Problem 3.7

(a) The first step in the solution to this problem is to plot $m(t)$, or use a root-finding algorithm in order to determine the minimum value of $m(t)$. We find that the minimum of $m(t) = -11.9523$, and the minimum falls at $t = 0.0352$ and $t = 0.0648$. Thus the normalized message signal is

$$m_n(t) = \frac{1}{11.9523}\left[9\cos 20\pi t - 7\cos 60\pi t\right]$$

With the given value of $c(t)$ and the index a, we have

$$x_c(t) = 100\left[1 + 0.5m_n(t)\right]\cos 200\pi t$$

This yields

$$\begin{aligned} x_c(t) &= -14.6415\cos 140\pi t + 18.8428\cos 180\pi t \\ &\quad +100\cos 200\pi t \\ &\quad +18.8248\cos 220\pi t - 14.6415\cos 260\pi t \end{aligned}$$

We will need this later to plot the spectrum.

(b) The value of $\left\langle m_n^2(t)\right\rangle$ is

$$\left\langle m_n^2(t)\right\rangle = \left(\frac{1}{11.9523}\right)^2\left(\frac{1}{2}\right)\left[(9)^2 + (7)^2\right] = 0.455$$

(c)This gives the efficiency

$$E = \frac{(0.5)^2(0.455)}{1 + (0.5)^2(0.455)} = 10.213\%$$

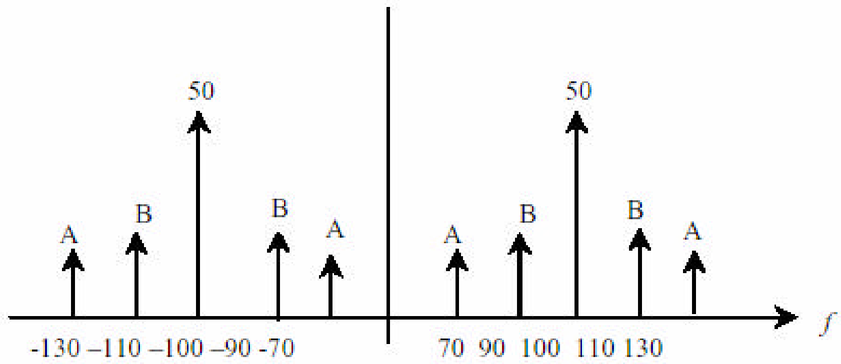

Figure 3.4:

(d) The two-sided amplitude spectrum is shown in Figure 3.4.
where

$$A = \frac{14.4615}{2} = 7.2307$$

and

$$B = \frac{18.8248}{2} = 9.4124$$

The phase spectrum results by noting that A is negative and all other terms are positive.
(e) By inspection the signal is of the form

$$x_c(t) = a(t)c(t)$$

where $a(t)$ is lowpass and $c(t($ is highpass. The spectra of $a(t)$ and $c(t)$ are do not overlap. Thus the Hilbert transform of $c(t)$ is

$$\widehat{x}_c(t) = a(t)\widehat{c}(t)$$

in which $c(t) = 100\cos 200\pi t$. This the envelope is

$$e(t) = 100\sqrt{a^2(t)\cos^2 200\pi t + a^2(t)\sin^2 200\pi t} = 100a(t)$$

where

$$a(t) = [1 + 0.5m_n(t)]$$

Problem 3.8

(a) $m_n(t) = \frac{1}{16}\left[9\cos 20\pi t + 7\cos 60\pi t\right]$
(b) $\langle m_n^2(t)\rangle = \left(\frac{1}{16}\right)^2\left(\frac{1}{2}\right)\left[(9)^2 + (7)^2\right] = 0.2539$
(c) $E_{ff} = \frac{0.25(0.2539)}{1+0.25(0.2539)} = 0.05969 = 5.969\%$
(d) The expression for $x_c(t)$ is

$$\begin{aligned} x_c(t) &= 100\left[1 + \frac{1}{32}(9\cos 20\pi t + \cos 60\pi t)\right]\cos 200\pi t \\ &= 10.9375\cos 140\pi t + 14.0625\cos 180\pi t \\ &\quad +100\cos 200\pi t \\ &\quad +14.0625\cos 220\pi t + 10.9375\cos 260\pi t \end{aligned}$$

Note that all terms are positive so the phase spectrum is everywhere zero. The amplitude spectrum is identical to that shown in the previous problem except that

$$\begin{aligned} A &= \frac{1}{2}(10.9375) = 5.46875 \\ B &= \frac{1}{2}(14.0625) = 7.03125 \end{aligned}$$

