Problem 2.3
(a) Not periodic.
(b) Periodic. To find the period, note that

$$\frac{6\pi}{2\pi} = n_1 f_0 \text{ and } \frac{20\pi}{2\pi} = n_2 f_0$$

Therefore

$$\frac{10}{3} = \frac{n_2}{n_1}$$

Hence, take $n_1 = 3$, $n_2 = 10$, and $f_0 = 1$ Hz.
(c) Periodic. Using a similar procedure as used in (b), we find that $n_1 = 2$, $n_2 = 7$, and $f_0 = 1$ Hz.
(d) Periodic. Using a similar procedure as used in (b), we find that $n_1 = 2$, $n_2 = 3$, $n_3 = 11$, and $f_0 = 1$ Hz.

Problem 2.7
(a), (c), (e), and (f) are periodic. Their periods are 1 s, 4 s, 3 s, and 2/7 s, respectively. The waveform of part (c) is a periodic train of impulses extending from $-\infty$ to ∞ spaced by 4 s. The waveform of part (a) is a complex sum of sinusoids that repeats (plot). The waveform of part (e) is a doubly-infinite train of square pulses, each of which is one unit high and one unit wide, centered at $\cdots, -6, -3, 0, 3, 6, \cdots$. Waveform (f) is a raised cosine of minimum and maximum amplitudes 0 and 2, respectively.

Problem 2.9
(a) Power. Since it is a periodic signal, we obtain

$$P_1 = \frac{1}{T_0}\int_0^{T_0} 4\sin^2(8\pi t + \pi/4)\, dt = \frac{1}{T_0}\int_0^{T_0} 2\left[1 - \cos(16\pi t + \pi/2)\right] dt = 2 \text{ W}$$

where $T_0 = 1/8$ s is the period.
(b) Energy. The energy is

$$E_2 = \int_{-\infty}^{\infty} e^{-2\alpha t} u^2(t) dt = \int_0^{\infty} e^{-2\alpha t} dt = \frac{1}{2\alpha}$$

(c) Energy. The energy is

$$E_3 = \int_{-\infty}^{\infty} e^{2\alpha t} u^2(-t) dt = \int_{-\infty}^{0} e^{2\alpha t} dt = \frac{1}{2\alpha}$$

(d) Neither energy or power.

$$E_4 = \lim_{T\to\infty}\int_{-T}^{T} \frac{dt}{(\alpha^2 + t^2)^{1/4}} = \infty$$

$P_4 = 0$ since $\lim_{T\to\infty}\frac{1}{T}\int_{-T}^{T}\frac{dt}{(\alpha^2+t^2)^{1/4}} = 0$.(e) Energy. Since it is the sum of $x_1(t)$ and $x_2(t)$, its energy is the sum of the energies of these two signals, or $E_5 = 1/\alpha$.

(f) Power. Since it is an aperiodic signal (the sine starts at $t = 0$), we use

$$\begin{aligned} P_6 &= \lim_{T\to\infty} \frac{1}{2T} \int_0^T \sin^2(5\pi t)\, dt = \lim_{T\to\infty} \frac{1}{2T} \int_0^T \frac{1}{2} [1 - \cos(10\pi t)]\, dt \\ &= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{T}{2} - \frac{1}{2} \frac{\sin(20\pi t)}{20\pi} \right]_0^T = \frac{1}{4} \text{ W} \end{aligned}$$

Problem 2.10

(a) Power. Since the signal is periodic with period π/ω, we have

$$P = \frac{\omega}{\pi} \int_0^{\pi/\omega} A^2 \left|\sin(\omega t + \theta)\right|^2 dt = \frac{\omega}{\pi} \int_0^{\pi/\omega} \frac{A^2}{2} \{1 - \cos[2(\omega t + \theta)]\}\, dt = \frac{A^2}{2}$$

(b) Neither. The energy calculation gives

$$E = \lim_{T\to\infty} \int_{-T}^{T} \frac{(A\tau)^2\, dt}{\sqrt{\tau + jt}\sqrt{\tau - jt}} = \lim_{T\to\infty} \int_{-T}^{T} \frac{(A\tau)^2\, dt}{\sqrt{\tau^2 + t^2}} \to \infty$$

The power calculation gives

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{(A\tau)^2\, dt}{\sqrt{\tau^2 + t^2}} = \lim_{T\to\infty} \frac{(A\tau)^2}{2T} \ln\left(\frac{1 + \sqrt{1 + T^2/\tau^2}}{-1 + \sqrt{1 + T^2/\tau^2}} \right) = 0$$

