This implies that the system is time-invariant.

(c) If $|x[n]| < B$, then

$$y[n] \leq (2n_0 + 1)B$$

Therefore, $C \leq (2n_0 + 1)B$.

1.19. (a) (i) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = t^2 x_1(t-1)$$

$$x_2(t) \longrightarrow y_2(t) = t^2 x_2(t-1)$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= t^2 x_3(t-1) \\ &= t^2(ax_1(t-1) + bx_2(t-1)) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Therefore, the system is **linear**.

(ii) Consider an arbitrary input $x_1(t)$. Let

$$y_1(t) = t^2 x_1(t-1)$$

be the corresponding output. Consider a second input $x_2(t)$ obtained by shifting $x_1(t)$ in time:

$$x_2(t) = x_1(t-t_0)$$

The output corresponding to this input is

$$y_2(t) = t^2 x_2(t-1) = t^2 x_1(t-1-t_0)$$

Also note that

$$y_1(t-t_0) = (t-t_0)^2 x_1(t-1-t_0) \neq y_2(t)$$

Therefore, the system is **not time-invariant**.

(b) (i) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \longrightarrow y_1[n] = x_1^2[n-2]$$

$$x_2[n] \longrightarrow y_2[n] = x_2^2[n-2]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is,

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= x_3^2[n-2] \\ &= (ax_1[n-2] + bx_2[n-2])^2 \\ &= a^2 x_1^2[n-2] + b^2 x_2^2[n-2] + 2abx_1[n-2]x_2[n-2] \\ &\neq ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is **not linear**.

(ii) Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = x_1^2[n-2]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_0]$$

The output corresponding to this input is

$$y_2[n] = x_2^2[n-2] = x_1^2[n-2-n_0]$$

Also note that

$$y_1[n-n_0] = x_1^2[n-2-n_0]$$

Therefore,

$$y_2[n] = y_1[n-n_0]$$

This implies that the system is **time-invariant.**

(c) (i) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \longrightarrow y_1[n] = x_1[n+1] - x_1[n-1]$$

$$x_2[n] \longrightarrow y_2[n] = x_2[n+1] - x_2[n-1]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is,

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= x_3[n+1] - x_3[n-1] \\ &= ax_1[n+1] + bx_1[n+1] - ax_1[n-1] - bx_2[n-1] \\ &= a(x_1[n+1] - x_1[n-1]) + b(x_2[n+1] - x_2[n-1]) \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is **linear.**

(ii) Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = x_1[n+1] - x_1[n-1]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_0]$$

The output corresponding to this input is

$$y_2[n] = x_2[n+1] - x_2[n-1] = x_1[n+1-n_0] - x_1[n-1-n_0]$$

Also note that

$$y_1[n-n_0] = x_1[n+1-n_0] - x_1[n-1-n_0]$$

Therefore,

$$y_2[n] = y_1[n-n_0]$$

This implies that the system is **time-invariant**.

(d) (i) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = \mathcal{O}d\{x_1(t)\}$$

$$x_2(t) \longrightarrow y_2(t) = \mathcal{O}d\{x_2(t)\}$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= \mathcal{O}d\{x_3(t)\} \\ &= \mathcal{O}d\{ax_1(t) + bx_2(t)\} \\ &= a\mathcal{O}d\{x_1(t)\} + b\mathcal{O}d\{x_2(t)\} = ay_1(t) + by_2(t) \end{aligned}$$

Therefore, the system is **linear**.

(ii) Consider an arbitrary input $x_1(t)$. Let

$$y_1[t) = \mathcal{O}d\{x_1(t)\} = \frac{x_1(t) - x_1(-t)}{2}$$

be the corresponding output. Consider a second input $x_2(t)$ obtained by shifting $x_1[n]$ in time:

$$x_2(t) = x_1(t-t_0)$$

The output corresponding to this input is

$$\begin{aligned} y_2(t) &= \mathcal{O}d\{x_2(t)\} = \frac{x_2(t) - x_2(-t)}{2} \\ &= \frac{x_1(t-t_0) - x_1(-t-t_0)}{2} \end{aligned}$$

Also note that

$$y_1(t-t_0) = \frac{x_1(t-t_0) - x_1(-t+t_0)}{2} \neq y_2(t)$$

Therefore, the system is **not time-invariant**.

1.20. (a) Given

$$x(t) = e^{j2t} \longrightarrow y(t) = e^{j3t}$$
$$x(t) = e^{-j2t} \longrightarrow y(t) = e^{-j3t}$$

Since the system is linear,

$$x_1(t) = \frac{1}{2}(e^{j2t} + e^{-j2t}) \longrightarrow y_1(t) = \frac{1}{2}(e^{j3t} + e^{-j3t})$$

Therefore,

$$x_1(t) = \cos(2t) \longrightarrow y_1(t) = \cos(3t)$$

(b) We know that

$$x_2(t) = \cos\left(2(t - \frac{1}{2})\right) = \frac{e^{-j}e^{j2t} + e^{j}e^{-j2t}}{2}$$

Using the linearity property, we may once again write

$$x_1(t) = \frac{1}{2}(e^{-j}e^{j2t} + eje^{-j2t}) \longrightarrow y_1(t) = \frac{1}{2}(e^{-j}e^{j3t} + e^{j}e^{-j3t}) = \cos(3t - 1)$$

