**Problem 2.38**

Fourier transform both sides of the differential equation using the differentiation theorem of Fourier transforms to get

$$[j2\pi f + a]\, Y(f) = [j2\pi bf + c]\, X(f)$$

Therefore, the transfer function is

$$H(f) = \frac{Y(f)}{X(f)} = \frac{c + j2\pi bf}{a + j2\pi f}$$

The amplitude response function is

$$|H(f)| = \frac{\sqrt{c^2 + (2\pi bf)^2}}{\sqrt{a^2 + (2\pi f)^2}}$$

and the phase response is

$$\arg[H(f)] = \tan^{-1}\left(\frac{2\pi bf}{c}\right) - \tan^{-1}\left(\frac{2\pi f}{a}\right)$$

Amplitude and phase responses for various values of the constants are plotted below.

**Problem 2.39**

(a) The find the unit impulse response, write $H(f)$ as

$$H(f) = 1 - \frac{5}{5 + j2\pi f}$$

Inverse Fourier transforming gives

$$h(t) = \delta(t) - 5e^{-5t}u(t)$$

(b) Use the transform pair

$$Ae^{-\alpha t}u(t) \longleftrightarrow \frac{A}{\alpha + j2\pi f}$$

and the time delay theorem to find the unit impulse as

$$h(t) = \frac{2}{5}e^{-\frac{8}{15}(t-3)}u(t-3)$$

**Problem 2.42**

(a) By long division

$$H(f) = 1 - \frac{R_1/L}{\frac{R_1+R_2}{L} + j2\pi f}$$

Using the transforms of a delta function and a one-sided exponential, we obtain

$$h(t) = \delta(t) - \frac{R_1}{L} \exp\left(-\frac{R_1+R_2}{L}t\right) u(t)$$

(b) Substituting the ac-equivalent impedance for the inductor and using voltage division, the transfer function is

$$H(f) = \frac{R_2}{R_1+R_2} \frac{j2\pi f L}{\frac{R_1 R_2}{R_1+R_2} + j2\pi f L} = \frac{R_2}{R_1+R_2}\left(1 - \frac{(R_1 \parallel R_2)/L}{(R_1 \parallel R_2)/L + j2\pi f}\right)$$

Therefore, the impulse response is

$$h(t) = \frac{R_2}{R_1+R_2}\left[\delta(t) - \frac{R_1 R_2}{(R_1+R_2)L} \exp\left(-\frac{R_1 R_2}{(R_1+R_2)L}t\right) u(t)\right]$$

**Problem 2.44**

(a) The condition for stability is

$$\begin{aligned}\int_{-\infty}^{\infty} |h_1(t)|\, dt &= \int_{-\infty}^{\infty} |\exp(-\alpha t)\cos(2\pi f_0 t)\, u(t)|\, dt \\ &= \int_0^{\infty} \exp(-\alpha t)\, |\cos(2\pi f_0 t)|\, dt < \int_0^{\infty} \exp(-\alpha t)\, dt = \frac{1}{\alpha} < \infty\end{aligned}$$

which follows because $|\cos(2\pi f_0 t)| \leq 1$.

(b) The condition for stability is

$$\begin{aligned}\int_{-\infty}^{\infty} |h_2(t)|\, dt &= \int_{-\infty}^{\infty} |\cos(2\pi f_0 t)\, u(t)|\, dt \\ &= \int_0^{\infty} |\cos(2\pi f_0 t)|\, dt \to \infty\end{aligned}$$

which follows by integrating one period of $|\cos(2\pi f_0 t)|$ and noting that the total integral is the limit of one period of area times $N$ as $N \to \infty$.

(c) The condition for stability is

$$\begin{aligned}\int_{-\infty}^{\infty} |h_3(t)|\, dt &= \int_{-\infty}^{\infty} \frac{1}{t} u(t-1)\, dt \\ &= \int_1^{\infty} \frac{dt}{t} = \ln(t)\big|_1^{\infty} \to \infty\end{aligned}$$

**Problem 2.49**

(a) Amplitude distortion; no phase distortion.

(b) No amplitude distortion; phase distortion.

(c) No amplitude distortion; no phase distortion.

(d) No amplitude distortion; no phase distortion.