(e) As in the previous problem, the signal is of the form

$$x_c(t) = a(t)c(t)$$

where $a(t)$ is lowpass and $c(t($ is highpass. The spectra of $a(t)$ and $c(t)$ are do not overlap. Thus the Hilbert transform of $c(t)$ is

$$\widehat{x}_c(t) = a(t)\widehat{c}(t)$$

where $c(t) = 100\cos 200\pi t$. This the envelope is

$$\begin{aligned} e(t) &= 100\sqrt{a^2(t)\cos^2 200\pi t + a^2(t)\sin^2 200\pi t} \\ &= 100a(t) = 100\left[1 + \frac{1}{32}(9\cos 20\pi t + \cos 60\pi t)\right] \end{aligned}$$

Problem 3.9
The modulator output

$$x_c(t) = 40\cos 2\pi(200)t + 4\cos 2\pi(180)t + 4\cos 2\pi(220)t$$

can be written

$$x_c(t) = [40 + 8\cos 2\pi(20)t]\cos 2\pi(200)t$$

or

$$x_c(t) = 40\left[1 + \frac{8}{40}\cos 2\pi(20)t\right]\cos 2\pi(200)t$$

By inspection, the modulation index is

$$a = \frac{8}{40} = 0.2$$

Since the component at 200 Hertz represents the carrier, the carrier power is

$$P_c = \frac{1}{2}(40)^2 = 800 \quad \text{Watts}$$

The components at 180 and 220 Hertz are sideband terms. Thus the sideband power is

$$P_{sb} = \frac{1}{2}(4)^2 + \frac{1}{2}(4)^2 = 16 \quad \text{Watts}$$

Thus, the efficiency is

$$E_{ff} = \frac{P_{sb}}{P_c + P_{sb}} = \frac{16}{800 + 16} = 0.0196 = 1.96\%$$

Problem 3.12

(a) By plotting $m(t)$ or by using a root-finding algorithm we see that the minimum value of $m(t)$ is $M = -3.432$. Thus

$$m_n(t) = 0.5828\cos(2\pi f_m t) + 0.2914\cos(4\pi f_m t) + 0.5828\cos(10\pi f_m t)$$

The AM signal is

$$\begin{aligned} x_c(t) &= A_c[1 + 0.7m_n(t)]\cos 2\pi f_c t \\ &= 0.2040A_c\cos 2\pi(f_c - 5f_m)t \\ &\quad +0.1020A_c\cos 2\pi(f_c - 2f_m)t \\ &\quad +0.2040A_c\cos 2\pi(f_c - f_m)t \\ &\quad +A_c\cos 2\pi f_c t \\ &\quad +0.2040A_c\cos 2\pi(f_c + f_m)t \\ &\quad +0.1020A_c\cos 2\pi(f_c + 2f_m)t \\ &\quad +0.2040A_c\cos 2\pi(f_c + 5f_m)t \end{aligned}$$

The spectrum is drawn from the expression for $x_c(t)$. It contains 14 discrete components as shown

Comp	Freq	Amp	Comp	Freq	Amp
1	$-f_c - 5f_m$	$0.102A_c$	8	$f_c - 5f_m$	$0.102A_c$
2	$-f_c - 2f_m$	$0.051A_c$	9	$f_c - 2f_m$	$0.051A_c$
3	$-f_c - f_m$	$0.102A_c$	10	$f_c - f_m$	$0.102A_c$
4	$-f_c$	$0.5A_c$	11	f_c	$0.5A_c$
5	$-f_c + f_m$	$0.102A_c$	12	$f_c + f_m$	$0.102A_c$
6	$-f_c + 2f_m$	$0.051A_c$	13	$f_c + 2f_m$	$0.051A_c$
7	$-f_c + 5f_m$	$0.102A_c$	14	$f_c + 5f_m$	$0.102A_c$

(b) The efficiency is 15.8%.

