

Maxwell Equations

INEL 4151 ch 9

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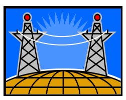
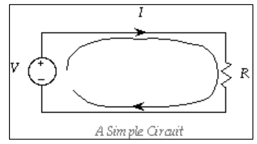


Electricity => Magnetism

- In 1820 Oersted discovered that a steady current produces a magnetic field while teaching a physics class.



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Would magnetism would produce electricity?

- Eleven years later, and at the same time, Mike Faraday in London and Joe Henry in New York discovered that a time-varying magnetic field would produce an electric current!

Faraday's Law:

$$V_{emf} = -N\frac{d\Psi}{dt}$$

$$V_{emf} = \oint_L E\cdot dl = -\frac{\partial}{\partial t}\int_s B\cdot dS$$

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Len's Law = (-)

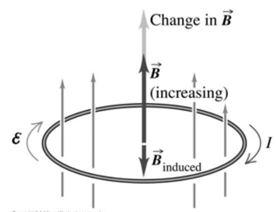
- If N=1 (1 loop)

$$V_{emf} = -\frac{d\Psi}{dt}$$

- The time change

$$V_{emf} = \oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{S}$$

can refer to B or S



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Two cases of $V_{emf} = \oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{S}$

- B changes

$$\oint_L \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

Stoke's theorem

$$\int_S \left(\nabla\times\vec{E}\right)\cdot d\vec{S} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$$\boxed{\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

- S (area) changes

$$\oint_L \vec{E}\cdot dl = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{S}$$

$$\vec{E}_m = \vec{F}_m / Q = \vec{u}\times\vec{B}$$

$$\int_S \left(\nabla\times E\right)\cdot dS = \oint_L \left(u\times\vec{B}\right)\cdot dl$$

$$\boxed{\nabla\times\vec{E} = \nabla\times\vec{u}\times\vec{B}}$$

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3 cases:

- Stationary loop in time-varying B field

$$\boxed{\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

- Moving loop in static B field

$$\boxed{\nabla\times\vec{E} = \nabla\times\vec{u}\times\vec{B}}$$

- Moving loop in time-varying B field

$$\boxed{\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} + \nabla\times\vec{u}\times\vec{B}}$$

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PE 9.1 If B=

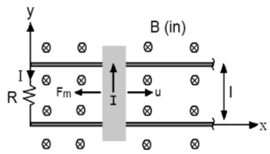

Figure 9.3 Induced emf due to a moving loop in a static **B** field.

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{l} = \oint_L \vec{u} \times \vec{B} \cdot dl$$

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PE 9.1

A conducting bar can slide freely over two conducting rails as shown in Figure 9.6. Calculate the induced voltage in the bar

(a) If the bar is stationed at $y = 8$ cm and $\mathbf{B} = 4 \cos 10^6 t\, \mathbf{a}_z$ mWb/m^2

(b) If the bar slides at a velocity $\mathbf{u} = 20\mathbf{a}_y$ m/s and $\mathbf{B} = 4\mathbf{a}_z$ mWb/m^2

(c) If the bar slides at a velocity $\mathbf{u} = 20\mathbf{a}_y$ m/s and $\mathbf{B} = 4 \cos (10^6 t - y)\, \mathbf{a}_z$ mWb/m^2

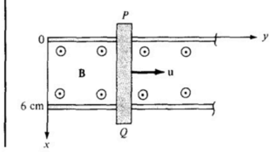


Figure 9.6 For Example 9.1.

$$V_{emf} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = 4 \cdot 10^6 \sin(10^6 t)(.08).06) = 19.2 \sin(10^6 t) kV$$

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{l} = \oint_L \vec{u} \times \vec{B} \cdot dl$$

