ACT ON THE PLAN

First, we need $p_1(t)$, so we first calculate

$$p_1(t) = iv_1 = (De^{-t/30} \times 10^{-3}\ \text{A})(Be^{-t/30}\ \text{V})$$
$$= DBe^{-t/30} \times 10^{-3}\ \text{W} = DBe^{-t/30}\ \text{mW}$$

Second, we need to find w_1 for the first 60 s as

$$w_1 = \int_0^{60} (DBe^{-t/30} \times 10^{-3})\,dt = \frac{DB \times 10^{-3} e^{-t/30}}{-1/30}\Bigg|_0^{60}$$
$$= -30DB \times 10^{-3}(e^{-2} - 1) = 25.9DB \times 10^{-3}\ \text{J}$$

Since we require $w_1 = 40$ mJ,

$$40 = 25.9DB$$

Next, select the limiting value, D = 1, to get

$$B = \frac{40}{(25.9)(1)} = 1.54\ \text{V}$$

Thus, we select a 2-V battery so that the magnitude of the current is less than 1 mA.

VERIFY THE PROPOSED SOLUTION

We must verify that at least 40 mJ is supplied using the 2-V battery. Since $i = e^-$ and $v_2 = 2e^{-t/60}$ V, the energy supplied by the battery is

$$w = \int_0^{60} (2e^{-t/60})(e^{-t/60} \times 10^{-3})\,dt = \int_0^{60} 2e^{-t/30} \times 10^{-3}\,dt = 51.8\ \text{mJ}$$

Thus, we have verified the solution, and we communicate it by recording the req for a 2-V battery.

1.10 SUMMARY

- The uses of electric power are diverse and very important to modern societies. However, electrical science developed slowly over the centuries, with many studies focusing on the nature of charge. As scientists became aware of the ability to store and control charge, they formulated the idea of a circuit.
- A circuit consists of electrical elements linked together in a closed path so that charge may flow.
- Charge is the intrinsic property of matter responsible for electric phenomena. The quantity of charge q can be expressed in terms of the charge on one electron, which is -1.602×10^{-19} coulombs. Current i is the time rate of change of charge past a given point. We can exp as $i = \frac{dq}{dt}$. The unit of current is the ampere (A): is 1 coulomb per second.
- The SI units are used by today's engineers an Using decimal prefixes, we may simply expre quantities with a wide range of magnitudes.
- The voltage across an element is the work requi a unit of charge through the element. When convention is used to assign the reference di product of the element current and the elen gives the power absorbed by the element.

PROBLEMS

Section 1.3 Electric Circuits and Current Flow

P1.3-1 The total charge that has entered a circuit element is $q(t) = 4(1 - e^{-5t})$ when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t \geq 0$.

Answer: $i(t) = 20e^{-5t}$ A

P1.3-2 The current in a circuit element is $i(t) =$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determ charge that has entered a circuit element for $t \geq 0$.

Hint: $q(0) = \int_{-\infty}^{0} i(\tau)\,d\tau = \int_{-\infty}^{0} 0\,d\tau = 0$

Answer: $q(t) = 4t + 0.8\,e^{-5t} - 0.8$ C for $t \geq 0$

P 1.3-3 The current in a circuit element is

$$i(t) = \begin{cases} 0 & t < 2 \\ 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & 8 < t \end{cases}$$

where the units of current are A and the units of time are s. Determine the total charge that has entered a circuit element for $t \geq 0$.

Answer: $q(t) = \begin{cases} 0 & t < 2 \\ 2t - 4 & 2 < t < 4 \\ 8 - t & 4 < t < 8 \\ 0 & 8 < t \end{cases}$ where the units of charge are C.

P 1.3-4 An electroplating bath, as shown in Figure P 1.3-4, is used to plate silver uniformly onto objects such as kitchen ware and plates. A current of 600 A flows for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams?

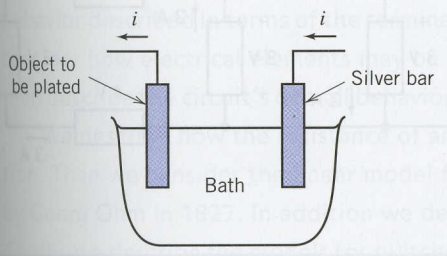

FIGURE P 1.3-4
An electroplating bath.

Section 1.6 Power and Energy

P 1.6-1 An electric range has a constant current of 10 A entering the positive voltage terminal with a voltage of 110 V. The range is operated for two hours. (a) Find the charge in coulombs that passes through the range. (b) Find the power absorbed by the range. (c) If electric energy costs 6 cents per kilowatt-hour, determine the cost of operating the range for two hours.

P 1.6-2 A walker's cassette tape player uses four AA batteries in series to provide 6 V to the player circuit. The four alkaline battery cells store a total of 200 watt-seconds of energy. If the cassette player is drawing a constant 10 mA from the battery pack, how long will the cassette operate at normal power?

P 1.6-3 The current through and voltage across an element vary with time as shown in Figure P 1.6-3. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.

