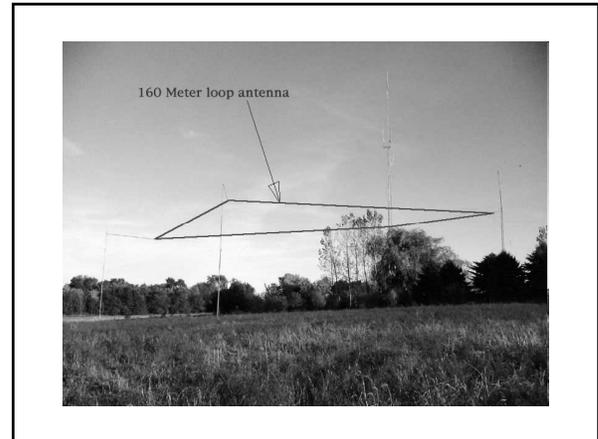
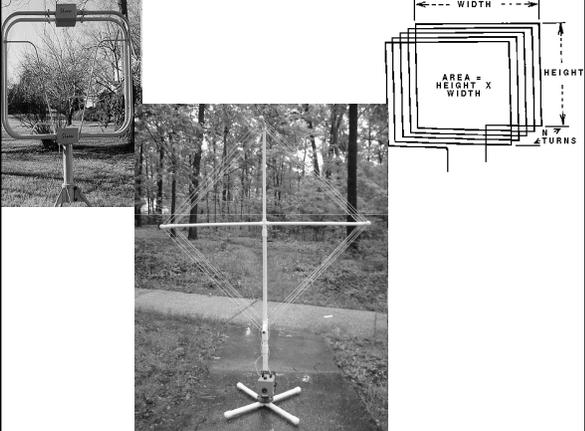




Antena Lazo

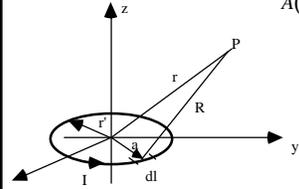
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AREA = HEIGHT x WIDTH
N TURNS

The Loop antenna

- Has many practical applications
- Can be considered as a sum (integration) of Hertzian dipole elements

$$A(x,y,z) = \frac{\mu}{4\pi} \int \frac{I(x',y',z')}{R} e^{-jkR} dl'$$


Prime variables refer to distance to the source from the origin.
R is the distance from antenna to observation.

Lazo

$\vec{R} = \hat{r} - \hat{r}'$

Para simplificar, y aprovechándome de la simetría del problema escojo un punto de observación en el plano xz.

donde: $\hat{r} = x\hat{x} + z\hat{z}$
 $\hat{r}' = x'\hat{x} + y'\hat{y}$

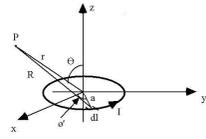
Sustituyendo:
 $x' = a \cos \phi'$
 $y' = a \sin \phi'$
 $x = r \sin \theta$
 $y = r \cos \theta$

Entonces queda:

$$R = |\vec{R}| = \sqrt{(r \sin \theta - a \cos \phi')^2 + (a \sin \phi')^2 + (r \cos \theta)^2}$$

$$R = r \sqrt{1 - \frac{2a}{r} \cos \phi' \sin \theta + \frac{a^2}{r^2}}$$

Aplicando la Serie de Taylor:
 $R \cong r - a \sin \theta \cos \phi'$



The Loop antenna

- La corriente viaja en dirección de azimut:

$$I = I_o \hat{\phi}$$

$$= I_o (-\hat{x} \sin \phi' + \hat{y} \cos \phi')$$

- Y el elemento de largo es:

$$dl' = a d\phi'$$

- Sustituyendo queda:

$$A = \frac{\mu I_o}{4\pi} \int_0^{2\pi} \frac{a d\phi'}{r} e^{-jk r} e^{jka \sin \theta \cos \phi'} (-\hat{x} \sin \phi' + \hat{y} \cos \phi')$$

Dos Casos:

- Lazo pequeño ($a/\lambda \ll 1$)
- Lazo de tamaño arbitrario

Recall: Integral of trigonometric funct.

Given that m and n are integers:

$$\int_0^\pi \sin m\phi \cos n\phi d\phi = \begin{cases} 0 & m+n \text{ even} \\ \frac{2m}{m^2-n^2} & m+n \text{ odd} \end{cases}$$

$$\int_0^\pi \cos m\phi \cos n\phi d\phi = \begin{cases} 0 & m \neq n \\ \frac{\pi}{2} & m = n \end{cases}$$

$$\int_0^\pi \sin m\phi \sin n\phi d\phi = \begin{cases} 0 & m \neq n \\ \frac{\pi}{2} & m = n \end{cases}$$

