INEL 4075: Fundamentos de Ingeniería Eléctrica

Primer semestre 2006-2007
Nayda G. Santiago

Libro: Introduction to Electric Circuits, 6th Edition
Problemas del capítulo 3

Hacer

Ejercicios
3.4-1, 3.4-2, 3.4-3, 3.4-4
3.7-1, 3.7-2, 3.7-3

Problemas
3.3-1
3.3-2
3.3-3
3.3-5
3.3-6
3.3-7
3.3-9
3.3-10

} Estos problemas para el examen 1

3.4-1, 3.4-2, 3.4-3, 3.4-4
3.5-1, 3.5-2, 3.5-3, 3.5-5
3.7-1, 3.7-2

Example 3.4-2

For the circuit of Figure 3.4-4*a*, find the current measured by the ammeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

Solution

Figure 3.4-4*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m. Applying KVL gives

$$15 + 5i_m + 10i_m = 0$$

The current measured by the ammeter is

$$i_m = -\frac{15}{5+10} = -1\text{ A}$$

(Why is i_m negative? Why can't we just divide the source voltage by the equivalent resistance? Recall that when we use Ohm's law, the voltage and current must adhere to the passive convention. In this case, the current calculated by dividing the source voltage by the equivalent resistance does not have the same reference direction as i_m, and so we need a minus sign.)

The total power absorbed by the two resistors is

$$p_R = 5i_m^2 + 10i_m^2 = 15(1^2) = 15\text{ W}$$

The power supplied by the source is

$$p_s = -v_s\, i_m = -15(-1) = 15\text{ W}$$

Thus, the power supplied by the source is equal to that absorbed by the series connection of resistors.

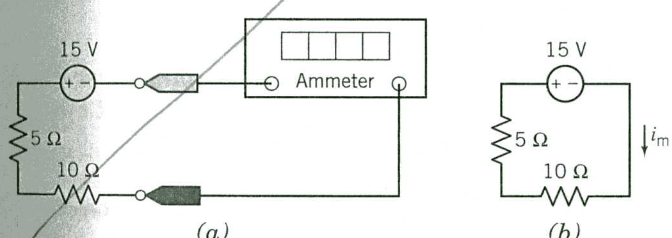

FIGURE 3.4-4 (*a*) A circuit containing series resistors. (*b*) The circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

Try It Yourself! More Problems and Worked Examples Are in the Electric Circuit Study Applets

Exercise 3.4-1 For the circuit of Figure E 3.4-1, find the voltage v_3 and the current i and show that the power delivered to the three resistors is equal to that supplied by the source.
Answer: $v_3 = 3$ V, $i = 1$ A

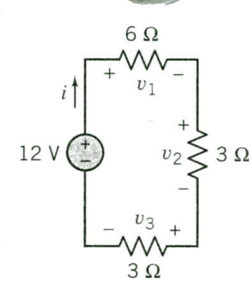

FIGURE E 3.4-1 Circuit with three series resistors (for Exercise 3.4-1).

Exercise 3.4-2 Consider the voltage divider shown in Figure E 3.4-2 when $R_1 = 6\ \Omega$. It is desired that the output power absorbed by $R_1 = 6\ \Omega$ be 6 W. Find the voltage v_o and the required source v_s.
Answer: $v_s = 14$ V, $v_o = 6$ V

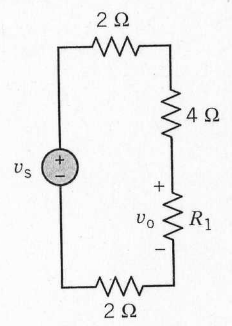

FIGURE E 3.4-2
Voltage divider for Exercise 3.4-2.

Exercise 3.4-3 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.4-3*a*.

Hint: Figure E 3.4-3*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Answer: $v_m = 2$ V

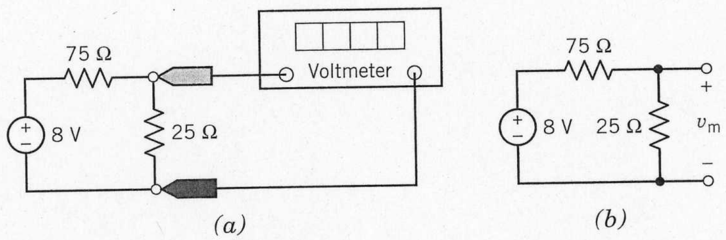

FIGURE E 3.4-3
(*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Exercise 3.4-4 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.4-4*a*.