(c) Energy:

$$E = \int_0^\infty A^2 t^4 \exp(-2t/\tau)\, dt = \frac{3}{4} A^2 \tau^5 \quad \text{(use table of integrals)}$$

(d) Energy:

$$E = 2\left(\int_0^{\tau/2} 2^2 dt + \int_{\tau/2}^{\tau} 1^2 dt \right) = 5\tau$$

Problem 2.11

(a) This is a periodic train of "boxcars", each 3 units in width and centered at multiples of 6:

$$P_a = \frac{1}{6} \int_{-3}^{3} \Pi^2\left(\frac{t}{3}\right) dt = \frac{1}{6} \int_{-1.5}^{1.5} dt = \frac{1}{2} \text{ W}$$

(b) This is a periodic train of unit-high isoceles triangles, each 4 units wide and centered at multiples of 5:

$$P_b = \frac{1}{5}\int_{-2.5}^{2.5} \Lambda^2\left(\frac{t}{2}\right) dt = \frac{2}{5}\int_0^2 \left(1-\frac{t}{2}\right)^2 dt = -\frac{2}{5}\frac{2}{3}\left(1-\frac{t}{2}\right)^3\Big|_0^2 = \frac{4}{15}\text{ W}$$

(c) This is a backward train of sawtooths (right triangles with the right angle on the left), each 2 units wide and spaced by 3 units:

$$P_c = \frac{1}{3}\int_0^2 \left(1-\frac{t}{2}\right)^2 dt = -\frac{1}{3}\frac{2}{3}\left(1-\frac{t}{2}\right)^3\Big|_0^2 = \frac{2}{9}\text{ W}$$

(d) This is a full-wave rectified cosine wave of period 1/5 (the width of each cosine pulse):

$$P_d = 5\int_{-1/10}^{1/10} |\cos(5\pi t)|^2 dt = 2\times 5\int_0^{1/10}\left[\frac{1}{2}+\frac{1}{2}\cos(10\pi t)\right] dt = \frac{1}{2}\text{ W}$$

Problem 2.17

Parts (a) through (c) were discussed in the text. For (d), break the integral for $x(t)$ up into a part for $t<0$ and a part for $t>0$. Then use the odd half-wave symmetry contition.

Problem 2.18

This is a matter of integration. Only the solution for part (b) will be given here. The integral for the Fourier coefficients is (note that the period really is $T_0/2$)

$$\begin{aligned} X_n &= \frac{A}{T_0}\int_0^{T_0/2} \sin(\omega_0 t)\, e^{-jn\omega_0 t} dt \\ &= -\frac{Ae^{-jn\omega_0 t}}{\omega_0 T_0(1-n^2)}\left[jn\sin(\omega_0 t)+\cos(\omega_0 t)\right]\Big|_0^{T_0/2} \\ &= \frac{A\left(1+e^{-jn\pi}\right)}{\omega_0 T_0(1-n^2)},\ n\neq\pm 1 \end{aligned}$$

For $n=1$, the integral is

$$X_1 = \frac{A}{T_0}\int_0^{T_0/2} \sin(\omega_0 t)\left[\cos(jn\omega_0 t) - j\sin(jn\omega_0 t)\right] dt = -\frac{jA}{4} = -X_{-1}^*$$

This is the same result as given in Table 2.1.

Problem 2.20

(a) The integral for Y_n is

$$Y_n = \frac{1}{T_0}\int_{T_0} y(t)\, e^{-jn\omega_0 t} dt = \frac{1}{T_0}\int_0^{T_0} x(t - t_0)\, e^{-jn\omega_0 t} dt$$

Let $t' = t - t_0$, which results in

$$Y_n = \left[\frac{1}{T_0}\int_{-t_0}^{T_0 - t_0} x(t')\, e^{-jn\omega_0 t'} dt'\right] e^{-jn\omega_0 t_0} = X_n e^{-jn\omega_0 t_0}$$

(b) Note that

$$y(t) = A\cos\omega_0 t = A\sin(\omega_0 t + \pi/2) = A\sin[\omega_0(t + \pi/2\omega_0)]$$

Thus, t_0 in the theorem proved in part (a) here is $-\pi/2\omega_0$. By Euler's theorem, a sine wave can be expressed as

$$\sin(\omega_0 t) = \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$$

Its Fourier coefficients are therefore $X_1 = \frac{1}{2j}$ and $X_{-1} = -\frac{1}{2j}$. According to the theorem proved in part (a), we multiply these by the factor

$$e^{-jn\omega_0 t_0} = e^{-jn\omega_0(-\pi/2\omega_0)} = e^{jn\pi/2}$$

For $n = 1$, we obtain

$$Y_1 = \frac{1}{2j}e^{j\pi/2} = \frac{1}{2}$$

For $n = -1$, we obtain

$$Y_{-1} = -\frac{1}{2j}e^{-j\pi/2} = \frac{1}{2}$$

which gives the Fourier series representation of a cosine wave as

$$y(t) = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} = \cos\omega_0 t$$

We could have written down this Fourier representation directly by using Euler's theorem.