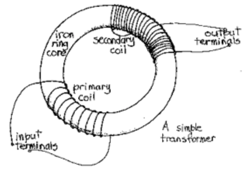
Example 9.3 Circuit with 10^{-3} m^2 cross-section, radius =10cm, $i_1(t)$=3 sin ωt [A], 50 Hz, N_1=200 turns, N_2= 100 , μ=500μ_0, find V_{emf} on both coils

$$\omega = 2\pi f$$

- Find reluctance and use Faraday's Law

$$V_{emf} = -N_2 \frac{d\Psi}{dt}$$

$$\Psi = \mathscr{F} / \mathscr{R} = N_1 I_1 \frac{\mu S}{l}$$



$$V_{emf} = -100(200)(3\omega \cos\omega t)\frac{500\mu_o 10^{-3}}{2\pi(0.10)} = -6\pi \cos 100\pi t$$

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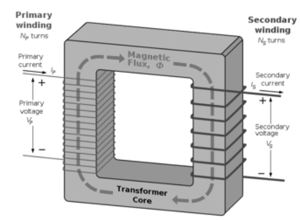
Example PE 9.3 A magnetic core of uniform cross-section 4 cm^2 is connected to a 120V, 60Hz generator. Calculate the induced emf V_2 in the secondary coil. N_1= 500, N_2=300

- **Use Faraday's Law**

$$V_1 = -N_1 \frac{d\Psi}{dt}$$

$$V_2 = -N_2 \frac{d\Psi}{dt}$$

$$V_2 = N_2 \frac{V_1}{N_1}$$



Answer: 72 cos(120πt) V

Some terms

- *E* = electric field intensity [V/m]
- *D* = electric field density [C/m^2]
- *H* = magnetic field intensity, [A/m]
- *B* = magnetic field density, [Teslas]

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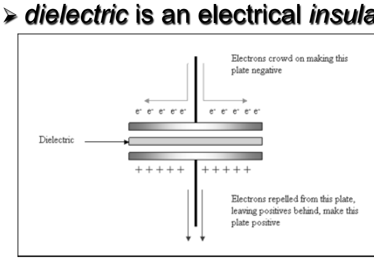
Take a look at a capacitor

- *dielectric* is an electrical *insulator*



- *d.c., C = open circuits*
- What's the current in an open circuit?

Maxwell's Eqs.

- the equation of continuity $\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$
- Maxwell added the term $\frac{\partial D}{\partial t}$ to Ampere's Law so that it not only works for **static** conditions but also for **time-varying** situations.
- This added term is called the displacement current density, J_d, while $J=J_c$ is the conduction current.

For static fields we had: $\nabla \times \vec{H} = \vec{J}$

- But the divergence of the curl of ANY vector ≡0 $\nabla \cdot \nabla \times \vec{H} = 0 = \nabla \cdot \vec{J}$

The continuity of the current requires:

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0$$ *This doesn't work for time-varying fields!*

We need to define: $\nabla \times \vec{H} = \vec{J} + \vec{J}_d$

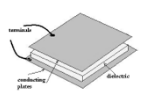
Now when we take the divergence of curl

$$\nabla \cdot \nabla \times \vec{H} = 0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_d$$

Therefore: $\nabla \cdot J_d = -\nabla \cdot J$

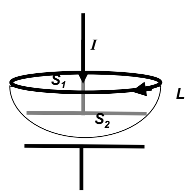
Maxwell put them together

If we didn't have the displacement current, J_d, then Ampere's Law:



$$\oint_L H \cdot dl = \int_{S_1} J \cdot dS = I_{enc} = I$$

$$\oint_L H \cdot dl = \int_{S_2} J \cdot dS = 0$$



$$\oint_L H \cdot dl = \int_{S_2} J_d \cdot dS = \frac{d}{dt}\int_{S_2} D \cdot dS = \frac{dQ}{dt} = I$$

At low frequencies $J \gg J_d$, but at radio frequencies both terms are comparable in magnitude.