Hint: Figure E 3.4-4*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Answer: $v_m = -2$ V

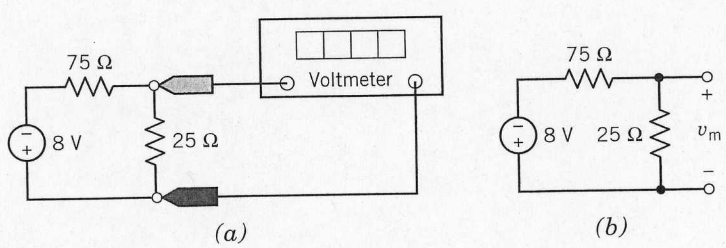

FIGURE E 3.4-4
(*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

3.5 Parallel Resistors and Current Division

Circuit elements, such as resistors, are connected in *parallel* when the voltage across each element is identical. The resistors in Figure 3.5-1 are connected in *parallel*. Notice, for example, that resistors R_1 and R_2 are each connected to both node a and node b. Consequently, $v_1 = v_2$, so both resistors have the same voltage. A similar argument shows that resistors R_2 and R_3 are also connected in parallel. Noticing that R_2 is connected in parallel with both R_1 and R_3, we say that all three resistors are connected in parallel. The order of parallel resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.5-1 will not change if we interchange the positions R_2 and R_3.

In Figure 3.7-5*c* the series combination of the two 20-Ω resistors has been replaced with the equivalent 40-Ω resistor. Now the 40-Ω resistor is parallel to the 10-Ω resistor. The equivalent resistance is

$$\frac{40 \cdot 10}{40 + 10} = 8\,\Omega$$

In Figure 3.7-5*d* the parallel combination of the 40-Ω and 10-Ω resistors has been replaced with the equivalent 8-Ω resistor. Thus, the ohmmeter measures a resistance equal to 8 Ω.

Try It Yourself! More Problems and Worked Examples Are in the **Electric Circuit Study Applets**

In general, we may find the equivalent resistance (or conductance) for a portion of a circuit consisting only of resistors and then replace that portion of the circuit with the equivalent resistance. For example, consider the circuit shown in Figure 3.7-6. We use $R_{\text{eq x-y}}$ to denote the equivalent resistance seen looking into terminals x–y. We note that the equivalent resistance to the right of terminals c–d is

$$R_{\text{eq c-d}} = \frac{15(6+4)}{15+(6+4)} = \frac{150}{25} = 6\,\Omega$$

Then the equivalent resistance of the circuit to the right of terminals a–b is

$$R_{\text{eq a-b}} = 4 + R_{\text{eq c-d}} = 4 + 6 = 10\,\Omega$$

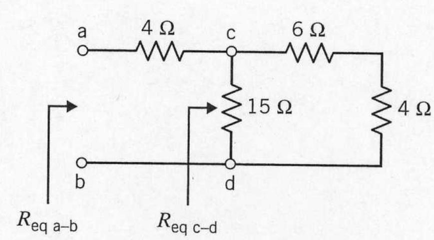

FIGURE 3.7-6
The equivalent resistance looking into terminals c–d is denoted as $R_{\text{eq c-d}}$.

Exercise 3.7-1 Determine the resistance measured by the ohmmeter in Figure E 3.7-1.

Answer: $\dfrac{(30+30)\cdot 30}{(30+30)+30} + 30 = 50\,\Omega$

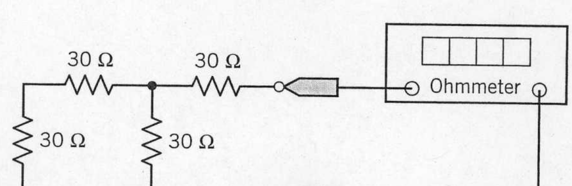

FIGURE E 3.7-1

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E 3.7-1.

Exercise 3.7-2 Determine the resistance measured by the ohmmeter in Figure E 3.7-2.

Answer: $12 + \dfrac{40 \cdot 10}{40 + 10} + 4 = 24\ \Omega$

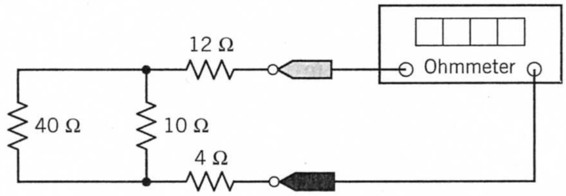

FIGURE E 3.7-2

Exercise 3.7-3 Determine the resistance measured by the ohmmeter in Figure E 3.7-3.

Answer: $\dfrac{(60 + 60 + 60) \cdot 60}{(60 + 60 + 60) + 60} = 45\ \Omega$

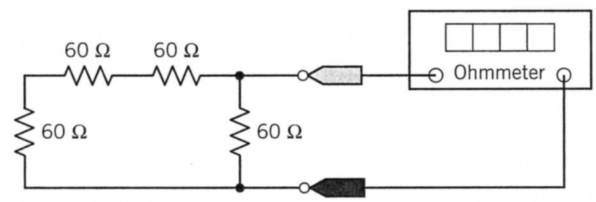

FIGURE E 3.7-3

3.8 Analyzing Resistive Circuits Using MATLAB

The computer program MATLAB is a tool for making mathematical calculations. In this section MATLAB is used to solve the equations encountered when analyzing a resistive circuit. Consider the resistive circuit shown in Figure 3.8-1*a*. The goal is to determine the value of the input voltage, V_s, required to cause the current I to be 1 A.