Problem 3.13

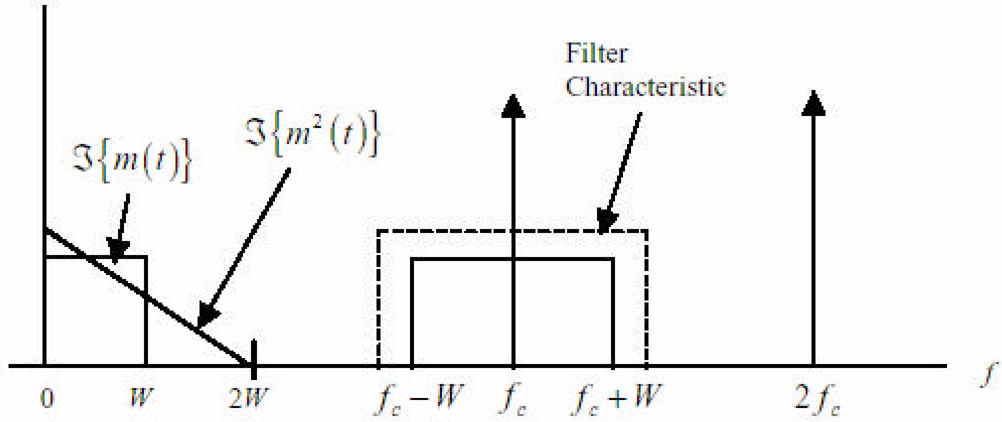

Figure 3.5:

(a) From Figure 3.75

$$x(t) = m(t) + \cos \omega_c t$$

With the given relationship between $x(t)$ and $y(t)$ we can write

$$y(t) = 4\{m(t) + \cos \omega_c t\} + 10\{m(t) + \cos \omega_c t\}^2$$

which can be written

$$y(t) = 4m(t) + 4\cos \omega_c t + 10m^2(t) + 20m(t)\cos \omega_c t + 5 + 5\cos 2\omega_c t$$

The equation for $y(t)$ is more conveniently expressed

$$y(t) = 5 + 4m(t) + 10m^2(t) + 4[1 + 5m(t)]\cos \omega_c t + 5\cos 2\omega_c t$$

(b) The spectrum illustrating the terms involved in $y(t)$ is shown in Figure 3.5. The center frequency of the filter is f_c and the bandwidth must be greater than or equal to $2W$. In addition, $f_c - W > 2W$ or $f_c > 3W$, and $f_c + W < 2f_c$. The last inequality states that $f_c > W$, which is redundant since we know that $f_c > 3W$.

(c) From the definition of $m(t)$ we have

$$m(t) = Mm_n(t)$$

so that

$$g(t) = 4[1 + 5Mm_n(t)]\cos \omega_c t$$

It follows that

$$a = 0.8 = 5M$$

Thus

$$M = \frac{0.8}{5} = 0.16$$

(d) This method of forming a DSB signal avoids the need for a multiplier.

Problem 3.22
By definition

$$x_c(t) = A_c \cos[\omega_c t + k_p m(t)] = A_c \cos[\omega_c t + k_p u(t - t_0)]$$

The waveforms for the three values of k_p are shown in Figure 3.7. The top pane is for $k_p = \pi$, the middle pane is for $k_p = -\pi/2$, and the bottom pane is for $k_p = \pi/4$.

Problem 3.33
The frequency deviation in Hertz is the plot shown in Fig. 3.76 with the ordinate values multiplied by 25.The phase deviation in radians is given

$$\phi(t) = 2\pi f_d \int^t m(\alpha)\, d\alpha = 50\pi \int^t m(\alpha)\, d\alpha$$

For $0 \leq t \leq 1$, we have

$$\phi(t) = 50\pi \int_0^t 2\alpha d\alpha = 50\pi t^2$$

For $1 \leq t \leq 2$

$$\begin{aligned} \phi(t) &= \phi(1) + 50\pi \int_1^t (5 - \alpha)\, d\alpha = 50\pi + 250\pi(t-1) - 25\pi\left(t^2 - 1\right) \\ &= -175\pi + 250\pi t - 25\pi t^2 \end{aligned}$$

For $2 \leq t \leq 3$

$$\phi(t) = \phi(2) + 50\pi \int_2^t 3d\alpha = 225\pi + 150\pi(t-2)$$

For $3 \leq t \leq 4$

$$\phi(t) = \phi(3) + 50\pi \int_3^t 2d\alpha = 375\pi + 100\pi(t-3)$$

Finally, for $t > 4$ we recognize that $\phi(t) = \phi(4) = 475\pi$. The required figure results by plotting these curves.