Therefore,

$$x_1(t) = \cos(2(t - 1/2)) \longrightarrow y_1(t) = \cos(3t - 1)$$

1.21. The signals are sketched in Figure S1.21.

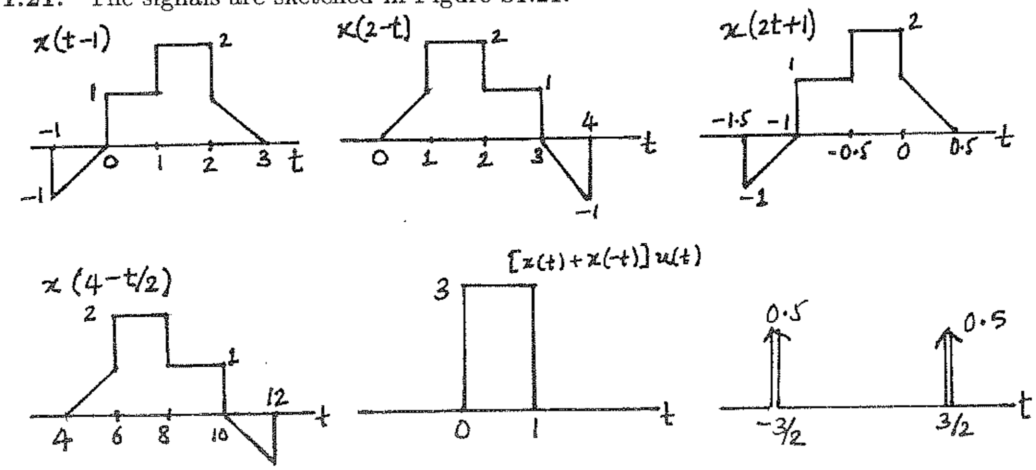

Figure S1.21

1.22. The signals are sketched in Figure S1.22.

1.23. The even and odd parts are sketched in Figure S1.23.

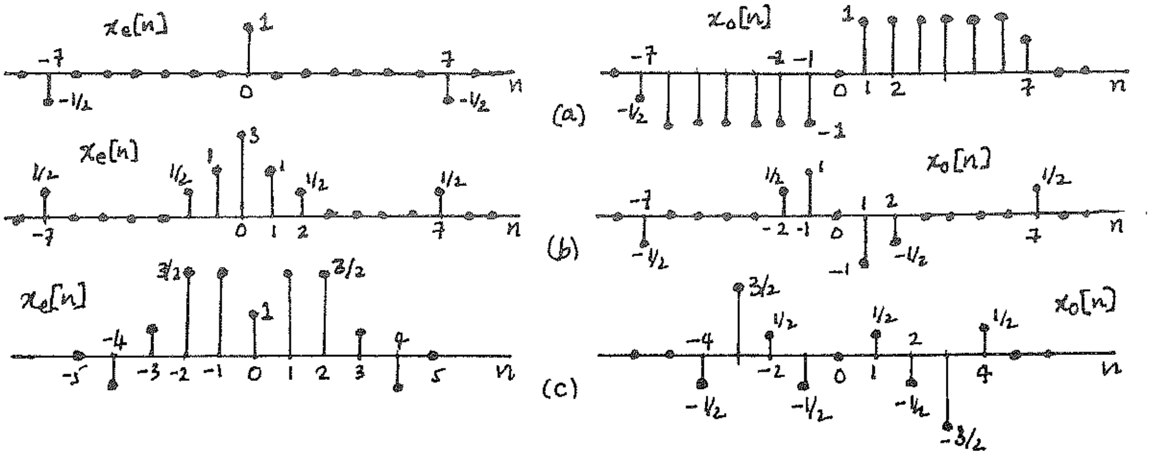

Figure S1.24

1.24. The even and odd parts are sketched in Figure S1.24.

1.25. (a) Periodic, period $= 2\pi/(4) = \pi/2$.

(b) Periodic, period $= 2\pi/(\pi) = 2$.

(c) $x(t) = [1 + \cos(4t - 2\pi/3)]/2$. Periodic, period $= 2\pi/(4) = \pi/2$.

(d) $x(t) = \cos(4\pi t)/2$. Periodic, period $= 2\pi/(4\pi) = 1/2$.

(e) $x(t) = [\sin(4\pi t)u(t) - \sin(4\pi t)u(-t)]/2$. Not periodic.

(f) Not periodic.

1.26. (a) Periodic, period $= 7$.

(b) Not periodic.

(c) Periodic, period $= 8$.

(d) $x[n] = (1/2)[\cos(3\pi n/4) + \cos(\pi n/4)]$. Periodic, period $= 8$.

(e) Periodic, period $= 16$.

1.27. (a) Linear, stable.

(b) Memoryless, linear, causal, stable.

(c) Linear

(d) Linear, causal, stable.

(e) Time invariant, linear, causal, stable.

(f) Linear, stable.

(g) Time invariant, linear, causal.

1.28. (a) Linear, stable.

(b) Time invariant, linear, causal, stable.

(c) Memoryless, linear, causal.

(d) Linear, stable.

(e) Linear, stable.

(f) Memoryless, linear, causal, stable.

(g) Linear, stable.

1.29. (a) Consider two inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \mathcal{R}e\{x_1[n]\} \quad \text{and} \quad x_2[n] \xrightarrow{S} y_2[n] = \mathcal{R}e\{x_2[n]\}.$$

Now consider a third input $x_3[n] = x_1[n] + x_2[n]$. The corresponding system output will be

$$\begin{aligned} y_3[n] &= \mathcal{R}e\{x_3[n]\} \\ &= \mathcal{R}e\{x_1[n] + x_2[n]\} \\ &= \mathcal{R}e\{x_1[n]\} + \mathcal{R}e\{x_2[n] \\ &= y_1[n] + y_2[n] \end{aligned}$$

Therefore, we may conclude that the system is additive.