**Problem 2.50**

The transfer function corresponding to this impulse response is

$$H(f) = \frac{2}{3 + j2\pi f} = \frac{2}{\sqrt{9 + (2\pi f)^2}} \exp\left[-j \tan\left(\frac{2\pi f}{3}\right)\right]$$

The group delay is

$$\begin{aligned}T_g(f) &= -\frac{1}{2\pi}\frac{d}{df}\left[-\tan\left(\frac{2\pi f}{3}\right)\right] \\ &= \frac{3}{9 + (2\pi f)^2}\end{aligned}$$

The phase delay is

$$T_p(f) = -\frac{\theta(f)}{2\pi f} = \frac{\tan\left(\frac{2\pi f}{3}\right)}{2\pi f}$$

**Problem 2.51**
The group and phase delays are, respectively,

$$\begin{aligned} T_g(f) &= \frac{0.1}{1+(0.2\pi f)^2} - \frac{0.333}{1+(0.667\pi f)^2} \\ T_p(f) &= \frac{1}{2\pi f}\left[\tan(0.2\pi f) - \tan(0.667\pi f)\right] \end{aligned}$$

**Problem 2.55**
Write the transfer function as

$$H(f) = H_0 e^{-j2\pi f t_0} - H_0 \Pi\left(\frac{f}{2B}\right) e^{-j2\pi f t_0}$$

Use the inverse Fourier transform of a constant, the delay theorem, and the inverse Fourier transform of a rectangular pulse function to get

$$h(t) = H_0 \delta(t - t_0) - 2BH_0 \text{sinc}\left[2B(t - t_0)\right]$$

**Problem 2.58**
(a) 0.5 seconds; (b) and (c) - use sketches to show.

**Problem 2.61**
The lowpass recovery filter can cut off in the range $1.9^+$ kHz to $2.1^-$ kHz.

**Problem 2.62**
For bandpass sampling and recovery, all but (b) and (e) will work theoretically, although an ideal filter with bandwidth exactly equal to the unsampled signal bandwidth is necessary. For lowpass sampling and recovery, only (f) will work.

**Problem 3.2**

Multiplying the AM signal

$$x_c(t) = A_c[1 + am_n(t)] \cos \omega_c t$$

by $x_c(t) = A_c[1 + am_n(t)] \cos \omega_c t$ and lowpass filtering to remove the double frequency $(2\omega_c)$ term yields

$$y_D(t) = A_c[1 + am_n(t)] \cos \theta(t)$$

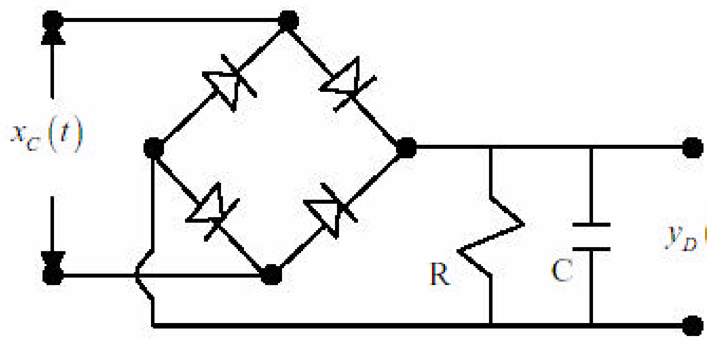


Figure 3.1:

For negligible demodulation phase error, $\theta(t) \approx 0$, this becomes

$$y_D(t) = A_c + A_c am_n(t)$$

The dc component can be removed resulting in $A_c am_n(t)$, which is a signal proportional to the message, $m(t)$. This process is not generally used in AM since the reason for using AM is to avoid the necessity for coherent demodulation.

**Problem 3.4**

| Part | $\langle m_n^2(t) \rangle$ | $a = 0.4$ | $a = 0.6$ | $a = 1$ |
|---|---|---|---|---|
| a | 1/3 | $E_{ff} = 5,1\%$ | $E_{ff} = 10.7\%$ | $E_{ff} = 25\%$ |
| b | 1/3 | $E_{ff} = 5.1\%$ | $E_{ff} = 10.7\%$ | $E_{ff} = 25\%$ |
| c | 1 | $E_{ff} = 13.8\%$ | $E_{ff} = 26.5\%$ | $E_{ff} = 50\%$ |