(Subscripts can't be used in the MATLAB input file. Thus V_s and R_p in Figure 3.8-1 become Vs and Rp in the MATLAB input file. We have been using lowercase letters to represent element voltages and currents, but in MATLAB examples we will use capital

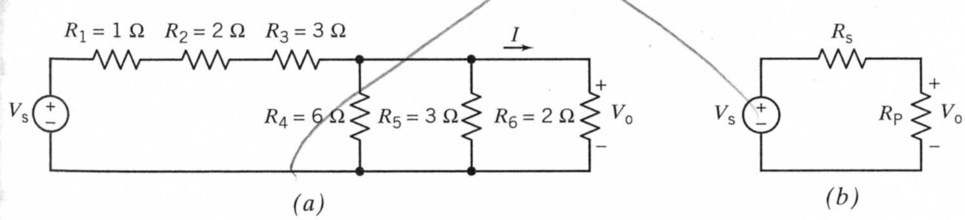

FIGURE 3.8-1
(*a*) A resistive circuit and (*b*) an equivalent circuit.

PROBLEMS

Section 3.3 Kirchhoff's Laws

P 3.3-1 Consider the circuit shown in Figure P 3.3-1. Determine the values of the power supplied by branch *B* and the power supplied by branch *F*.

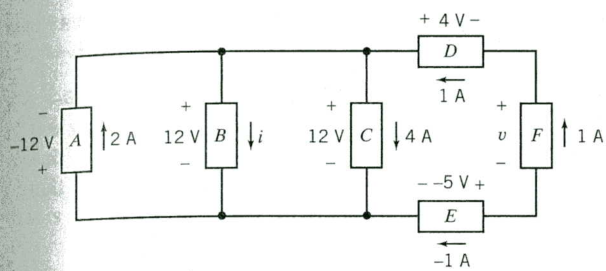

FIGURE P 3.3-1

P 3.3-2 Determine the values of i_2, i_4, v_2, v_3, and v_6 in Figure P 3.3-2.

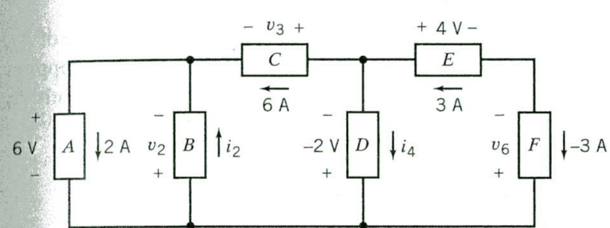

FIGURE P 3.3-2

P 3.3-3 Consider the circuit shown in Figure P 3.3-3.

(a) Suppose that $R_1 = 6\ \Omega$ and $R_2 = 3\ \Omega$. Find the current *i* and the voltage *v*.

(b) Suppose, instead, that $i = 1.5$ A and $v = 2$ V. Determine the resistances R_1 and R_2.

(c) Suppose, instead, that the voltage source supplies 24 W of power and that the current source supplies 9 W of power. Determine the current *i*, the voltage *v*, and the resistances R_1 and R_2.

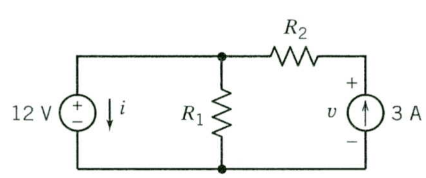

FIGURE P 3.3-3

P 3.3-4 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.3-4.
Answer: The 4-Ω resistor absorbs 100 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 72 W.

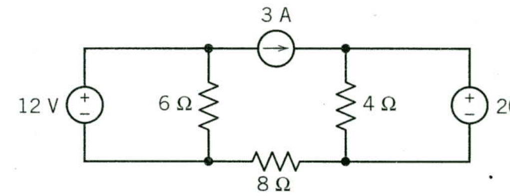

FIGURE P 3.3-4

P 3.3-5 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.3-5.
Answer: The 4-Ω resistor absorbs 16 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 8 W.

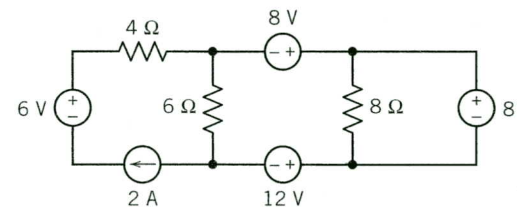

FIGURE P 3.3-5

P 3.3-6 Determine the power supplied by each current source in the circuit of Figure P 3.3-6.
Answer: The 2-mA current source supplies 6 mW and the 1-mA current source supplies −7 mW.

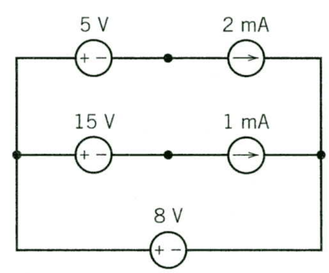

FIGURE P 3.3-6

P 3.3-7 Determine the power supplied by each voltage source in the circuit of Figure P 3.3-7.
Answer: The 2-V voltage source supplies 2 mW and the 3-V voltage source supplies −6 mW.