Let us now assume that the input-output relationship is changed to $y[n] = \mathcal{R}e\{e^{j\pi/4}x[n]\}$. Also, consider two inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \mathcal{R}e\{e^{j\pi/4}x_1[n]\}$$

and

$$x_2[n] \xrightarrow{S} y_2[n] = \mathcal{R}e\{e^{j\pi/4}x_2[n]\}.$$

Now consider a third input $x_3[n] = x_1[n] + x_2[n]$. The corresponding system output will be

$$\begin{aligned} y_3[n] &= \mathcal{R}e\{e^{j\pi/4}x_3[n]\} \\ &= \cos(\pi n/4)\mathcal{R}e\{x_3[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_3[n]\} \\ &\quad + \cos(\pi n/4)\mathcal{R}e\{x_1[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_1[n]\} \\ &+ \cos(\pi n/4)\mathcal{R}e\{x_2[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_2[n]\} \\ &= \mathcal{R}e\{e^{j\pi/4}x_1[n]\} + \mathcal{R}e\{e^{j\pi/4}x_2[n]\} \\ &= y_1[n] + y_2[n] \end{aligned}$$

Therefore, we may conclude that the system is additive.

1.30. (a) Invertible. Inverse system: $y(t) = x(t+4)$.

(b) Non invertible. The signals $x(t)$ and $x_1(t) = x(t) + 2\pi$ give the same output.

(c) Non invertible. $\delta[n]$ and $2\delta[n]$ give the same output.

(d) Invertible. Inverse system: $y(t) = dx(t)/dt$.

(e) Invertible. Inverse system: $y[n] = x[n+1]$ for $n \geq 0$ and $y[n] = x[n]$ for $n < 0$.

(f) Non invertible. $x[n]$ and $-x[n]$ give the same result.

(g) Invertible. Inverse system: $y[n] = x[1-n]$.

(h) Invertible. Inverse system: $y(t) = x(t) + dx(t)/dt$.

(i) Invertible. Inverse system: $y[n] = x[n] - (1/2)x[n-1]$.

(j) Non invertible. If $x(t)$ is any constant, then $y(t) = 0$.

(k) Non invertible. $\delta[n]$ and $2\delta[n]$ result in $y[n] = 0$.

(l) Invertible. Inverse system: $y(t) = x(t/2)$.

(m) Non invertible. $x_1[n] = \delta[n] + \delta[n-1]$ and $x_2[n] = \delta[n]$ give $y[n] = \delta[n]$.

(n) Invertible. Inverse system: $y[n] = x[2n]$.

1.31. (a) Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is as shown in Figure S1.31.

(b) Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$. This is as shown in Figure S1.31.

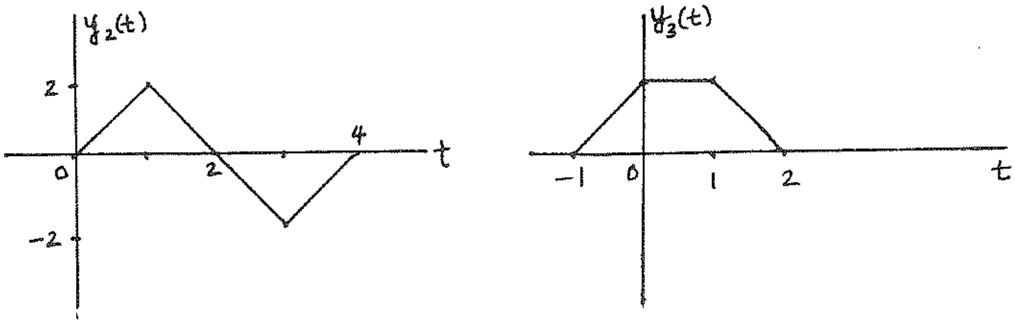

Figure S1.31

1.32. All statements are true.

(1) $x(t)$ periodic with period T; $y_1(t)$ periodic, period $T/2$.

(2) $y_1(t)$ periodic, period T; $x(t)$ periodic, period $2T$.

(3) $x(t)$ periodic, period T; $y_2(t)$ periodic, period $2T$.

(4) $y_2(t)$ periodic, period T; $x(t)$ periodic, period $T/2$.

1.33. (1) True. $x[n] = x[n+N]$; $y_1[n] = y_1[n+N_0]$. i.e. periodic with $N_0 = N/2$ if N is even, and with period $N_0 = N$ if N is odd.

Chapter 2 Answers

2.1. (a) We know that

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \qquad \text{(S2.1–1)}$$

The signals $x[n]$ and $h[n]$ are as shown in Figure S2.1.

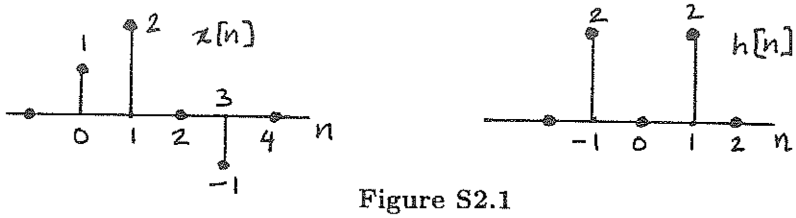

Figure S2.1

From this figure, we can easily see that the above convolution sum reduces to

$$\begin{aligned} y_1[n] &= h[-1]x[n+1] + h[1]x[n-1] \\ &= 2x[n+1] + 2x[n-1] \end{aligned}$$

