1 a $x_1[n]$

$$X_1[k] = \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

b. $$X_2[k] = \begin{bmatrix} W_3^0 & W_3^0 & W_3^0 \\ W_3^0 & W_3^1 & W_3^2 \\ W_3^0 & W_3^2 & W_3^4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -.5 - j\,2.598 \\ -.5 + j\,2.598 \end{bmatrix}$$

c. $$X_3[k] = \begin{bmatrix} 2 \\ .5 - j\,2.598 \\ .5 + j\,2.598 \end{bmatrix}$$

d $$X_4[k] = \begin{bmatrix} 6 \\ 2.545 - j\,2.127 \\ -3.045 + j\,1.314 \\ -3.045 - j\,1.314 \\ 2.545 + j\,2.127 \end{bmatrix}$$

2. a. $X(e^{j\omega}) = 1 + 2e^{-j\omega} - e^{-j2\omega}$

b. same as 1.c

c. $X[k] = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{3}}$

5.28 Real Valued a, d
Im Valued b

5.34 $N = 10$ $x[n]$ is real, so $X[k] = X^*[\langle -k \rangle_N]$

a. $X[0] = \sum_{n=0}^{9} x(n) = 10$

b. $X[6] = \sum_{n=0}^{9} (-1)^n x(n) = 0$

c. $\sum_{k=0}^{9} X[k] = 10\, x(0) = -30$

d. $\sum_{k=0}^{9} e^{-j\frac{2\pi k}{5}} X[k]$. The inverse DFT of $e^{-j\frac{2\pi k}{5}}$ is

$x[\langle n-2 \rangle_{10}]$, so $\sum_{k=0}^{9} e^{-j\frac{2\pi k}{5}} X[k] = 10\, x[\langle 0-2 \rangle_{10}] = 10\, x[8] = -100$

e. From Parsevals Theorem $\sum_{k=0}^{9} |X[k]|^2 = 10 \sum_{n=0}^{9} |x[n]|^2 = 3860$

5.38 $X[k] = X^*[\langle -k \rangle_9]$ Error in the problem $X[8] = 4.5 + j1.6$

$X[1] = X^*[\langle -1 \rangle_9] = X^*[8] = 4.5 - j1.6$

$X[4] = X^*[\langle -4 \rangle_9] = X^*[5] = -3.1 - j8.2$

$X[6] = X^*[3] = -7.2 + j4.1$

$X[7] = X^*[2] = 1.2 + j2.3$

5.39 Similar to 5.38

5.36 $y_c[n] = \sum_{k=0}^{6} g[k]\, h[\langle n-k \rangle_7]$ $\qquad y_L[n] = \sum_{k=0}^{6} g[k]\, h[n-k]$

$y_c[0] = g[0]h[0] + g[1]h[6] + g[2]h[5] + g[3]h[4] + g[4]h[3] + g[5]h[2] + g[6]h[1]$

$y_L[0] = g[0]h[0]$

$y_L[6] = g[0]\cancel{h[7]}^{0} + g[1]h[6] + g[2]h[5] + g[3]h[4] + g[4]h[3] + g[5]h[2] + g[6]h[1]$

So $y_c[0] = y_L[0] + y_L[6]$. Similar for other $\underline{n}$.

5.45 a) $y_L[n] = \{-4 \quad 10 \quad -6 \quad 8 \quad 7 \quad -3\}$

b. $y_c[n] = g[n] \; ④ \; h[n] = \sum_{k=0}^{3} g[k]\, h[\langle n-k \rangle_4]$

$y_c[0] = g[0]h[0] + g[1]h[3] + g[2]h[2] + g[3]h[1] = 3$

$y_c[1] = g[0]h[1] + g[1]h[0] + g[2]h[3] + g[3]h[2] = 7$

$y_c[2] = -6$

$y_c[3] = 8$

c. $G[k] = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^0 & W_4^9 \end{bmatrix} \begin{bmatrix} g[0] \\ g[1] \\ g[2] \\ g[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1+j \\ 6 \\ -1-j \end{bmatrix}$