Problem 3.34
The frequency deviation in Hertz is the plot shown in Fig. 3.77 with the ordinate values multiplied by 10. The phase deviation is given by

$$\phi(t) = 2\pi f_d \int^t m(\alpha)\, d\alpha = 20\pi \int^t m(\alpha)\, d\alpha$$

For $0 \leq t \leq 1$, we have

$$\phi(t) = 20\pi \int_0^t \alpha d\alpha = 10\pi t^2$$

For $1 \le t \le 2$

$$\begin{aligned}\phi(t) &= \phi(1) + 20\pi \int_1^t (\alpha - 2)\, d\alpha = 10\pi + 10\pi\left(t^2 - 1\right) - 40\pi(t-1) \\ &= 10\pi\left(t^2 - 4t + 4\right) = 10\pi\,(t-2)^2\end{aligned}$$

For $2 \le t \le 4$

$$\begin{aligned}\phi(t) &= \phi(2) + 20\pi \int_2^t (6 - 2\alpha) d\alpha = 0 + 20\pi\,(6)\,(t-2) - 20\pi\left(t^2 - 4\right) \\ &= -20\pi(t^2 - 6t + 8)\end{aligned}$$

Finally, for $t > 4$ we recognize that $\phi(t) = \phi(4) = 0$. The required figure follows by plotting these expressions.

Problem 3.30

(a) Since the carrier frequency is 1000 Hertz, the general form of $x_c(t)$ is

$$x_c(t) = A_c \cos\left[2\pi\,(1000)\,t + \phi(t)\right]$$

The phase deviation, $\phi(t)$, is therefore given by

$$\phi(t) = 20t^2 \qquad \text{rad}$$

The frequency deviation is

$$\frac{d\phi}{dt} = 40t \qquad \text{rad/sec}$$

or

$$\frac{1}{2\pi}\frac{d\phi}{dt} = \frac{20}{\pi}t \qquad \text{Hertz}$$

(b) The phase deviation is

$$\phi(t) = 2\pi\,(500)\,t^2 - 2\pi\,(1000)\,t \qquad \text{rad}$$

and the frequency deviation is

$$\begin{aligned}\frac{d\phi}{dt} &= 4\pi\,(500)\,t - 2\pi\,(1000) \qquad \text{rad/sec} \\ &= 2000\pi\,(t-1) \qquad \text{rad/sec}\end{aligned}$$

or

$$\frac{1}{2\pi}\frac{d\phi}{dt} = 1000\,(t-1) \qquad \text{Hertz}$$

(c) The phase deviation is

$$\phi(t) = 2\pi(100)t = 200\pi t \qquad \text{rad}$$

and the frequency deviation is

$$\frac{d\phi}{dt} = 200\pi \qquad \text{rad/sec}$$

or

$$\frac{1}{2\pi}\frac{d\phi}{dt} = 100 \qquad \text{Hertz}$$

which should be obvious from the expression for $x_c(t)$.
(d) The phase deviation is

$$\phi(t) = 200\pi t + 10\sqrt{t} \qquad \text{rad}$$

and the phase deviation is

$$\frac{d\phi}{dt} = 200\pi + \frac{1}{2}(10)t^{-\frac{1}{2}} = 200\pi + \frac{5}{\sqrt{t}} \qquad \text{rad/sec}$$

or

$$\frac{1}{2\pi}\frac{d\phi}{dt} = 100 + \frac{5}{2\pi\sqrt{t}} \qquad \text{Hertz}$$

Problem 3.31
(a) The phase deviation is

$$\phi(t) = 2\pi(30)\int_0^t (8)dt = 480\pi t, \qquad t \le 4$$

The maximum phase deviation is $\phi(4) = 480\pi(4) = 1920\pi$. The required plot is simply

$$\phi(t) = \begin{cases} 0, & t < 0 \\ 480\pi t, & 0 \le t < 4 \\ 1920\pi, & t \ge 4 \end{cases}$$

(b) The frequency deviation, in Hz, is

$$\frac{1}{2\pi}\frac{d\phi}{dt} = 30m(t) = \begin{matrix} 0, & t < 0 \\ 240, & 0 \le t < 4 \\ 0 & t \ge 4 \end{matrix}$$

The required sketch follows simply.
(c) The peak frequency deviation is $8f_d = 8\,(30) = 240$ Hertz
(d) The peak phase deviation is

$$2\pi\,(30)\int_0^4 (8)\,dt = 2\pi\,(30)\,(8)\,(4) = 1920\pi \qquad \text{rad}$$