This gives

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

(b) We know that

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

Comparing with eq. (S2.1-1), we see that

$$y_2[n] = y_1[n+2]$$

(c) We may rewrite eq. (S2.1-1) as

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Similarly, we may write

$$y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} x[k]h[n+2-k]$$

Comparing this with eq. (S2.1), we see that

$$y_3[n] = y_1[n+2]$$

2.2. Using the given definition for the signal $h[n]$, we may write

$$h[k] = \left(\frac{1}{2}\right)^{k-1} \{u[k+3] - u[k-10]\}$$

1) The signal $h[k]$ is non zero only in the range $-3 \le k \le 9$. From this we know that the signal $h[-k]$ is non zero only in the range $-9 \le k \le 3$. If we now shift the signal $h[-k]$ by n to the right, then the resultant signal $h[n-k]$ will be non zero in the range $(n-9) \le k \le (n+3)$. Therefore,

$$A = n - 9, \qquad B = n + 3$$

2.3. Let us define the signals

$$x_1[n] = \left(\frac{1}{2}\right)^{n} u[n]$$

and

$$h_1[n] = u[n].$$

We note that

$$x[n] = x_1[n-2] \qquad \text{and} \qquad h[n] = h_1[n+2]$$

Now,

$$\begin{aligned} y[n] &= x[n] * h[n] = x_1[n-2] * h_1[n+2] \\ &= \sum_{k=-\infty}^{\infty} x_1[k-2] h_1[n-k+2] \end{aligned}$$

By replacing k with $m+2$ in the abovr summation, we obtain

$$y[n] = \sum_{m=-\infty}^{\infty} x_1[m] h_1[n-m] = x_1[n] * h_1[n]$$

Using the results of Example 2.1 in the text book, we may write

$$y[n] = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u[n]$$

2.4. We know that

$$y[n] = x[n] * h[n] = \sum_{k=\infty}^{\infty} x[k] h[n-k]$$

The signals $x[n]$ and $y[n]$ are as shown in Figure S2.4. From this figure, we see that the above summation reduces to

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$

Figure S2.4

2.5. The signal $y[n]$ is

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

In this case, this summation reduces to

$$y[n] = \sum_{k=0}^{9} x[k]h[n-k] = \sum_{k=0}^{9} h[n-k]$$

From this it is clear that $y[n]$ is a summation of shifted replicas of $h[n]$. Since the last replica will begin at $n = 9$ and $h[n]$ is zero for $n > N$, $y[n]$ is zero for $n > N + 9$. Using this and the fact that $y[14] = 0$, we may conclude that N can *at most* be 4. Furthermore, since $y[4] = 5$, we can conclude that $h[n]$ has *at least* 5 non-zero points. The only value of N which satifies both these conditions is 4.

2.6. From the given information, we have:

$$\begin{aligned}
y[n] &= x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \sum_{k=-\infty}^{\infty} (\frac{1}{3})^{-k} u[-k-1]u[n-k-1] \\
&= \sum_{k=-\infty}^{-1} (\frac{1}{3})^{-k} u[n-k-1] \\
&= \sum_{k=1}^{\infty} (\frac{1}{3})^{k} u[n+k-1]
\end{aligned}$$

Replacing k by $p - 1$,

$$y[n] = \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} u[n+p] \qquad \text{(S2.6–1)}$$

For $n \geq 0$ the above equation reduces to,

$$y[n] = \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} = \frac{1}{3}\frac{1}{1-\frac{1}{3}} = \frac{1}{2}.$$

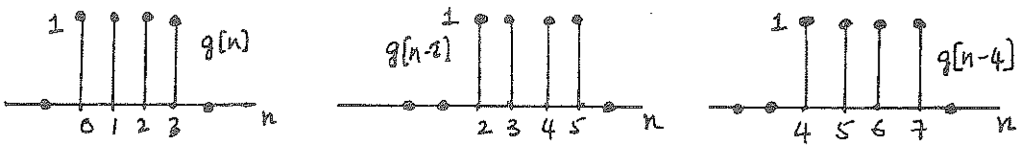

Figure S2.7

2.8. Using the convolution integral,

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$, the above integral reduces to

$$x(t) * y(t) = x(t+2) + 2x(t+1)$$

The signals $x(t+2)$ and $2x(t+1)$ are plotted in Figure S2.8.

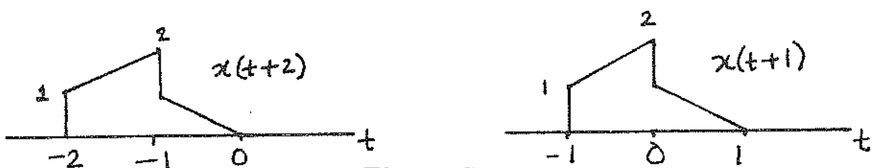

Figure S2.8

Using these plots, we can easily show that

$$y(t) = \begin{cases} t+3, & -2 < t \leq -1 \\ t+4, & -1 < t \leq 0 \\ 2-2t, & 0 < t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

2.9. Using the given definition for the signal $h(t)$, we may write

$$h(\tau) = e^{2\tau}u(-\tau+4) + e^{-2\tau}u(\tau-5) = \begin{cases} e^{-2\tau}, & \tau > 5 \\ e^{2\tau}, & \tau < 4 \\ 0, & 4 < \tau < 5 \end{cases}$$

Therefore,

$$h(-\tau) = \begin{cases} e^{2\tau}, & \tau < -5 \\ e^{-2\tau}, & \tau > -4 \\ 0, & -5 < \tau < -4 \end{cases}$$

If we now shift the signal $h(-\tau)$ by t to the right, then the resultant signal $h(t-\tau)$ will be

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)}, & \tau < t-5 \\ e^{2(t-\tau)}, & \tau > t-4 \\ 0, & (t-5) < \tau < (t-4) \end{cases}$$

Therefore,

$$A = t - 5, \qquad B = t - 4.$$

2.10. From the given information, we may sketch $x(t)$ and $h(t)$ as shown in Figure S2.10.

(a) With the aid of the plots in Figure S2.10, we can show that $y(t) = x(t) * h(t)$ is as shown in Figure S2.10.