$H[k] = \begin{bmatrix} 3 \\ -4-j5 \\ -3 \\ -4+j5 \end{bmatrix}$ $\qquad G[k]H[k] = \begin{bmatrix} 12 \\ 9+j \\ -18 \\ 9-j \end{bmatrix}$

$$y_c[n] = \frac{1}{4}\begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^{-1} & W_4^{-2} & W_4^{-3} \\ W_4^0 & W_4^{-2} & W_4^{-4} & W_4^{-6} \\ W_4^0 & W_4^{-3} & W_4^{-6} & W_4^{-9} \end{bmatrix}\begin{bmatrix} 12 \\ 9+j \\ -18 \\ 9-j \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -6 \\ 8 \end{bmatrix}$$

d. same as part a

1. a. $x[n] * h[n] = \{ \underset{\uparrow}{2} \;\; 8 \;\; 11 \;\; 7 \;\; 2 \}$

b. $x[n] \;④\; h[n] = \{4 \;\; 8 \;\; 7 \;\; 11\}$

c. $x[n] \;⑤\; h[n] = \{2 \;\; 8 \;\; 11 \;\; 7 \;\; 2\}$

5.17 $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$

$$Y[k] = \sum_{n=0}^{MN-1} x[n]\, W_{MN}^{kn} = \sum_{n=0}^{N-1} x[n]\, W_{MN}^{kn} \qquad x[n]=0 \quad n \geq N.$$

so, $Y[kM] = \sum_{n=0}^{N-1} x[n]\, W_{MN}^{kMn} = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} = X[k]$

since $W_{MN}^{kMn} = e^{-j\frac{2\pi k M n}{MN}} = e^{-j\frac{2\pi k n}{N}} = W_N^{kn}$.

1. $x[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

a. $X(e^{j\omega}) = 1 + 2e^{-j\omega} + e^{-j2\omega}$

b. $X[k] = \begin{bmatrix} 4 \\ -.5 - j.866 \\ -.5 + j.866 \end{bmatrix} \qquad N = 3$

c. $X[k] = \begin{bmatrix} 4 \\ 1.5 - j2.598 \\ -.5 - j.866 \\ 0 \\ -.5 + j.866 \\ 1.5 + j2.598 \end{bmatrix} \qquad N = 6$

3. $x[n]$ has length 100

a. $X[10]$ samples $X(e^{j\omega})$ at $\omega_k = \frac{2\pi k}{N}$

$$\omega_{10} = \frac{2\pi(10)}{100} = .2\pi \ \frac{rad}{sam}$$

b. $X[11]$ = samples $X(e^{j\omega})$ at $\omega_{11} = \frac{2\pi(11)}{100} = .22\pi \ \frac{rad}{sam}$

c. Spacing is $.02\pi \ \frac{rad}{sam}$ $\left(\frac{2\pi}{N}\right)$

d. $\frac{2\pi k}{N} = 1.8 \ \frac{rad}{sam} \Rightarrow k = 28.65$.

Since k must be an integer, $k = 29$ $\omega_{29} = 1.822 \ \frac{rad}{sam}$

4 a. $x[n] = x_a(t) = \big|_{t=nT} = A\cos\left(\frac{20}{30}\pi n\right)$

$$\omega_0 = \frac{2}{3}\pi \ \frac{rad}{sam}$$

$|X(e^{j\omega})|$ will have a peak at $\frac{2\pi}{3} \ \frac{rad}{sam}$

b. $\omega_k = \frac{2\pi k}{N}$ $\quad \frac{2\pi}{3} = \frac{2\pi k}{100} \Rightarrow k = 33.3$

rounding to nearest integer $k = 33$

$|X[k]|$ will have a peak at $k = 33$

5. $F_s = 1\,kHz$ $\quad N = 200$

a. $K = 59$ $\quad \omega_{59} = \frac{2\pi k}{N} = \frac{2\pi(59)}{200} = .59\pi \ \frac{rad}{sam}$

Since $x[n] = x_a(t)\big|_{t=nT}$ $\quad \omega_0 = \Omega_0 T = \frac{\Omega_0}{F_s}$ or