Problem 2.24

(a) This is the right half of a triangle waveform of width τ and height A, or $A(1 - t/\tau)$. Therefore, the Fourier transform is

$$\begin{aligned} X_1(f) &= A\int_0^{\tau}(1 - t/\tau)\, e^{-j2\pi f t} dt \\ &= \frac{A}{j2\pi f}\left[1 - \frac{1}{j2\pi f\tau}\left(1 - e^{-j2\pi f\tau}\right)\right] \end{aligned}$$

where a table of integrals has been used.

(b) Since $x_2(t) = x_1(-t)$ we have, by the time reversal theorem, that

$$\begin{aligned} X_2(f) &= X_1^*(f) = X_1(-f) \\ &= \frac{A}{-j2\pi f}\left[1 + \frac{1}{j2\pi f\tau}\left(1 - e^{j2\pi f\tau}\right)\right] \end{aligned}$$

(c) Since $x_3(t) = x_1(t) - x_2(t)$ we have, after some simplification, that

$$\begin{aligned} X_3(f) &= X_1(f) - X_2(f) \\ &= \frac{jA}{\pi f}\operatorname{sinc}(2f\tau) \end{aligned}$$

(d) Since $x_4(t) = x_1(t) + x_2(t)$ we have, after some simplification, that

$$\begin{aligned} X_4(f) &= X_1(f) + X_2(f) \\ &= A\tau\frac{\sin^2(\pi f\tau)}{(\pi f\tau)^2} \\ &= A\tau\operatorname{sinc}^2(f\tau) \end{aligned}$$

This is the expected result, since $x_4(t)$ is really a triangle function.

Problem 2.27
(a) This is an odd signal, so its Fourier transform is odd and purely imaginary.
(b) This is an even signal, so its Fourier transform is even and purely real.
(c) This is an odd signal, so its Fourier transform is odd and purely imaginary.
(d) This signal is neither even nor odd signal, so its Fourier transform is complex.
(e) This is an even signal, so its Fourier transform is even and purely real.
(f) This signal is even, so its Fourier transform is real and even.

Problem 2.29
(a) The Fourier transform of this signal is

$$X_1(f) = \frac{2(1/3)}{1 + (2\pi f/3)^2} = \frac{2/3}{1 + [f/(3/2\pi)]^2}$$

Thus, the energy spectral density is

$$G_1(f) = \left\{\frac{2/3}{1 + [f/(3/2\pi)]^2}\right\}^2$$

(b) The Fourier transform of this signal is

$$X_2(f) = \frac{2}{3}\Pi\left(\frac{f}{30}\right)$$

Thus, the energy spectral density is

$$X_2(f) = \frac{4}{9}\Pi^2\left(\frac{f}{30}\right) = \frac{4}{9}\Pi\left(\frac{f}{30}\right)$$

(c) The Fourier transform of this signal is

$$X_3(f) = \frac{4}{5}\text{sinc}\left(\frac{f}{5}\right)$$

so the energy spectral density is

$$G_3(f) = \frac{16}{25}\text{sinc}^2\left(\frac{f}{5}\right)$$

(d) The Fourier transform of this signal is

$$X_4(f) = \frac{2}{5}\left[\text{sinc}\left(\frac{f-20}{5}\right) + \text{sinc}\left(\frac{f+20}{5}\right)\right]$$

so the energy spectral density is

$$G_4(f) = \frac{4}{25}\left[\text{sinc}\left(\frac{f-20}{5}\right) + \text{sinc}\left(\frac{f+20}{5}\right)\right]^2$$

Problem 2.31

(a) The convolution operation gives

$$y_1(t) = \begin{cases} 0, \ t \leq \tau - 1/2 \\ \frac{1}{\alpha}\left[1 - e^{-\alpha(t-\tau+1/2)}\right], \ \tau - 1/2 < t \leq \tau + 1/2 \\ \frac{1}{\alpha}\left[e^{-\alpha(t-\tau-1/2)} - e^{-\alpha(t-\tau+1/2)}\right], \ t > \tau + 1/2 \end{cases}$$