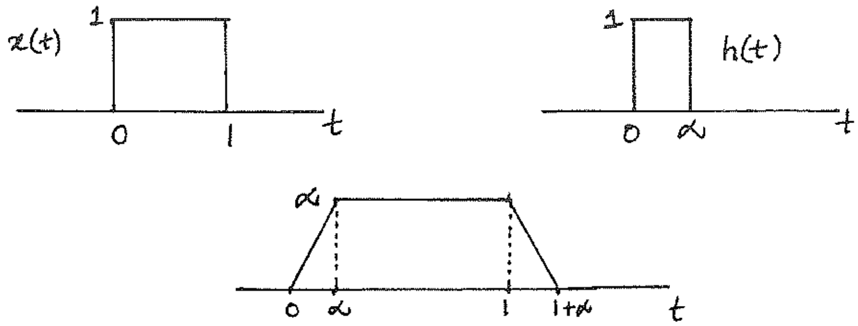

Figure S2.10

Therefore,

$$y(t) = \begin{cases} t, & 0 \le t \le \alpha \\ \alpha, & \alpha \le t \le 1 \\ 1 + \alpha - t, & 1 \le t \le (1 + \alpha) \\ 0, & \text{otherwise} \end{cases}$$

(b) From the plot of $y(t)$, it is clear that $\frac{dy(t)}{dt}$ has discontinuities at 0, α, 1, and $1 + \alpha$. If we want $\frac{dy(t)}{dt}$ to have only three discontinuities, then we need to ensure that $\alpha = 1$.

2.11. (a) From the given information, we see that $h(t)$ is non zero only for $0 \le t \le \infty$. Therefore,

$$\begin{aligned} y(t) &= x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau \\ &= \int_{0}^{\infty} e^{-3\tau}(u(t - \tau - 3) - u(t - \tau - 5))d\tau \end{aligned}$$

We can easily show that $(u(t - \tau - 3) - u(t - \tau - 5))$ is non zero only in the range $(t - 5) < \tau < (t - 3)$. Therefore, for $t \le 3$, the above integral evaluates to zero. For $3 < t \le 5$, the above integral is

$$y(t) = \int_{0}^{t-3} e^{-3\tau} d\tau = \frac{1 - e^{-3(t-3)}}{3}$$

For $t > 5$, the integral is

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau} d\tau = \frac{(1 - e^{-6})e^{-3(t-5)}}{3}$$

Therefore, the result of this convolution may be expressed as

$$y(t) = \begin{cases} 0, & -\infty < t \le 3 \\ \frac{1-e^{-3(t-3)}}{3}, & 3 < t \le 5 \\ \frac{(1-e^{-6})e^{-3(t-5)}}{3}, & 5 < t \le \infty \end{cases}$$

(b) By differentiating $x(t)$ with respect to time we get

$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$$

Therefore,

$$g(t) = \frac{dx(t)}{dt} * h(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5).$$

(c) From the result of part (a), we may compute the derivative of $y(t)$ to be

$$\frac{dy(t)}{dt} = \begin{cases} 0, & -\infty < t \le 3 \\ e^{-3(t-3)}, & 3 < t \le 5 \\ (e^{-6}-1)e^{-3(t-5)}, & 5 < t \le \infty \end{cases}$$

This is exactly equal to $g(t)$. Therefore, $g(t) = \frac{dy(t)}{dt}$.

2.12. The signal $y(t)$ may be written as

$$y(t) = \cdots + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t) + e^{-(t-3)}u(t-3) + e^{-(t-6)}u(t-6) + \cdots$$

In the range $0 \le t < 3$, we may write $y(t)$ as

$$\begin{aligned} y(t) &= \cdots + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t) \\ &= e^{-t} + e^{-(t+3)} + e^{-(t+6)} + \cdots \\ &= e^{-t}(1 + e^{-3} + e^{-6} + \cdots) \\ &= e^{-t}\frac{1}{1-e^{-3}} \end{aligned}$$

Therefore, $A = \frac{1}{1-e^{-3}}$.

2.13. (a) We require that

$$\left(\frac{1}{5}\right)^n u[n] - A\left(\frac{1}{5}\right)^{(n-1)} u[n-1] = \delta[n]$$

Putting $n = 1$ and solving for A gives $A = \frac{1}{5}$.

(b) From part (a), we know that

$$\begin{aligned} h[n] - \frac{1}{5}h[n-1] &= \delta[n] \\ h[n] * (\delta[n] - \frac{1}{5}\delta[n-1]) &= \delta[n] \end{aligned}$$

From the definition of an inverse system, we may argue that

$$g[n] = \delta[n] - \frac{1}{5}\delta[n-1].$$

2.14. (a) We first determine if $h_1(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_1(\tau)| d\tau = \int_{0}^{\infty} e^{-t} d\tau = 1$$

Therefore, $h_1(t)$ is the impulse response of a stable LTI system.

(b) We determine if $h_2(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_2(\tau)| d\tau = \int_{0}^{\infty} e^{-t} |\cos(2t)| d\tau$$

This integral is clearly finite-valued because $e^{-t}|\cos(2t)|$ is an exponentially decaying function in the range $0 \le t \le \infty$. Therefore, $h_2(t)$ is the impulse response of a stable LTI system.

2.15. (a) We determine if $h_1[n]$ is absolutely summable as follows

$$\sum_{k=-\infty}^{\infty} |h_1[k]| = \sum_{k=0}^{\infty} k|\cos(\frac{\pi}{4}k)|$$

This sum does not have a finite value because the function $k|\cos(\frac{\pi}{4}k)|$ increases as the value of k increases. Therefore, $h_1[n]$ cannot be the impulse response of a stable LTI system.

(b) We determine if $h_2[n]$ is absolutely summable as follows

$$\sum_{k=-\infty}^{\infty} |h_2[k]| = \sum_{k=-\infty}^{10} 3^k \approx 3^{11}/2$$

Therefore, $h_2[n]$ is the impulse response of a stable LTI system.