$\Omega_0 = \omega_0 F_s$

$$\Omega = (.59\pi)(1k) = 590\pi \ \frac{rad}{sec} \Rightarrow 295\,Hz.$$

b. $k = 159$ $\qquad \omega_{159} = \frac{2\pi(159)}{200} = 1.59\pi \ \frac{rad}{sam}$

but $\cos(1.59\pi n) = \cos(-.41\pi n) = \cos(.41\pi n)$

$\Omega = (.41\pi)(1k) = 410\pi \ \frac{rad}{sec} \Rightarrow 205 \text{ Hz}.$

(Using $\Omega = (1.59\pi)(1k) = 1590\pi \ \frac{rad}{sec} \Rightarrow 795 \text{Hz}$ does not give the correct answer. If we sample at 1kHz, we can only have frequencies to 500Hz)

1. a. $x[n] \Leftrightarrow X[z] = 1$ $\qquad$ ROC = all z plane

b. $3\delta[n-1] \Leftrightarrow 3z^{-1}$ $\qquad$ ROC $|z| > 0$

c. $.1\delta[n] + .3\delta[n-1] + .1\delta[n-2] \Leftrightarrow .1 + .3z^{-1} + .1z^{-2}$ $\qquad$ ROC $|z| > 0$

d. $.3^n \mu[n] \Leftrightarrow \frac{1}{1 - .3z^{-1}}$ $\qquad |z| > .3$

e. $-.3^n \mu[-n-1] \Leftrightarrow \frac{1}{1 - .3z^{-1}}$ $\qquad |z| < .3$

6.2 c) $r^n \sin(\omega_0 n)\mu[n] = r^n \frac{1}{2j}\left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right)\mu[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} \frac{1}{2j} r^n \left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right)\mu[n] z^{-n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left(re^{j\omega_0} z^{-1}\right)^n - \frac{1}{2j} \sum_{n=0}^{\infty} \left(re^{-j\omega_0} z^{-1}\right)^n$$

$$= \frac{1}{2j} \frac{1}{1 - rz^{-1}e^{j\omega_0}} - \frac{1}{2j} \frac{1}{1 - re^{-j\omega_0}z^{-1}}$$

$\left|re^{j\omega_0}z^{-1}\right| < 1$

$|z| > |r|$

$$= \frac{1}{2j}\left[\frac{(1-rz^{-1}e^{-j\omega_0}) - (1-rz^{-1}e^{j\omega_0})}{(1-rz^{-1}e^{j\omega_0})(1-rz^{-1}e^{-j\omega_0})}\right]$$

$$= \frac{1}{2j}\left[\frac{-rz^{-1}(-2j\sin(\omega_0))}{1-rz^{-1}e^{-j\omega_0}-rz^{-1}e^{j\omega_0}+(rz^{-1})^2}\right]$$

$$= \frac{rz^{-1}\sin(\omega_0)}{1+2rz^{-1}\cos(\omega_0)+(rz^{-1})} \qquad |z|>|r|$$

6.3. d) $x_4[n] = \alpha^n \mu[-n]$

$$X_4(z) = \sum_{n=-\infty}^{\infty} x_4[n]z^{-n} = \sum_{n=-\infty}^{\infty} \alpha^n \mu[-n]\, z^{-n} = \sum_{n=-\infty}^{0} \alpha^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (\alpha^{-1}z)^n = \frac{1}{1-\alpha^{-1}z} \qquad |\alpha^{-1}z|<1$$

$$|z|<|\alpha|$$

6.5

a) i) $|z|>.3$ ii) $|z|>.7$ iii) $|z|>.4$ iv) $|z|<.4$

b. i) $ROC_{Y_1} = ROC_{X_1} \cap ROC_{X_2} = |z|>.7$

ii) $|z|>.4$ iii) $.3<|z|<.4$ iv) $|z|>.7$ v) ϕ

vi) ϕ

6.7

a. $X_1(z) = \sum_{n=-\infty}^{\infty} x_1[n] z^{-n} = \sum_{n=-\infty}^{\infty} (\alpha^n+\beta^n)\mu[n+2]\, z^{-n}$

$$= \sum_{n=-2}^{\infty} (\alpha z^{-1})^n + \sum_{n=-2}^{\infty} (\beta z^{-1})^n = \frac{(\alpha z^{-1})^{-2}}{1-\alpha z^{-1}} + \frac{(\beta z^{-1})^{-2}}{1-\beta z^{-1}}$$