Small Loop,

- Since $ka \ll 1$ $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
 $\cong 1 + x$ para $x \ll 1$
- Then $e^{jka \sin \theta \cos \phi'} \cong 1 + jka \sin \theta \cos \phi'$
- And solving:

$$A = \frac{\mu I_o a}{4\pi r} \int_0^{2\pi} \frac{a d\phi'}{r} e^{-jkr} [1 + jka \sin \theta \cos \phi'] (-\hat{x} \sin \phi' + \hat{y} \cos \phi')$$

- We obtain that: $A_y = \frac{jka^2 \mu I_o}{4r} \sin \theta e^{-jkr} = A_\phi$
 $A_x = 0$

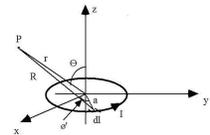
Again, if we know A we can find H and E

$$H = \frac{1}{\mu} \nabla \times A = \frac{1}{\mu} \hat{r} \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} A_\phi \sin \theta - \frac{1}{\mu} \hat{\theta} \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi)$$

$$H_\theta = \frac{-jka^2}{4r} I_o \sin \theta \frac{\partial}{\partial r} (e^{-jkr})$$

$$H_\theta = \frac{-I_o (ka)^2}{4r} e^{-jkr} \sin \theta$$

$$E_\phi = -\eta H_\theta$$



?Como lucira' el patron?

Arbitrary Size Loop Antenna

- We can't approximate, so:

$$A = \frac{\mu I_o a}{4\pi r} e^{-jkr} \int_0^{2\pi} e^{jka \sin \theta \cos \phi'} (-\hat{x} \sin \phi' + \hat{y} \cos \phi') d\phi'$$

- For the x-component:

$$A_x \propto - \int_0^{2\pi} e^{jka \sin \theta \cos \phi'} \sin \phi' d\phi'$$

- If we let: $u = jka \sin \theta \cos \phi'$

- then $A_x \propto \frac{1}{jka \sin \theta} \int_{jka \sin \theta}^{-jka \sin \theta} e^u du = 0$

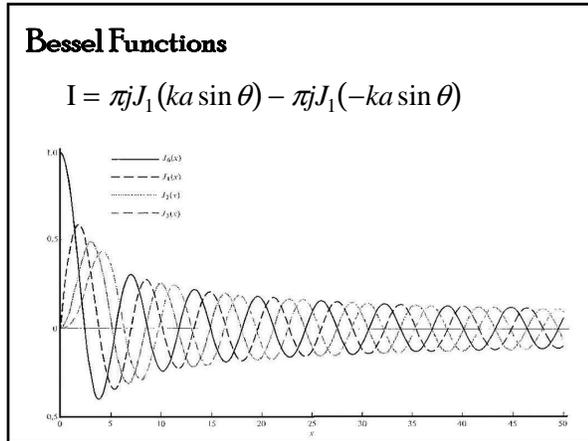
So we are left with y-component only

- Were the integral: $A_y = A_\phi = \frac{\mu I_o a}{4\pi r} e^{-jkr} \int_0^{2\pi} e^{jka \sin \theta \cos \phi'} \cos \phi' d\phi'$
 $A_y = A_\phi = \frac{\mu I_o a}{4\pi r} e^{-jkr} I$

- Can be identified as the Bessel function of first kind of order n.

$$I = \int_0^\pi e^{jka \sin \theta \cos \phi'} \cos \phi' d\phi' - \int_0^\pi e^{-jka \sin \theta \cos \phi'} \cos \phi' d\phi'$$

$$\pi j^n J_n(z) \cong \int_0^\pi e^{jz \cos \phi'} \cos n\phi' d\phi'$$



Loop Antenna of arbitrary length

- $J_1(z)$ es una función impar, [i.e., $-J_1(-z) = J_1(z)$], la integral se puede simplificar a,

$$I = 2\pi j J_1(ka \sin \theta)$$

$$A_\phi = \frac{j\mu_0 a}{2r} e^{-jkr} J_1(ka \sin \theta)$$

$$A_\phi = \underbrace{\left[\frac{jka^2 \mu_0 \sin \theta}{4r} e^{-jkr} \right]}_{\text{Igual al lazo pequeño}} \underbrace{\left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]}_{\text{Jinc}(x)}$$

Campos E y H

- Se obtienen

$$E_\phi = \frac{60\pi ka I_0 e^{-jkr}}{r} J_1(ka \sin \theta)$$

$$H_\theta = \frac{ka I_0 e^{-jkr}}{2r} J_1(ka \sin \theta) = -\frac{E_\phi}{\eta}$$

- Donde definimos:

$$ka = \frac{2\pi}{\lambda} a = \frac{C}{\lambda} = C_\lambda$$

Como trazar el patron de un Lazo

1. Trazamos líneas verticales desde $J(x)$ hasta un arco de radio C_λ ,
2. Se coloca un punto radialmente desde el origen hasta este arco a una distancia proporcional al valor de $|J(x)|$ en ese punto
3. Se repite esto para varios puntos entre $x=0$ y $x=C_\lambda$ formando así un patrón polar del lazo (no normalizado).