(b) The convolution of these two signals gives

$$y_2(t) = \Lambda(t) + \text{tr}(t)$$

where tr(t) is a trapezoidal function given by

$$\text{tr}(t) = \begin{cases} 0, \ t < -3/2 \text{ or } t > 3/2 \\ 1, \ -1/2 \leq t \leq 1/2 \\ 3/2 + t, \ -3/2 \leq t < -1/2 \\ 3/2 - t, \ 1/2 \leq t < 3/2 \end{cases}$$

(c) The convolution results in

$$y_3(t) = \int_{-\infty}^{\infty} e^{-\alpha|\lambda|}\Pi(\lambda - t)\,d\lambda = \int_{t-1/2}^{t+1/2} e^{-\alpha|\lambda|}d\lambda$$

Sketches of the integrand for various values of t gives the following cases:

$$y_3(t) = \begin{cases} \int_{t-1/2}^{t+1/2} e^{\alpha\lambda}d\lambda, \ t \leq -1/2 \\ \int_{t-1/2}^{0} e^{\alpha\lambda}d\lambda + \int_{0}^{t+1/2} e^{-\alpha\lambda}d\lambda, \ -1/2 < t \leq 1/2 \\ \int_{t-1/2}^{t+1/2} e^{-\alpha\lambda}d\lambda, \ t > 1/2 \end{cases}$$

Integration of these three cases gives

$$y_3(t) = \begin{cases} \frac{1}{\alpha}\left[e^{\alpha(t+1/2)} - e^{\alpha(t-1/2)}\right], \ t \leq -1/2 \\ \frac{1}{\alpha}\left[e^{-\alpha(t-1/2)} - e^{-\alpha(t+1/2)}\right], \ -1/2 < t \leq 1/2 \\ \frac{1}{\alpha}\left[e^{-\alpha(t-1/2)} - e^{-\alpha(t+1/2)}\right], \ t > 1/2 \end{cases}$$

(d) The convolution gives

$$y_4(t) = \int_{-\infty}^{t} x(\lambda)\,d\lambda$$

Problem 2.26

(a) Two differentiations give

$$\frac{d^2x_1(t)}{dt^2} = \frac{d\delta(t)}{dt} - \delta(t-2) + \delta(t-3)$$

Application of the differentiation theorem of Fourierr transforms gives

$$(j2\pi f)^2 X_1(f) = (j2\pi f)(1) - 1\cdot e^{-j4\pi f} + 1\cdot e^{-j6\pi f}$$

where the time delay theorem and the Fourier transform of a unit impulse have been used. Dividing both sides by $(j2\pi f)^2$, we obtain

$$X_1(f) = \frac{1}{j2\pi f} - \frac{e^{-j4\pi f} - e^{-j6\pi f}}{(j2\pi f)^2} = \frac{1}{j2\pi f} - \frac{e^{-j5\pi f}}{j2\pi f}\text{sinc}(2f)$$

(b) Two differentiations give

$$\frac{d^2x_2(t)}{dt^2} = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

Application of the differentiation theorem gives

$$(j2\pi f)^2 X_2(f) = 1 - 2e^{-j2\pi f} + e^{-j4\pi f}$$

Dividing both sides by $(j2\pi f)^2$, we obtain

$$X_2(f) = \frac{1 - 2e^{-j2\pi f} + e^{-j4\pi f}}{(j2\pi f)^2} = \text{sinc}^2(f)\, e^{-j2\pi f}$$

(c) Two differentiations give

$$\frac{d^2x_3(t)}{dt^2} = \delta(t) - \delta(t-1) - \delta(t-2) + \delta(t-3)$$

Application of the differentiation theorem gives

$$(j2\pi f)^2 X_3(f) = 1 - e^{-j2\pi f} - e^{-j4\pi f} + e^{-j6\pi f}$$

Dividing both sides by $(j2\pi f)^2$, we obtain

$$X_3(f) = \frac{1 - e^{-j2\pi f} - e^{-j4\pi f} + e^{-j6\pi f}}{(j2\pi f)^2}$$

(d) Two differentiations give

$$\frac{d^2x_4(t)}{dt^2} = 2\Pi(t-1/2) - 2\delta(t-1) - 2\frac{d\delta(t-2)}{dt}$$

Application of the differentiation theorem gives

$$(j2\pi f)^2 X_4(f) = 2\text{sinc}(f)\, e^{-j\pi f} - 2e^{-j2\pi f} - 2(j2\pi f)\, e^{-j4\pi f}$$

Dividing both sides by $(j2\pi f)^3$, we obtain

$$X_4(f) = \frac{2e^{-j2\pi f} + (j2\pi f)\, e^{-j4\pi f} - \text{sinc}(f)\, e^{-j\pi f}}{2(\pi f)^2}$$