$|z|>|\alpha| \cap |z|>|\beta| = |z|>|\beta|$

b. $X_2(z) = \sum_{n=-\infty}^{\infty} x_2[n] z^{-n} = \sum_{n=-\infty}^{\infty} \left(\alpha^n\mu[-n-2] + \beta^n \mu[n-1]\right) z^{-n}$

$$= \sum_{n=-\infty}^{-2} (\alpha z^{-1})^n + \sum_{n=1}^{\infty}(\beta z^{-1})^n$$

$$= \sum_{n=2}^{\infty}(\alpha^{-1}z)^n + \sum_{n=1}^{\infty}(\beta z^{-1})^n = \frac{1}{1-\alpha^{-1}z} + \frac{1}{1-\beta z^{-1}}$$

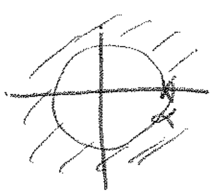
$|\alpha^{-1}z|<1$ $\quad$ $|\beta z^{-1}|<1$

$|\beta|<|z|<|\alpha| \Rightarrow \phi$

6.14 $x_b[n] = \alpha^n\left(\mu[n]-\mu[n-8]\right)$ $\quad |\alpha|<1$

$X_b(z) = \sum_{n=\infty}^{\infty} x_b[n]z^{-n} = \sum_{n=-\infty}^{\infty}\alpha^n\left(\mu[n]-\mu[n-8]\right)z^{-n}$

$$= \sum_{n=0}^{7} (\alpha^1 z^{-1})^n = \frac{1-(\alpha z^{-1})^8}{1-\alpha z^{-1}} \qquad |z|>|\alpha|$$



$x_c[n] \Leftrightarrow X_c(z) = \sum_{n=\infty}^{\infty} n\alpha^n\mu[n]z^{-n} + \sum_{n=-\infty}^{\infty}\alpha^n\mu[n]z^{-n}$

$=$

$$= \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} + \frac{1}{1-\alpha z^{-1}} \qquad |z| > |\alpha|$$

6.8 $$G(z) = \frac{(z^2 + .2z + .1)(z^2 - z + .5)}{(z^2 + .3z - .18)(z^2 - 2z + 4)}$$

$G(z)$ has 4 poles

$-.6$

$.3$

$1 + j1.732$

$1 - j1.732$

The possible ROC are

$|z| < .3$ — left sided inverse (sequence goes from $-\infty$ to -1)

$.3 < |z| < .6$ — two sided inverse

$.6 < |z| < 2$ — two sided inverse

$|z| > 2$ — right sided inverse (0 to ∞)

6.25 $X_1(z) = \log(1 - \alpha z^{-1}) \qquad |z| > \alpha$

Expanding $\log(1-\alpha z^{-1})$ in a power series we get

$$X_1(z) = -\alpha z^{-1} - \frac{\alpha^2 z^{-2}}{2} - \frac{\alpha^3 z^{-3}}{3} + \cdots = -\sum_{n=1}^{\infty} \frac{\alpha^n}{n} z^{-n}$$

so $\quad x_1[n] = -\frac{\alpha^n}{n}\mu[n-1]$

6.20

$$X_a(z) \quad \frac{3z}{z^2+.3z-.18} = \frac{3z}{(z+.6)(z-.3)}$$

Two poles $z = -.6, .3$

The possible ROC are $|z|<.3$

$.3<|z|<.6$

$|z|>.6$

$$X_a(z) = A_0 + \frac{A_1 z}{z+.6} + \frac{A_2 z}{z-.3}$$

$$A_0 = X_a(z)\Big|_{z=0} = 0$$

$$A_1 = X_a(z)\left.\frac{z+.6}{z}\right|_{z=-.6} = \left.\frac{3}{z-.3}\right|_{z=-.6} = \frac{3}{-.6-.3} = \frac{3}{-.9} = -3.33$$

$$A_2 = X_a(z)\left.\frac{z-.3}{z}\right|_{z=.3} = \left.\frac{3}{z+.6}\right|_{z=.3} = \frac{3}{.3+.6} = \frac{3}{.9} = 3.33$$