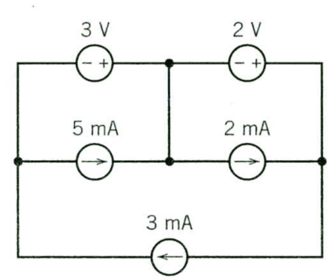

FIGURE P 3.3-7

P 3.3-8 What is the value of the resistance R in Figure P 3.3-8?
Hint: Assume an ideal ammeter. An ideal ammeter is equivalent to a short circuit.
Answer: $R = 4\ \Omega$

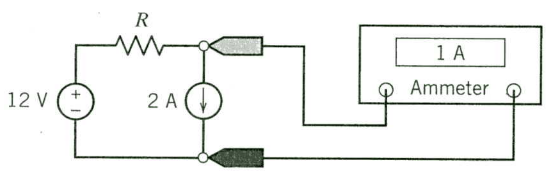

FIGURE P 3.3-8

P 3.3-9 The voltmeter in Figure P 3.3-9 measures the value of the voltage across the current source to be 56 V. What is the value of the resistance R?
Hint: Assume an ideal voltmeter. An ideal voltmeter is equivalent to an open circuit.
Answer: $R = 10\ \Omega$

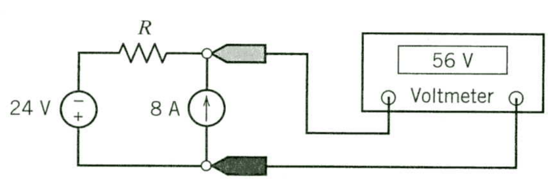

FIGURE P 3.3-9

P 3.3-10 Determine the values of the resistances R_1 and R_2 in Figure P 3.3-10.

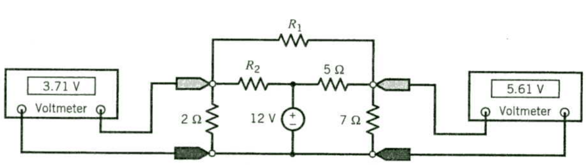

FIGURE P 3.3-10

Section 3.4 A Single-Loop Circuit—The Voltage Divider

P 3.4-1 Use voltage division to determine the voltages v_1, v_2, v_3, and v_4 in the circuit shown in Figure P 3.4-1.

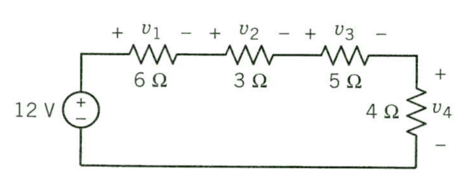

FIGURE P 3.4-1

P 3.4-2 Consider the circuits shown in Figure P 3.4-2.

(a) Determine the value of the resistance R in Figure P 3.4-2b that makes the circuit in Figure P 3.4-2b equivalent to the circuit in Figure P 3.4-2a.

(b) Determine the current i in Figure P 3.4-2b. Because the circuits are equivalent, the current i in Figure P 3.4-2a is equal to the current i in Figure P 3.4-2b.

(c) Determine the power supplied by the voltage source.

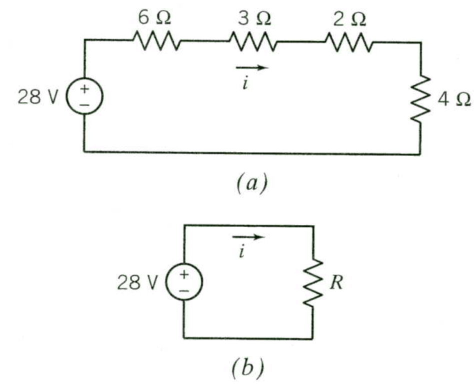

FIGURE P 3.4-2

P 3.4-3 The ideal voltmeter in the circuit shown in Figure P 3.4-3 measures the voltage v.

(a) Suppose $R_2 = 100\ \Omega$. Determine the value of R_1.

(b) Suppose, instead, $R_1 = 100\ \Omega$. Determine the value of R_2.

(c) Suppose, instead, that the voltage source supplies 1.2 W of power. Determine the values of both R_1 and R_2.

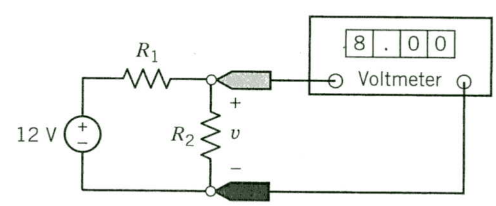

FIGURE P 3.4-3

P 3.4-4 Determine the voltage v in the circuit shown in Figure P 3.4-4.

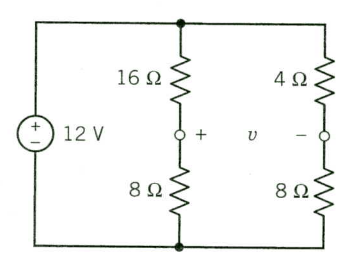

FIGURE P 3.4-4

P 3.4-5 The model of a cable and load resistor connected to a source is shown in Figure P 3.4-5. Determine the appropriate cable resistance, R, so that the output voltage, v_o, remains between 9 V and 13 V when the source voltage, v_s, varies between 20 V and 28 V. The cable resistance can only assume integer values in the range $20 < R < 100\ \Omega$.