2.16. (a) **True**. This may be easily argued by noting that convolution may be viewed as the process of carrying out the superposition of a number of echos of $h[n]$. The first such echo will occur at the location of the first non zero sample of $x[n]$. In this case, the first echo will occur at N_1. The echo of $h[n]$ which occurs at $n = N_1$ will have its first non zero sample at the time location $N_1 + N_2$. Therefore, for all values of n which are lesser that $N_1 + N_2$, the output $y[n]$ is zero.

(b) **False**. Consider

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \end{aligned}$$

From this,

$$\begin{aligned} y[n-1] &= \sum_{k=-\infty}^{\infty} x[k]h[n-1-k] \\ &= x[n] * h[n-1] \end{aligned}$$

This shows that the given statement is false.

2.18. Since the system is causal, $y[n] = 0$ for $n < 1$. Now,

$$\begin{aligned}
y[1] &= \frac{1}{4}y[0] + x[1] = 0 + 1 = 1 \\
y[2] &= \frac{1}{4}y[1] + x[2] = \frac{1}{4} + 0 = \frac{1}{4} \\
y[3] &= \frac{1}{4}y[2] + x[3] = \frac{1}{16} + 0 = \frac{1}{16} \\
&\vdots \\
y[m] &= (\frac{1}{4})^{m-1} \\
&\vdots
\end{aligned}$$

Therefore,

$$y[n] = (\frac{1}{4})^{n-1}u[n-1]$$

2.19. **(a)** Consider the difference equation relating $y[n]$ and $w[n]$ for S_2:

$$y[n] = \alpha y[n-1] + \beta w[n]$$

From this we may write

$$w[n] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1]$$

and

$$w[n-1] = \frac{1}{\beta}y[n-1] - \frac{\alpha}{\beta}y[n-2]$$

Weighting the previous equation by 1/2 and subtracting from the one before, we obtain

$$w[n] - \frac{1}{2}w[n-1] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1] - \frac{1}{2\beta}y[n-1] + \frac{\alpha}{2\beta}y[n-2]$$

Substituting this in the difference equation relating $w[n]$ and $x[n]$ for S_1,

$$\frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1] - \frac{1}{2\beta}y[n-1] + \frac{\alpha}{2\beta}y[n-2] = x[n]$$

That is,

$$y[n] = (\alpha + \frac{1}{2})y[n-1] - \frac{\alpha}{2}y[n-2] + \beta x[n]$$

Comparing with the given equation relating $y[n]$ and $x[n]$, we obtain

$$\alpha = \frac{1}{4}, \qquad \beta = 1$$

(b) The difference equations relating the input and output of the systems S_1 and S_2 are

$$w[n] = \frac{1}{2}w[n-1] + x[n] \quad \text{and} \quad y[n] = \frac{1}{4}y[n-1] + w[n]$$

From these, we can use the method specifed in Example 2.15 to show that the impulse responses of S_1 and S_2 are

$$h_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

and

$$h_2[n] = \left(\frac{1}{4}\right)^n u[n],$$

respectively. The overall impulse response of the system made up of a cascade of S_1 and S_2 will be

$$\begin{aligned} h[n] &= h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k]h_2[n-k] \\ &= \sum_{k=0}^{\infty}(\frac{1}{2})^k(\frac{1}{4})^{n-k}u[n-k] \\ &= \sum_{k=0}^{n}(\frac{1}{2})^k(\frac{1}{4})^{n-k} = \sum_{k=0}^{n}(\frac{1}{2})^{2(n-k)} \\ &= [2(\frac{1}{2})^n - (\frac{1}{4})^n]u[n] \end{aligned}$$

2.20. (a)

$$\int_{-\infty}^{\infty} u_0(t)\cos(t)dt = \int_{-\infty}^{\infty} \delta(t)dt = 1$$

(b)

$$\int_{0}^{5} \sin(2\pi t)\delta(t+3)dt = \sin(6\pi) = 0$$

(c) In order to evaluate the integral

$$\int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)d\tau,$$

consider the signal

$$x(t) = \cos(2\pi t)[u(t+5) - u(t-5)].$$

We know that

$$\begin{aligned} \frac{dx(t)}{dt} &= u_1(t) * x(t) = \int_{-\infty}^{\infty} u_1(t-\tau)x(\tau)d\tau \\ &= \int_{-5}^{5} u_1(t-\tau)\cos(2\pi\tau)d\tau \end{aligned}$$

Now,

$$\left.\frac{dx(t)}{dt}\right|_{t=1} = \int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)d\tau$$

which is the desired integral. We now evaluate the value of the integral as

$$\left.\frac{dx(t)}{dt}\right|_{t=1} = \sin(2\pi t)|_{t=1} = 0.$$

2.21. **(a)** The desired convolution is

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= \beta^n \sum_{k=0}^{n} (\alpha/\beta)^k \text{ for } n \geq 0 \\ &= [\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}]u[n] \text{ for } \alpha \neq \beta. \end{aligned}$$

(b) From (a),

$$y[n] = \alpha^n \left[\sum_{k=0}^{n} 1\right] u[n] = (n+1)\alpha^n u[n].$$

(c) For $n \leq 6$,

$$y[n] = 4^n \left\{\sum_{k=0}^{\infty}(-\frac{1}{8})^k - \sum_{k=0}^{3}(-\frac{1}{8})^k\right\}.$$

For $n > 6$,

$$y[n] = 4^n \left\{\sum_{k=0}^{\infty}(-\frac{1}{8})^k - \sum_{k=0}^{n-1}(-\frac{1}{8})^k\right\}.$$

Therefore,

$$y[n] = \begin{cases} (8/9)(-1/8)^4 4^n, & n \leq 6 \\ (8/9)(-1/2)^n, & n > 6 \end{cases}$$

(d) The desired convolution is

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3] + x[4]h[n-4] \\ &= h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4]. \end{aligned}$$

This is as shown in Figure S2.21.