$$X_a(z) = \frac{-3.33}{1+.6z^{-1}} + \frac{3.33}{1-.3z^{-1}}$$

$\underline{|z|<.3}$

$$x_a[n] = 3.33(-.6)^n \mu[-n-1] - 3.33(.3)^n \mu[-n-1]$$

$\underline{.3<|z|<.6}$

$$x_a[n] = 3.33(-.6)^n \mu[-n-1] + 3.33(.3)^n \mu[n]$$

$\underline{|z|>.6}$

$$x_a[n] = -3.33(-.6)^n \mu[n] + 3.33(.3)^n \mu[n]$$

6.27 Table 6.2

Conjugation

$$g[n] \Leftrightarrow G(z) = \sum_{n=-\infty}^{\infty} g[n] z^{-n}$$

$$\sum_{n=-\infty}^{\infty} g^*[n] z^{-n} = \left[\sum_{n=-\infty}^{\infty} g[n](z^{-n})^*\right]^* = G^*(z^*) \Leftrightarrow g^*[n]$$

Time Reversal

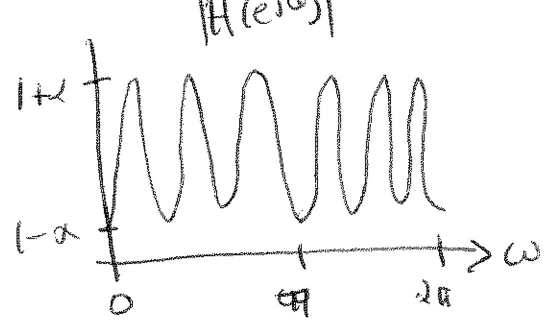
$$\sum_{n=-\infty}^{\infty} g[-n] z^{-n} = \sum_{n=+\infty}^{-\infty} g[n] z^{n} = \sum_{n=-\infty}^{\infty} g[n] \left(\tfrac{1}{z}\right)^{-n} = G\left(\tfrac{1}{z}\right)$$

3.51 $h[n] = \delta[n] - \alpha\,\delta[n-R] \qquad |\alpha|<1$

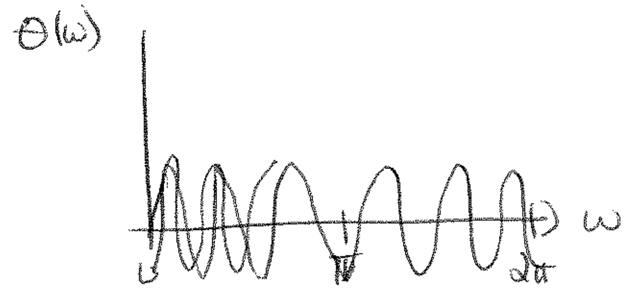
$H(z) = 1 - \alpha z^{-R}$ ROC whole z plane

$H(e^{j\omega}) = 1 - \alpha e^{-j\omega R}$

$R=6$



6 peaks



6 peaks

3.56 $H_1(z) = 1 + az^{-1} + bz^{-2}$

$H_2(z) = \dfrac{1}{1 - cz^{-1}} \qquad |z| > |c|$

$H_3(z) = \dfrac{1}{1 - dz^{-1}} \qquad |z| > |d|$

$H_{eq}(z) = H_1(z)\,H_2(z)\,H_3(z) = \dfrac{1 + az^{-1} + bz^{-2}}{(1 - cz^{-1})(1 - dz^{-1})}$

$= \dfrac{1 + az^{-1} + bz^{-2}}{1 - (c+d)z^{-1} + cdz^{-2}}$

$H_{eq}(e^{j\omega}) = \dfrac{1 + ae^{-j\omega} + be^{-j2\omega}}{1 - (c+d)e^{-j\omega} + cde^{-j2\omega}}$

for $|H_{eq}(e^{j\omega})| = 1$

$a = -c + d$

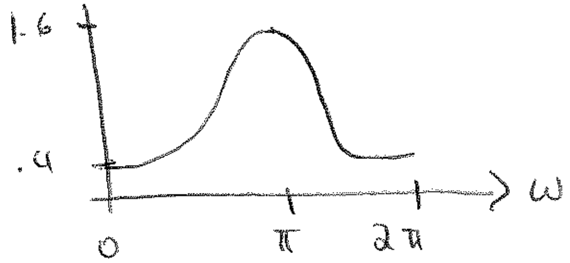
$b = cd$

3.54 $h[n] = a_1 \delta[n] + a_2 \delta[n-1] + a_3 \delta[n-2] + a_4 \delta[n-3]$
$+ a_5 \delta[n-4]$