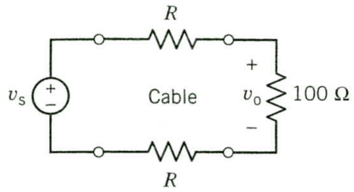

FIGURE P 3.4-5
Circuit with a cable.

P 3.4-6 The input to the circuit shown in Figure P 3.4-6 is the voltage of the voltage source, v_a. The output of this circuit is the voltage measured by the voltmeter, v_b. This circuit produces an output that is proportional to the input, that is

$$v_b = k\, v_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, v_b, when $R = 240\ \Omega$ and $v_a = 18$ V.

(b) Determine the value of the power supplied by the voltage source when $R = 240\ \Omega$ and $v_a = 18$ V.

(c) Determine the value of the resistance, R, required to cause the output to be $v_b = 2$ V when the input is $v_a = 18$ V.

(d) Determine the value of the resistance, R, required to cause $v_b = 0.2\, v_a$ (that is, the value of the constant of proportionality is $k = \frac{2}{10}$).

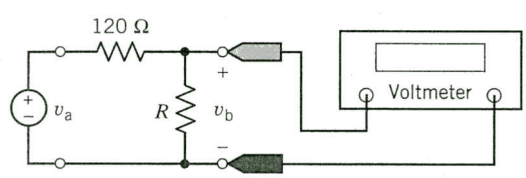

FIGURE P 3.4-6

Section 3.5 Parallel Resistors and Current Division

P 3.5-1 Use current division to determine the currents i_1, i_2, i_3, and i_4 in the circuit shown in Figure P 3.5-1.

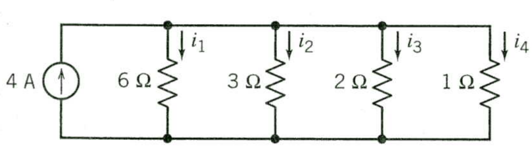

FIGURE P 3.5-1

P 3.5-2 Consider the circuits shown in Figure P 3.5-2.

(a) Determine the value of the resistance R in Figure P 3.5-2b that makes the circuit in Figure P 3.4-2b equivalent to the circuit in Figure P 3.5-2a.

(b) Determine the voltage v in Figure P 3.5-2b. Because the circuits are equivalent, the voltage v in Figure P 3.5-2a is equal to the voltage v in Figure P 3.5-2b.

(c) Determine the power supplied by the current source.

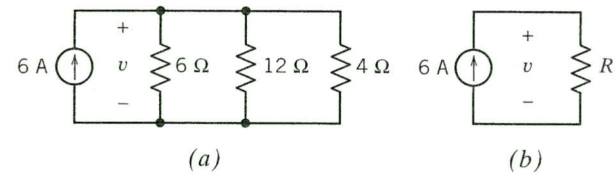

FIGURE P 3.5-2

P 3.5-3 The ideal voltmeter in the circuit shown in Figure P 3.5-3 measures the voltage v.

(a) Suppose $R_2 = 12\ \Omega$. Determine the value of R_1 and of the current i.

(b) Suppose, instead, $R_1 = 12\ \Omega$. Determine the value of R_2 and of the current i.

(c) Instead, choose R_1 and R_2 to minimize the power absorbed by any one resistor.

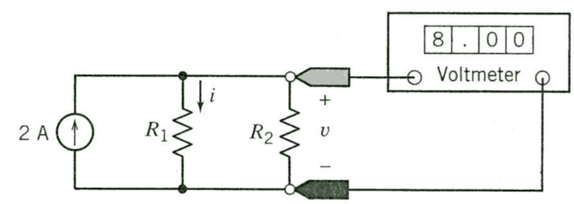

FIGURE P 3.5-3

P 3.5-4 Determine the current i in the circuit shown in Figure P 3.5-4.

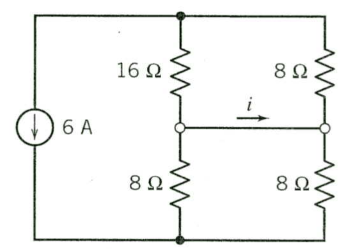

FIGURE P 3.5-4

P 3.5-5 Consider the circuit shown in Figure P 3.5-5 when $4\ \Omega \le R_1 \le 6\ \Omega$ and $R_2 = 10\ \Omega$. Select the source i_s so that v_o remains between 9 V and 13 V.

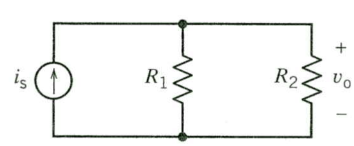

FIGURE P 3.5-5

P 3.5-6. The input to the circuit shown in Figure P 3.5-6 is the current of the current source, i_a. The output of this circuit is the current measured by the ammeter, i_b. This circuit produces an output that is proportional to the input, that is

$$i_b = k\, i_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, i_b, when $R = 24\ \Omega$ and $i_a = 1.8$ A.