The modulator output power is

$$P = \frac{1}{2}A_c^2 = \frac{1}{2}\,(100)^2 = 5000 \qquad \text{Watts}$$

Problem 3.32
(a) The message signal is

$$m\,(t) = \begin{cases} 0, & t < 4 \\ t-4, & 4 \le t \le 6 \\ 8-t & 6 \le t \le 8 \\ 0, & t > 8 \end{cases}$$

The phase deviation is

$$\begin{aligned} \phi\,(t) &= 2\pi f_d \int_4^t (t-4)\,dt \\ &= 60\pi\,(-4)\,(t-4) + 30\pi\left(t^2-16\right) \\ &= 30\pi\left(t^2-8t+16\right), \quad 4 \le t \le 6 \end{aligned}$$

$$\begin{aligned} \phi\,(t) &= 2\pi f_d \int_6^t (8-t)\,dt + \phi\,(6) \\ &= 120\pi + 60\pi\,(8)\,(t-6) - 30\pi\left(t^2-36\right) \\ &= 120\pi - 30\pi\left(t^2-16t+60\right), \quad 6 \le t \le 8 \end{aligned}$$

Also

$$\begin{aligned} \phi\,(t) &= 240\pi, \quad t > 8 \\ \phi\,(t) &= 0, \quad t < 4 \end{aligned}$$

The sketches follow immediately from the equations.
(b) The frequency deviation in Hertz is $30m(t)$.
(c) The peak phase deviation $= 240\pi$ rad.
(d) The peak frequency deviation $= 120$ Hertz.

(e) The carrier power is, assuming a sufficiently high carrier frequency,

$$P = \frac{1}{2}A_c^2 = \frac{1}{2}\,(100)^2 = 5000 \qquad \text{Watts}$$

Problem 3.35
The frequency deviation in Hertz is the plot shown in Fig. 3.78 with the ordinate values multiplied by 5. The phase deviation in radians is given by

$$\phi(t) = 2\pi f_d \int^t m(\alpha)\, d\alpha = 10\pi \int^t m(\alpha)\, d\alpha$$

For $0 \le t \le 1$, we have

$$\phi(t) = 10\pi \int_0^t (-2\alpha) d\alpha = -10\pi t^2$$

For $1 \le t \le 2$

$$\phi(t) = \phi(1) + 10\pi \int_1^t 2 d\alpha = -10\pi + 20\pi(t-1) = 10\pi(2t-3)$$

For $2 \le t \le 2.5$

$$\begin{aligned} \phi(t) &= \phi(2) + 10\pi \int_2^t (10-4\alpha) d\alpha = 10\pi + 10\pi(10)(t-2) - 10\pi(2)\left(t^2-4\right) \\ &= 10\pi(-2t^2+10t-11) \end{aligned}$$

For $2.5 \le t \le 3$

$$\begin{aligned} \phi(t) &= \phi(2.5) - 10\pi \int_{2.5}^t 2 d\alpha = 15\pi - 20\pi(t-2.5) \\ &= 10\pi(-2t+5.5) \end{aligned}$$

For $3 \le t \le 4$

$$\begin{aligned} \phi(t) &= \phi(3) + 10\pi \int_3^t (2\alpha - 8)\, d\alpha = 5\pi + 10\pi(t^2-9) - 10\pi(8)(t-3) \\ &= 10\pi(t^2 - 8t + 15.5) \end{aligned}$$

Finally, for $t > 4$ we recognize that $\phi(t) = \phi(4) = -5\pi$. The required figure follows by plotting these expressions.

Problem 3.36
(a) The peak deviation is $(12.5)(4) = 50$ and $f_m = 10$. Thus, the modulation index is $\frac{50}{10} = 5$.
(b) The magnitude spectrum is a Fourier-Bessel spectrum with $\beta = 5$. The $n = 0$ term falls at 1000 Hz and the spacing between components is 10 Hz. The sketch is that of Figure 3.24 in the text.
(c) Since β is not $\ll 1$, this is not narrowband FM. The bandwidth exceeds $2f_m$.
(d) For phase modulation, $k_p(4) = 5$ or $k_p = 1.25$.

Problem 3.40

(a) Peak frequency deviation = 80 Hz

(b) $\phi(t) = 8\sin(20\pi t)$

(c) $\beta = 8$

(d) $P_i = 50$ Watts, $P_0 = 16.76$ Watts

(e) The spectrum of the input signal is a Fourier_Bessel spectrum with $\beta = 8$. The $n = 0$ term is at the carrier frequency of 500 Hz and the spacing between components is 10 Hz. The output spectrum consistes of the $n = 0$ term and three terms each side of the $n = 0$ term. Thus the output spectrum has terms at 470, 480, 490, 500, 510, 520 and 530 Hz.