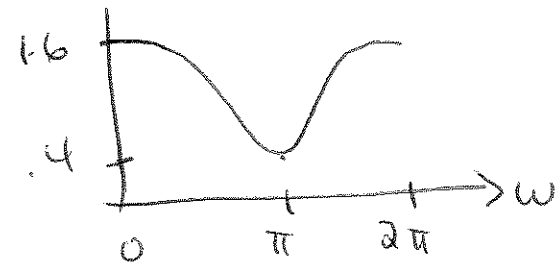
To have linear phase $h[n] = h[2\tau_0 - n]$
(if $h(n)$ is real)

so $a_1 = a_5$
$a_2 = a_4$

3.75 a. $|H_A(e^{j\omega})|$



$|H_D(e^{j\omega})|$



b. $h_c[n] = (-1)^n h_A[n] = e^{j\pi n} h_A[n]$

$H_c(e^{j\omega}) = \sum_{n=-\infty}^{\infty} e^{j\pi n} h_A[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} h_A[n] e^{-j(\omega-\pi)n}$

$= H_A(e^{j(\omega-\pi)})$ — H_c is H_A shifted by π.

3.83 $h[n] = (-.5)^n \mu[n]$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} = \frac{1}{1+.5e^{-j\omega}}$$

$$\left|H(e^{j\frac{\pi}{5}})\right| = .6969 \qquad \theta\left(\frac{\pi}{5}\right) = .2063$$

$$x[n] = \sin\left(\frac{\pi}{5}n\right)\mu[n] \qquad y[n] = .6969 \sin\left(\frac{\pi}{5}n + .2063\right)\mu[n]$$

1. $h_1[n] = .3\delta[n] + .8\delta[n-1] + .3\delta[n-2]$

a. $H_1(e^{j\omega}) = .3 + .8e^{-j\omega} + .3e^{-j2\omega}$

$$= e^{-j\omega}\left[.3e^{j\omega} + .8 + .3e^{-j\omega}\right] = \left[.8 + .6\cos(\omega)\right]e^{-j\omega}$$

2. $h_2[n] = .2\delta[n] + .8(.5)^n \mu[n]$

$$H_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_2[n] e^{-j\omega n} = .2 + .8\sum_{n=0}^{\infty}(.5)^n e^{-j\omega n}$$

$$= .2 + .8\,\frac{1}{1-.5e^{-j\omega}}$$

$$\left|H_2(e^{j\omega})\right| = \frac{\sqrt{\left[1-.1\cos(\omega)\right]^2 + \left[.1\sin(\omega)\right]^2}}{\sqrt{\left[1-.5\cos(\omega)\right]^2 + \left[.5\sin(\omega)\right]^2}}$$

$$\theta_2(\omega) = \tan^{-1}\left(\frac{.1\sin(\omega)}{1-.1\cos(\omega)}\right) - \tan^{-1}\left(\frac{.5\sin(\omega)}{1-.5\sin(\omega)}\right)$$

1. a. linear phase
   b. not linear phase
   c. not linear phase
   d. linear phase

2. a. $\theta(\omega) = -\omega \qquad \tau(\omega) = 1$

   b. $$\theta(\omega) = -\tan^{-1}\left(\frac{.3\sin(\omega)}{1-.3\sin(\omega)}\right)$$

   $$\tau(\omega) = \frac{.3\cos(\omega)\left[-1-.3\cos(\omega)\right] + .3^2\sin^2(\omega)}{\left[1-.3\cos(\omega)\right]^2 + \left[.3\sin(\omega)\right]^2}$$

   c. $$H(e^{j\omega}) = .2 - .3e^{-j\omega} - .2e^{-j2\omega}$$

   $$= e^{-j\omega}\left(.2e^{j\omega} - .3 - .2e^{-j\omega}\right) = \left(j2(.2)\sin(\omega) - .3\right)e^{-j\omega}$$

   $$= e^{-j\omega}\left(j(.4)\sin(\omega) - .3\right)$$

   $$\theta(\omega) = -\omega + \tan^{-1}\left(\frac{.4\sin(\omega)}{-.3}\right)$$

   d. $\theta(\omega) = -1.5\omega$

   $\tau(\omega) = 1.5$