(b) Determine the value of the resistance, R, required to cause the output to be $i_b = 1.6$ A when the input is $i_a = 2$ A.

(c) Determine the value of the resistance, R, required to cause $i_b = 0.4\, i_a$ (that is, the value of the constant of proportionality is $k = \frac{4}{10}$).

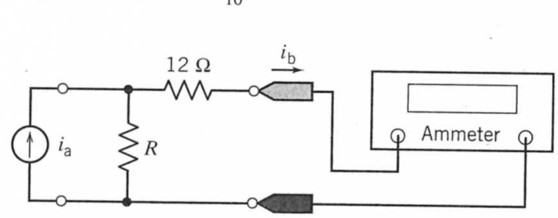

FIGURE P 3.5-6

Section 3.7 Circuit Analysis

P 3.7-1 The circuit shown in Figure P 3.7-1*a* has been divided into two parts. In Figure P 3.7-1*b*, the right-hand part has been replaced with an equivalent circuit. The left-hand part of the circuit has not been changed.

(a) Determine the value of the resistance R in Figure P 3.7-1*b* that makes the circuit in Figure P 3.7-1*b* equivalent to the circuit in Figure P 3.7-1*a*.

(b) Find the current i and the voltage v shown in Figure P 3.7-1*b*. Because of the equivalence, the current i and the voltage v shown in Figure P 3.7-1*a* are equal to the current i and the voltage v shown in Figure P 3.7-1*b*.

(c) Find the current i_2 shown in Figure P 3.7-1*a* using current division.

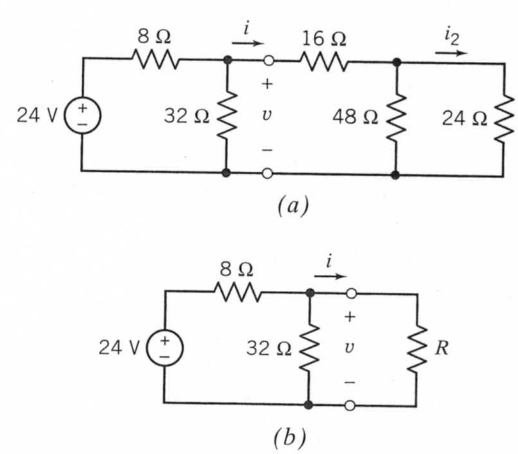

FIGURE P 3.7-1

P 3.7-2 The circuit shown in Figure P 3.7-2*a* has been divided into three parts. In Figure P 3.7-2*b*, the rightmost part has been replaced with an equivalent circuit. The rest of the circuit has not been changed. The circuit is simplified further in Figure 3.7-2*c*. Now the middle and rightmost parts have been replaced by a single equivalent resistance. The leftmost part of the circuit is still unchanged.

(a) Determine the value of the resistance R_1 in Figure P 3.7-2*b* that makes the circuit in Figure P 3.7-2*b* equivalent to the circuit in Figure P 3.7-2*a*.

(b) Determine the value of the resistance R_2 in Figure P 3.7-2*c* that makes the circuit in Figure P 3.7-2*c* equivalent to the circuit in Figure P 3.7-2*b*.

(c) Find the current i_1 and the voltage v_1 shown in Figure P 3.7-2*c*. Because of the equivalence, the current i_1 and the voltage v_1 shown in Figure P 3.7-2*b* are equal to the current i_1 and the voltage v_1 shown in Figure P 3.7-2*c*.

Hint: $24 = 6(i_1 - 2) + i_1 R_2$

(d) Find the current i_2 and the voltage v_2 shown in Figure P 3.7-2*b*. Because of the equivalence, the current i_2 and the voltage v_2 shown in Figure P 3.7-2*a* are equal to the current i_2 and the voltage v_2 shown in Figure P 3.7-2*b*.

Hint: Use current division to calculate i_2 from i_1.

(e) Determine the power absorbed by the 3-Ω resistance shown at the right of Figure P 3.7-2*a*.

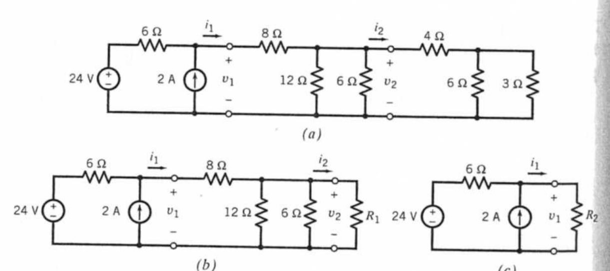

FIGURE P 3.7-2

P 3.7-3 Find i using appropriate circuit reductions and the current divider principle for the circuit of Figure P 3.7-3.