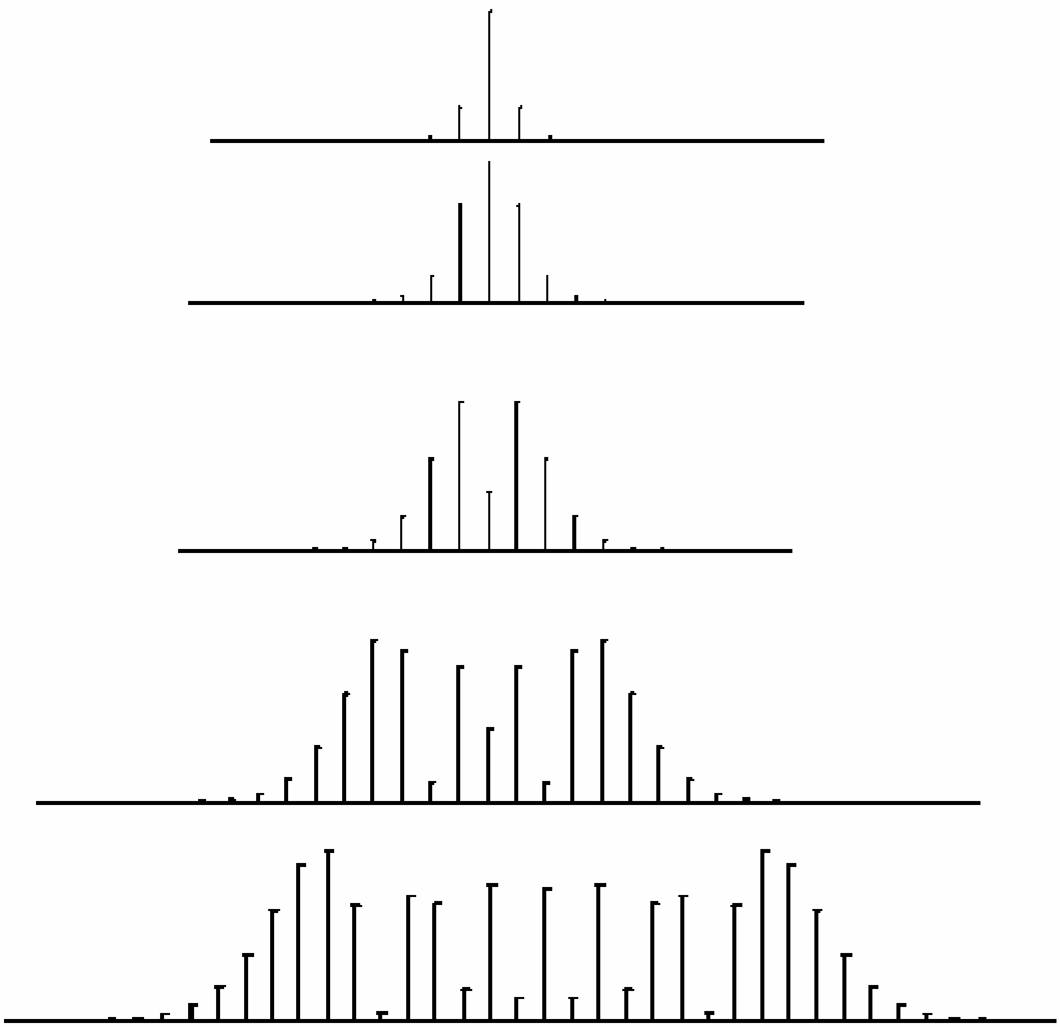

Figure 3.10:

Problem 3.41

The required spectra are given in Figure 3.10. The modulation indices are, from top to bottom, $\beta = 0.5$, $\beta = 1$, $\beta = 2$, $\beta = 5$, and $\beta = 10$.

Problem 3.42

We wish to find k such that

$$P_r = J_0^2(10) + 2\sum_{n=1}^{k} J_0^2(10) \geq 0.80$$

This gives $k = 9$, yielding a power ratio of $P_r = 0.8747$. The bandwidth is therefore

$$B = 2kf_m = 2(9)(150) = 2700 \text{ Hz}$$

For $P_r \geq 0.9$, we have $k = 10$ for a power ratio of 0.9603. This gives

$$B = 2kf_m = 2(10)(150) = 3000 \text{ Hz}$$