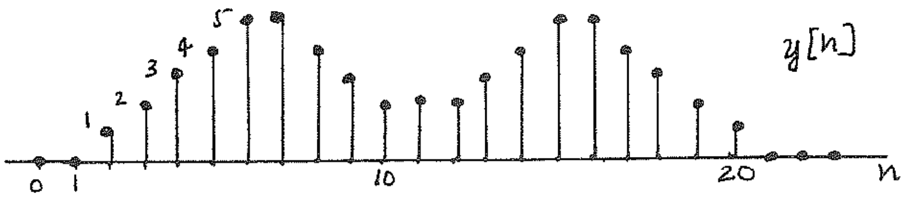

Figure S2.21

2.22. (a) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_0^t e^{-\alpha t} e^{-\beta(t-\tau)} d\tau, \; t \geq 0 \end{aligned}$$

Then

$$y(t) = \begin{cases} \frac{e^{-\beta t}\{e^{-(\alpha-\beta)t}-1\}}{\beta-\alpha} u(t) & \alpha \neq \beta \\ te^{-\beta t}u(t) & \alpha = \beta \end{cases}.$$

(b) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_0^2 h(t-\tau)d\tau - \int_2^5 h(t-\tau)d\tau. \end{aligned}$$

This may be written as

$$y(t) = \begin{cases} \int_0^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, & t \leq 1 \\ \int_{t-1}^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, & 1 \leq t \leq 3 \\ -\int_{t-1}^5 e^{2(t-\tau)}d\tau, & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$

Therefore,

$$y(t) = \begin{cases} (1/2)[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & t \leq 1 \\ (1/2)[e^2 + e^{2(t-5)} - 2e^{2(t-2)}], & 1 \leq t \leq 3 \\ (1/2)[e^{2(t-5)} - e^2], & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$

(c) The desired convolution is

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{0}^{2} \sin(\pi\tau)h(t-\tau)d\tau.$$

This gives us

$$y(t) = \begin{cases} 0, & t < 1 \\ (2/\pi)[1-\cos\{\pi(t-1)\}], & 1 < t < 3 \\ (2/\pi)[\cos\{\pi(t-3)\}-1], & 3 < t < 5 \\ 0, & 5 < t \end{cases}$$

(d) Let

$$h(t) = h_1(t) - \frac{1}{3}\delta(t-2),$$

where

$$h_1(t) = \begin{cases} 4/3, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

Now,

$$y(t) = h(t) * x(t) = [h_1(t) * x(t)] - \frac{1}{3}x(t-2).$$

We have

$$h_1(t) * x(t) = \int_{t-1}^{t} \frac{4}{3}(a\tau + b)d\tau = \frac{4}{3}[\frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 + bt - b(t-1)].$$

Therefore,

$$y(t) = \frac{4}{3}[\frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 + bt - b(t-1)] - \frac{1}{3}[a(t-2)+b] = at + b = x(t).$$

(e) $x(t)$ periodic implies $y(t)$ periodic. $\therefore$ determine 1 period only. We have

$$y(t) = \begin{cases} \int_{t-1}^{-\frac{1}{2}} (t-\tau-1)d\tau + \int_{-\frac{1}{2}}^{t} (1-t+\tau)d\tau = \frac{1}{4} + t - t^2, & -\frac{1}{2} < t < \frac{1}{2} \\ \int_{t-1}^{\frac{1}{2}} (1-t+\tau)d\tau + \int_{\frac{1}{2}}^{t} (t-1-\tau)d\tau = t^2 - 3t + 7/4, & \frac{1}{2} < t < \frac{3}{2} \end{cases}.$$

The period of $y(t)$ is 2.

2.23. $y(t)$ is sketched in Figure S2.23 for the different values of T.