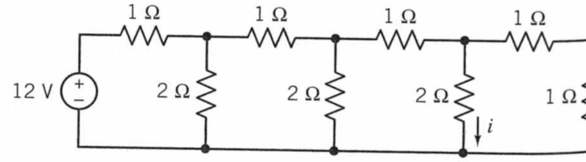

FIGURE P 3.7-3

P 3.7-4 (a) Determine values of R_1 and R_2 in Figure P 3.7-4*b* that make the circuit in Figure P 3.7-4*b* equivalent to the circuit in Figure P 3.7-4*a*.

(b) Analyze the circuit in Figure P 3.7-4*b* to determine the values of the currents i_a and i_b.

(c) Because the circuits are equivalent, the currents i_a and i_b shown in Figure P 3.7-4*b* are equal to the currents i_a and i_b shown in Figure P 3.7-4*a*. Use this fact to determine values of the voltage v_1 and current i_2 shown in Figure P 3.7-4*a*.

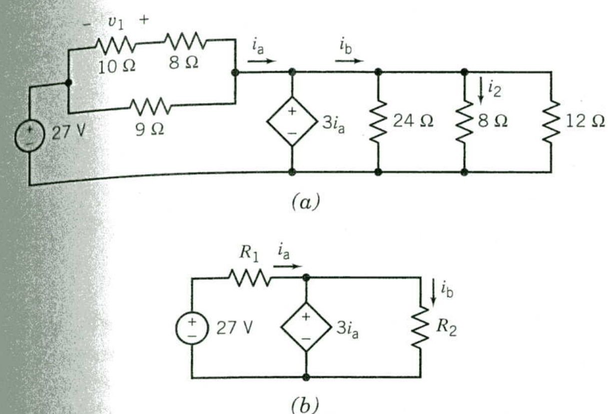

FIGURE P 3.7-4

P 3.7-5 The voltmeter in the circuit shown in Figure P 3.7-5 shows that the voltage across the 30-Ω resistor is 6 volts. Determine the value of the resistance R_1.
Hint: Use the voltage division twice.
Answer: $R_1 = 40\ \Omega$

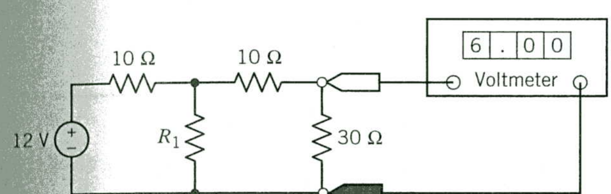

FIGURE P 3.7-5

P 3.7-6 Determine the voltages v_a and v_c and the currents i_b and i_d for the circuit shown in Figure P 3.7-6.
Answer: $v_a = -2$ V, $v_c = 6$ V, $i_b = -16$ mA, and $i_d = 2$ mA

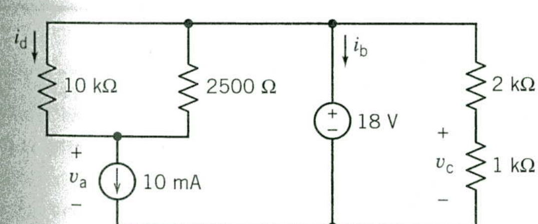

FIGURE P 3.7-6

P 3.7-7 Determine the value of the resistance R in Figure 3.7-7.
Answer: $R = 28$ kΩ

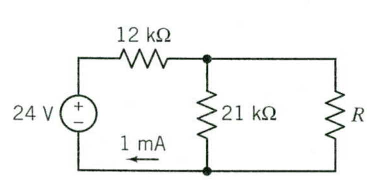

FIGURE P 3.7-7

P 3.7-8 Most of us are familiar with the effects of a mild electric shock. The effects of a severe shock can be devastating and often fatal. Shock results when current is passed through the body. A person can be modeled as a network of resistances. Consider the model circuit shown in Figure P 3.7-8. Determine the voltage developed across the heart and the current flowing through the heart of the person when he or she firmly grasps one end of a voltage source whose other end is connected to the floor. The heart is represented by R_h. The floor has resistance to current flow equal to R_f, and the person is standing barefoot on the floor. This type of accident might occur at a swimming pool or boat dock. The upper-body resistance R_u and lower-body resistance R_L vary from person to person.

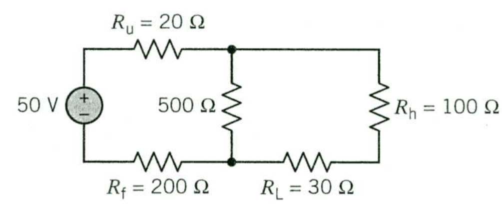

FIGURE P 3.7-8
The resistance of the heart. $R_h = 100\ \Omega$

P 3.7-9 Determine the value of the current i in Figure 3.7-9.
Answer: $i = 0.5$ mA

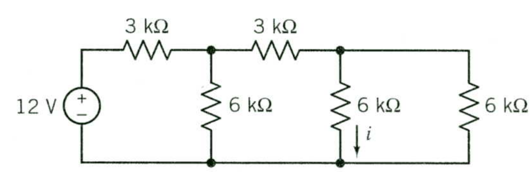

FIGURE P 3.7-9

P 3.7-10 Determine the values of i_a, i_b, and v_c in Figure P 3.7-10.