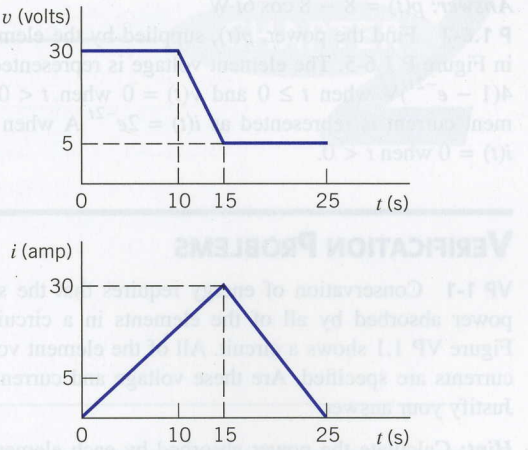

FIGURE P 1.6-3
(*a*) Voltage $v(t)$ and (*b*) current $i(t)$ for an element.

P 1.6-4 An automobile battery is charged with a constant current of 2 A for five hours. The terminal voltage of the battery is $v = 11 + 0.5t$ V for $t > 0$, where t is in hours. (a) Find the energy delivered to the battery during the five hours. (b) If electric energy costs 10 cents/kWh, find the cost of charging the battery for five hours.

Answer: (b) 1.23 cents

P 1.6-5 Find the power, $p(t)$, supplied by the element shown in Figure P 1.6-5 when $v(t) = 4 \cos 3t$ V and $i(t) = \dfrac{\sin 3t}{12}$ A. Evaluate $p(t)$ at $t = .5$ s and also at $t = 1$ s. Observe that the power supplied by this element has a positive value at some times and a negative value at other times.

Hint: $(\sin at)(\cos bt) = \dfrac{1}{2}(\sin(a + b)t + \sin(a - b)t)$

Answer: $p(t) = \dfrac{1}{6} \sin 6t$ W, $p(0.5) = 0.0235$ W, $p(1) = -0.0466$ W

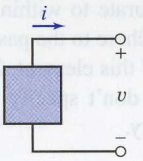

FIGURE P 1.6-5
An element.

P 1.6-6 Find the power, $p(t)$, supplied by the element shown in Figure P 1.6-5 when $v(t) = 8 \sin 3t$ V and $i(t) = 2 \sin 3t$ A.

Hint: $(\sin at)(\sin bt) = \dfrac{1}{2}(\cos(a - b)t - \cos(a + b)t)$

Answer: $p(t) = 8 - 8\cos 6t$ W

P 1.6-7 Find the power, $p(t)$, supplied by the element shown in Figure P 1.6-5. The element voltage is represented as $v(t) = 4(1 - e^{-2t})$V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The element current is represented as $i(t) = 2e^{-2t}$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$.

Answer: $p(t) = 8(1 - e^{-2t})e^{-2t}$ W

P 1.6-8 The battery of a flashlight develops 3 V, and the c
rent through the bulb is 200 mA. What power is absorbed
the bulb? Find the energy absorbed by the bulb in a five-mi
period.

Verification Problems

VP 1-1 Conservation of energy requires that the sum of the power absorbed by all of the elements in a circuit be zero. Figure VP 1.1 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.

Hint: Calculate the power absorbed by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.

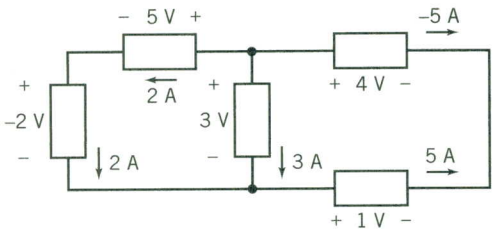

FIGURE VP 1.1

VP 1–2 Conservation of energy requires that the sum of power absorbed by all of the elements in a circuit be z
Figure VP 1.2 shows a circuit. All of the element voltages
currents are specified. Are these voltage and currents corr
Justify your answer.

Hint: Calculate the power absorbed by each element. Add
all of these powers. If the sum is zero, conservation of energ
satisfied and the voltages and currents are probably correc
the sum is not zero, the element voltages and currents ca
be correct.

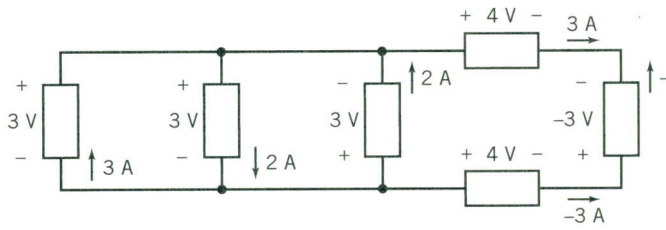

FIGURE VP 1.2

Design Problems

DP 1-1 A particular circuit element is available in three grades. Grade A guarantees that the element can safely absorb 1/2 W continuously. Similarly, Grade B guarantees that 1/4 W can be absorbed safely and Grade C guarantees that 1/8 W can be absorbed safely. As a rule, elements that can safely absorb more power are also more expensive and bulkier.

The voltage across an element is expected to be about 20 V and the current in the element is expected to be about 8 mA. Both estimates are accurate to within 25 percent. The voltage and current reference adhere to the passive convention.

Specify the grade of this element. Safety is the most important consideration, but don't specify an element that is more expensive than necessary.

DP 1-2 The voltage across a circuit element is $v(t) =$
$(1-e^{-8t})$ V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The cur
in this element is $i(t) = 30e^{-8t}$ mA when $t \geq 0$ and $i(t)$
when $t < 0$. The element current and voltage adhere to
passive convention. Specify the power that this device n
be able to absorb safely.

Hint: Use MATLAB, or a similar program, to plot the po