2.24. **(a)** We are given that $h_2[n] = \delta[n] + \delta[n-1]$. Therefore,

$$h_2[n] * h_2[n] = \delta[n] + 2\delta[n-1] + \delta[n-2].$$

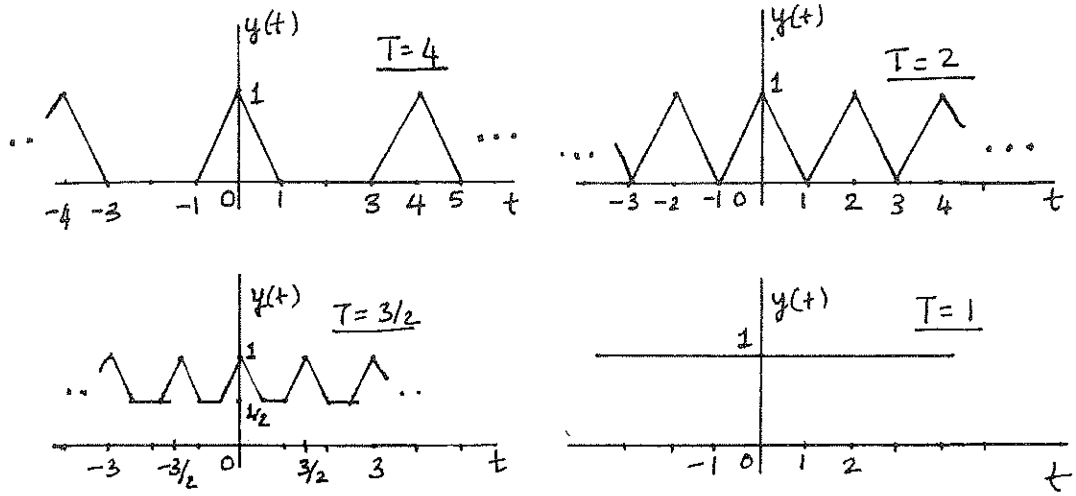

Figure S2.23

Since

$$h[n] = h_1[n] * [h_2[n] * h_2[n]],$$

we get

$$h[n] = h_1[n] + 2h_1[n-1] + h_1[n-2].$$

Therefore,

$$\begin{aligned}
h[0] &= h_1[0] & \Rightarrow \quad & h_1[0] = 1, \\
h[1] &= h_1[1] + 2h_1[0] & \Rightarrow \quad & h_1[1] = 3, \\
h[2] &= h_1[2] + 2h_1[1] + h_1[0] & \Rightarrow \quad & h_1[2] = 3, \\
h[3] &= h_1[3] + 2h_1[2] + h_1[1] & \Rightarrow \quad & h_1[3] = 2, \\
h[4] &= h_1[4] + 2h_1[3] + h_1[2] & \Rightarrow \quad & h_1[4] = 1 \\
h[5] &= h_1[5] + 2h_1[4] + h_1[3] & \Rightarrow \quad & h_1[5] = 0.
\end{aligned}$$

$h_1[n] = 0$ for $n < 0$ and $n \geq 5$.

(b) In this case,

$$y[n] = x[n] * h[n] = h[n] - h[n-1].$$

2.25. (a) We may write $x[n]$ as

$$x[n] = \left(\frac{1}{3}\right)^{|n|}.$$

Now, the desired convolution is

$$\begin{aligned} y[n] &= h[n] * x[n] \\ &= \sum_{k=-\infty}^{-1} (1/3)^{-k}(1/4)^{n-k}u[n-k+3] + \sum_{k=0}^{\infty}(1/3)^k(1/4)^{n-k}u[n-k+3] \\ &= (1/12)\sum_{k=0}^{\infty}(1/3)^k(1/4)^{n+k}u[n+k+4] + \sum_{k=0}^{\infty}(1/3)^k(1/4)^{n-k}u[n-k+3] \end{aligned}$$

By consider each summation in the above equation separately, we may show that

$$y[n] = \begin{cases} (12^4/11)3^n, & n < -4 \\ (1/11)4^4, & n = -4 \\ (1/4)^n(1/11) + -3(1/4)^n + 3(256)(1/3)^n, & n \geq -3 \end{cases}.$$

(b) Now consider the convolution

$$y_1[n] = [(1/3)^n u[n]] * [(1/4)^n u[n+3]].$$

We may show that

$$y_1[n] = \begin{cases} 0, & n < -3 \\ -3(1/4)^n + 3(256)(1/3)^n, & n \geq -3 \end{cases}.$$

Also, consider the convolution

$$y_2[n] = [(3)^n u[-n-1]] * [(1/4)^n u[n+3]].$$

We may show that

$$y_2[n] = \begin{cases} (12^4/11)3^n, & n < -4 \\ (1/4)^n(1/11), & n \geq -3 \end{cases}.$$

Clearly, $y_1[n] + y_2[n] = y[n]$ obtained in the previous part.

2.26. **(a)** We have

$$\begin{aligned} y_1[n] = x_1[n] * x_2[n] &= \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k] \\ &= \sum_{k=0}^{\infty}(0.5)^k u[n+3-k]. \end{aligned}$$

This evaluates to

$$y_1[n] = x_1[n] * x_2[n] = \begin{cases} 2\left\{1 - (1/2)^{n+4}\right\}, & n \geq -3 \\ 0, & \text{otherwise} \end{cases}.$$

(b) Now,

$$y[n] = x_3[n] * y_1[n] = y_1[n] - y_1[n-1].$$

Therefore,

$$y[n] = \begin{cases} 2\{1-(1/2)^{n+3}\} + 2\{1-(1/2)^{n+4}\} = (1/2)^{n+3}, & n \geq -2 \\ 1, & n = -3 \\ 0, & \text{otherwise} \end{cases}.$$

Therefore, $y[n] = (1/2)^{n+3}u[n+3]$.

(c) We have

$$y_2[n] = x_2[n] * x_3[n] = u[n+3] - u[n+2] = \delta[n+3].$$

(d) From the result of part (c), we get

$$y[n] = y_2[n] * x_1[n] = x_1[n+3] = (1/2)^{n+3}u[n+3].$$

2.27. The proof is as follows.

$$\begin{aligned} A_y &= \int_{-\infty}^{\infty} y(t)dt \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau dt \\ &= \int_{-\infty}^{\infty} x(\tau)\int_{-\infty}^{\infty} h(t-\tau)dtd\tau \\ &= \int_{-\infty}^{\infty} x(\tau)A_h d\tau \\ &= A_x A_h \end{aligned}$$

2.28. **(a)** Causal because $h[n] = 0$ for $n < 0$. Stable because $\sum_{n=0}^{\infty}(\frac{1}{5})^n = 5/4 < \infty$.

(b) Not causal because $h[n] \neq 0$ for $n < 0$. Stable because $\sum_{n=-2}^{\infty}(0.8)^n = 5 < \infty$.

(c) Anti-causal because $h[n] = 0$ for $n > 0$. Unstable because $\sum_{n=-\infty}^{0}(1/2)^n = \infty$

(d) Not causal because $h[n] \neq 0$ for $n < 0$. Stable because $\sum_{n=-\infty}^{3} 5^n = \frac{625}{4} < \infty$

(e) Causal because $h[n] = 0$ for $n < 0$. Unstable because the second term becomes infinite as $n \to \infty$.

(f) Not causal because $h[n] \neq 0$ for $n < 0$. Stable because $\sum_{n-\infty}^{\infty}|h[n]| = 305/3 < \infty$