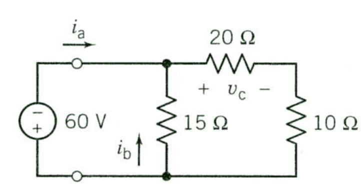

FIGURE P 3.7-10

P 3.7-11 Find i and $R_{eq\ a-b}$ if $v_{ab} = 40$ V in the circuit of Figure P 3.7-11.
Answer: $R_{eq\ a-b} = 8\ \Omega$, $i = 5/6$ A

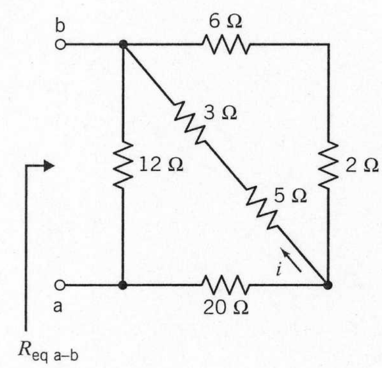

FIGURE P 3.7-11

P 3.7-12 The ohmmeter in Figure P 3.7-12 measures the equivalent resistance, R_{eq}, of the resistor circuit. The value of the equivalent resistance, R_{eq}, depends on the value of the resistance R.

(a) Determine the value of the equivalent resistance, R_{eq}, when $R = 18\ \Omega$.

(b) Determine the value of the resistance R required to cause the equivalent resistance to be $R_{eq} = 18\ \Omega$.

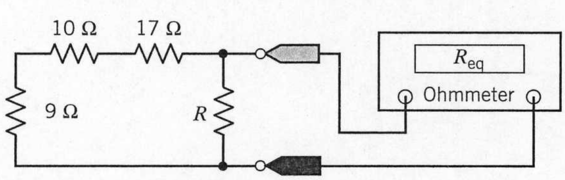

FIGURE P 3.7-12

P 3.7-13 The source $v_s = 240$ volts is connected to three equal resistors as shown in Figure P 3.7-13. Determine R when the voltage source delivers 1920 W to the resistors.
Answer: $R = 45\ \Omega$

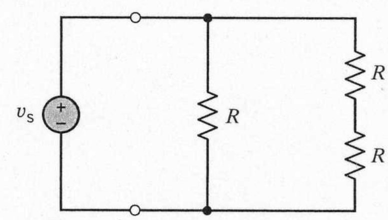

FIGURE P 3.7-13

P 3.7-14 Find the R_{eq} at terminals a–b in Figure P 3.7-14. Also determine i, i_1, and i_2.
Answer: $R_{eq} = 8\ \Omega$, $i = 5$ A, $i_1 = 5/3$ A, $i_2 = 5/2$ A

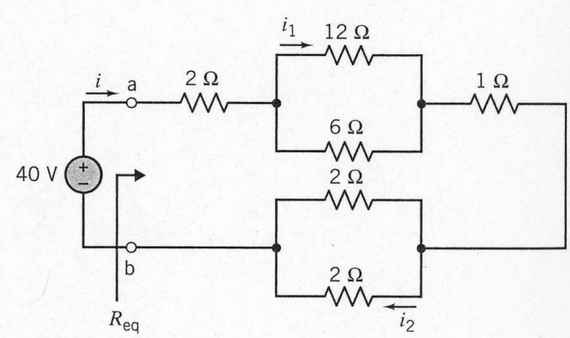

FIGURE P 3.7-14

VERIFICATION PROBLEMS

VP 3-1 A computer analysis program, used for the circuit of Figure VP 3.1, provides the following branch currents and voltages: i_1 A = −0.833, i_2 A = −0.333, i_3 A = −1.167, and $v = -2.0$ V. Are these answers correct?
Hint: Verify that KCL is satisfied at the center node and that KVL is satisfied around the outside loop consisting of the two 6-Ω resistors and the voltage source.

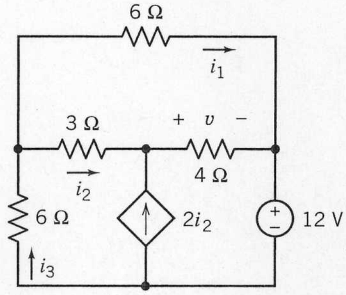

FIGURE VP 3.1

VP 3-2 The circuit of Figure VP 3.2 was assigned as a homework problem. The answer in the back of the textbook says the current, i, is 1.25 A. Verify this answer using current division.

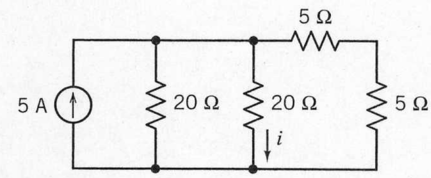

FIGURE VP 3.2

VP 3-3 The circuit of Figure VP 3.3 was built in the lab and v_o was measured to be 6.25 V. Verify this measurement using the voltage divider principle.