**Problem 3.5**

By inspection, the normalized message signal is as shown in Figure 3.3.
Thus

$$m_n(t) = \frac{2}{T}t, \qquad 0 \le t \le \frac{T}{2}$$

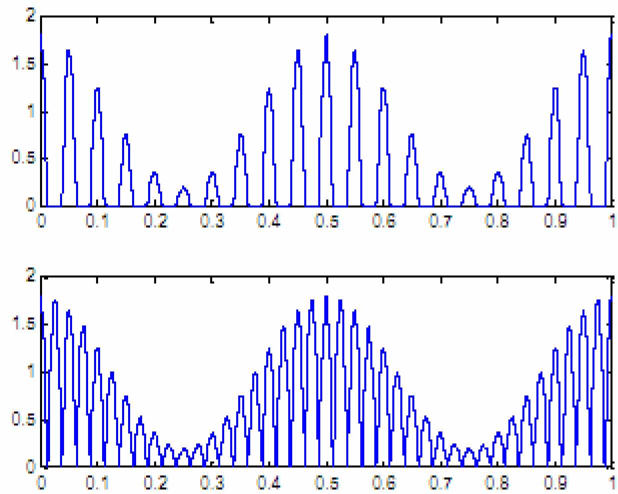

Figure 3.2:

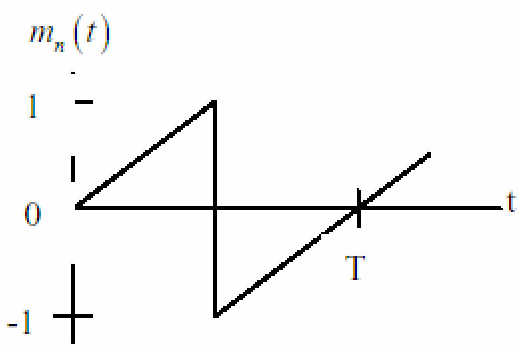


Figure 3.3:

and

$$\langle m_n^2(t)\rangle = \frac{2}{T}\int_0^{T/2}\left(\frac{2}{T}t\right)^2 dt = \frac{2}{T}\left(\frac{2}{T}\right)^2\frac{1}{3}\left(\frac{T}{2}\right)^3 = \frac{1}{3}$$

Also

$$\begin{aligned} A_c[1+a] &= 40 \\ A_c[1-a] &= 10 \end{aligned}$$

This yields

$$\frac{1+a}{1-a} = \frac{40}{10} = 4$$

or

$$\begin{aligned} 1+a &= 4-4a \\ 5a &= 3 \end{aligned}$$

Thus

$$a = 0.6$$

Since the index is 0.6, we can write

$$A_c[1+0.6] = 40$$

This gives

$$A_c = \frac{40}{1.6} = 25$$

This carrier power is

$$P_c = \frac{1}{2}A_c^2 = \frac{1}{2}(25)^2 = 312.5 \quad \text{Watts}$$

The efficiency is

$$E_{ff} = \frac{(0.6)^2\left(\frac{1}{3}\right)}{1+(0.6)^2\left(\frac{1}{3}\right)} = \frac{0.36}{3.36} = 0.107 = 10.7\%$$

Thus

$$\frac{P_{sb}}{P_c + P_{sb}} = 0.107$$

where $P_{sb}$ represents the power in the sidebands and $P_c$ represents the power in the carrier. The above expression can be written

$$P_{sb} = 0.107 + 0.107P_{sb}$$

This gives

$$P_{sb} = \frac{0.107}{1.0-0.107}P_c = 97.48 \quad \text{Watts}$$

## Problem 3.10

$$A = 14.14 \qquad B = 8.16 \qquad a = 1.1547$$

## Problem 3.11

The modulator output

$$x_c(t) = 20\cos 2\pi (150)\, t + 6\cos 2\pi (160)\, t + 6\cos 2\pi (140)\, t$$

is

$$x_c(t) = 20\left[1 + \frac{12}{20}\cos 2\pi (10)\, t\right]\cos 2\pi (150)\, t$$

Thus, the modulation index, $a$, is

$$a = \frac{12}{20} = 0.6$$

The carrier power is

$$P_c = \frac{1}{2}(20)^2 = 200 \quad \text{Watts}$$

and the sideband power is

$$P_{sb} = \frac{1}{2}(6)^2 + \frac{1}{2}(6)^2 = 36 \quad \text{Watts}$$

Thus, the efficiency is

$$E_{ff} = \frac{36}{200 + 36} = 0.1525$$