- For dynamic fields we had:

$$\nabla \cdot J_d = -\nabla \cdot J = \frac{\partial \rho_v}{\partial t} = \frac{\partial (\nabla \cdot D)}{\partial t} = \nabla \cdot \frac{\partial D}{\partial t}$$

Therefore:

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

Substituting in curl of H

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

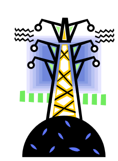
This is Maxwell's equation for Ampere's Law

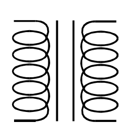
Maxwell Equations in General Form

Differential form	Integral Form	
$\nabla \cdot D = \rho_v$	$\oint_s D \cdot dS = \int_v \rho_v dv$	Gauss's Law for *E* field.
$\nabla \cdot B = 0$	$\oint_s B \cdot dS = 0$	Gauss's Law for *H* field. Nonexistence of monopole
$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_L E \cdot dl = -\frac{\partial}{\partial t}\int_s B \cdot dS$	Faraday's Law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_L H \cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$	Ampere's Circuit Law

Electromagnetics

- This is the principle of motors, hydro-electric generators and transformers operation.





This is what Oersted discovered accidentally:

$$\oint_L H \cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$$

Mention some examples of em waves

Phasors & complex #'s

Working with harmonic fields is easier, but requires knowledge of phasor, let's review

- **complex numbers** and
- **phasors**

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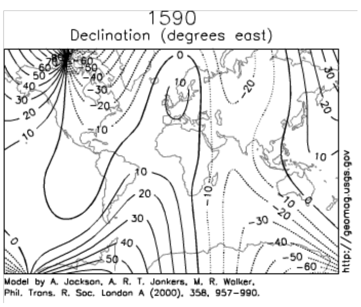
Maxwell Equations for Harmonic fields

Differential form*	
$\nabla \cdot \varepsilon E = \rho_v$	Gauss's Law for E field.
$\nabla \cdot \mu H = 0$	Gauss's Law for H field. No monopole
$\nabla \times E = -j\omega\mu H$	Faraday's Law
$\nabla \times H = J + j\omega\varepsilon E$	Ampere's Circuit Law

* (substituting $D = \varepsilon E$ and $H = \mu B$)

Earth Magnetic Field Declination from 1590 to 1990



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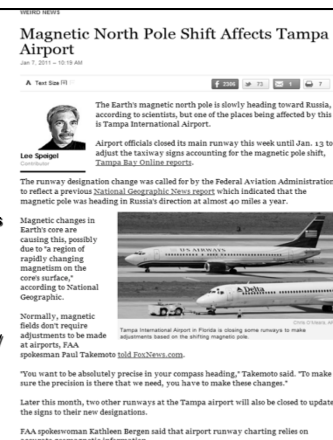
News

Jan 2011

- The Earth's magnetic north pole is slowly heading toward Russia, according to scientists, but one of the places being affected by this is Tampa International Airport.

  Airport officials closed its main runway this week until Jan. 13 to adjust the taxiway signs accounting for the magnetic pole shift, Tampa Bay Online reports.



WEIRD NEWS

Magnetic North Pole Shift Affects Tampa Airport

Jan 7, 2011 – 10:19 AM

Lee Speigel
Contributor

The Earth's magnetic north pole is slowly heading toward Russia, according to scientists, but one of the places being affected by this is Tampa International Airport.

Airport officials closed its main runway this week until Jan. 13 to adjust the taxiway signs accounting for the magnetic pole shift, Tampa Bay Online reports.

The runway designation change was called for by the Federal Aviation Administration to reflect a previous National Geographic News report which indicated that the magnetic pole was heading in Russia's direction at almost 40 miles a year.

Magnetic changes in Earth's core are causing this, possibly due to "a region of rapidly changing magnetism on the core's surface," according to National Geographic.

Chris O'Meara, AP

Tampa International Airport in Florida is closing some runways to make adjustments based on the shifting magnetic pole.

Normally, magnetic fields don't require adjustments to be made at airports, FAA spokesman Paul Takemoto told FoxNews.com.

"You want to be absolutely precise in your compass heading," Takemoto said. "To make sure the precision is there that we need, you have to make these changes."

Later this month, two other runways at the Tampa airport will also be closed to update the signs to their new designations.

FAA spokeswoman Kathleen Bergen said that airport runway charting relies on accurate geomagnetic information.