
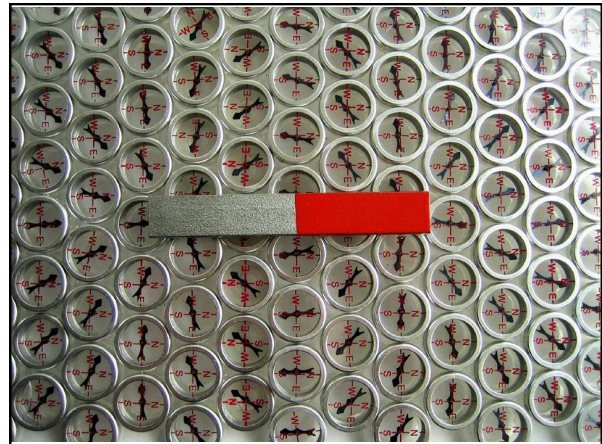


Magnetic Forces, Materials and Devices



INEL 4151 **ch 8**
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<http://www.treehugger.com/files/2008/10/spintronics-discover-could-lead-to-magnetic-batteries.php>



Section 8.2

MAGNETIC FORCE

Forces due to Magnetic fields

$B = \mu H$ = magnetic field density
 H = magnetic field intensity

Force can be due to:

- ① B on moving charge, Q
- ② B on current
- ③ Two currents

B is defined as force per unit current element

① Forces on a Charge

Analogous to the electric force: $\vec{F}_e = Q\vec{E}$

We have magnetic force: $\vec{F}_m = Q\vec{u} \times \vec{B}$

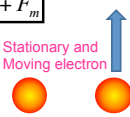
The total force is given by: $\vec{F} = \vec{F}_e + \vec{F}_m$

Note that F_m is perpendicular to both u and B

If the charge moving has a mass m , then:

$$\vec{F} = m \frac{d\vec{u}}{dt} = Q(\vec{E} + \vec{u} \times \vec{B})$$

Usually $F_m < F_e$



② Forces on a current element

The current element can be expressed as: $I d\vec{l} = \frac{dQ}{dt} d\vec{l} = dQ \frac{d\vec{l}}{dt} = dQ\vec{u}$

So we can write:

$$\vec{F}_m = Q\vec{u} \times \vec{B} = I\vec{l} \times \vec{B}$$

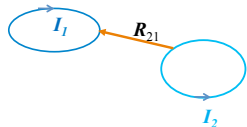
For closed path Line $I=[A]$ current element

$$\vec{F}_m = \oint_L I d\vec{l} \times \vec{B} \quad \vec{F}_m = \int_S K d\vec{S} \times \vec{B} \quad \vec{F}_m = \int_V J dv \times \vec{B}$$

For Surface $K=[A/m]$ current element For Volume $J=[A/m^2]$ current element

③ Force between two currents

- Each current produces a field B , which exerts a force on the other element. Let's find the Force on 1, due to the field produced by 2.

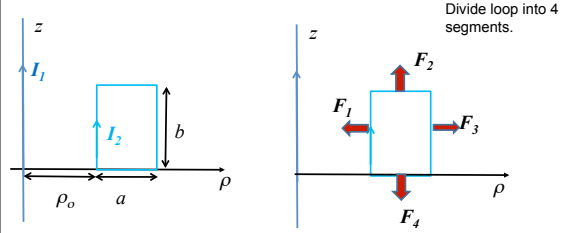


$$d\vec{F}_1 = I_1 d\vec{l}_1 \times \vec{B}_2$$

$$d\vec{B}_2 = \frac{\mu_o I_2 d\vec{l}_2 \times \hat{a}_{R_{21}}}{4\pi R_{21}^2}$$

$$\vec{F}_1 = \frac{\mu_o I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\vec{l}_1 \times (d\vec{l}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$$

P.E. 8.4 Find the force experienced by the loop due to the field produced by the line, if $I_1=10A$, $I_2=5A$, $\rho_o=20cm$, $a=1cm$, $b=30cm$



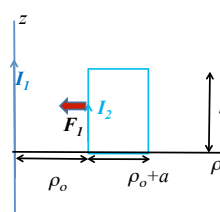
Divide loop into 4 segments.

Apply superposition

$$\vec{F}_l = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

For segment #1, Force #1

Since I_1 is infinite long wire:



$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi \rho_o}$$

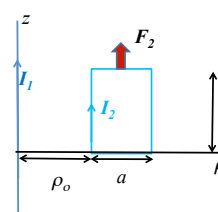
$$\vec{F}_1 = I_2 \int_{z=0}^b d\vec{l}_2 \times \vec{B}_1$$

$$\vec{F}_1 = I_2 \int_{z=0}^b dz \hat{a}_z \times \vec{B}_1$$

$$\vec{F}_1 = I_2 \frac{b \mu_o I_1}{2\pi \rho_o} (-\hat{a}_\rho)$$

For segment #2,

The B field at segment #2 due to line current 1.



$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi(\rho)}$$

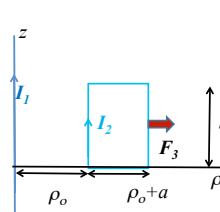
$$\vec{F}_2 = I_2 \int d\vec{l}_2 \times \vec{B}_1$$

$$\vec{F}_2 = I_2 \int_{\rho=\rho_o}^{\rho_o+a} d\rho \hat{\rho} \times \frac{\mu_o I_1}{2\pi \rho} \hat{a}_\phi$$

$$\vec{F}_2 = \frac{\mu_o I_1 I_2}{2\pi} \ln\left(\frac{\rho_o + a}{\rho_o}\right) (\hat{a}_z)$$

For segment #3, Force #3

The field at segment 3:



$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi(\rho_o + a)}$$

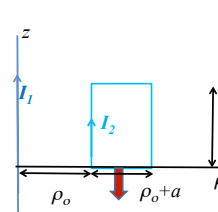
$$\vec{F}_3 = I_2 \int d\vec{l}_2 \times \vec{B}_1$$

$$\vec{F}_3 = I_2 \int_{z=b}^0 dz \hat{a}_z \times \frac{\mu_o I_1}{2\pi(\rho_o + a)} \hat{a}_\phi$$

$$\vec{F}_3 = \frac{\mu_o I_1 I_2 b}{2\pi(\rho_o + a)} (\hat{a}_\rho)$$

For segment #4,

The B field at segment #4 due to current 1.



$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi \rho}$$

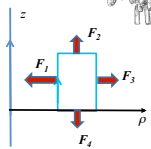
$$\vec{F}_4 = I_2 \int d\vec{l}_2 \times \vec{B}_1$$

$$\vec{F}_4 = I_2 \int_{\rho=\rho_o+a}^{\rho_o} d\rho \hat{\rho} \times \frac{\mu_o I_1}{2\pi \rho} \hat{a}_\phi$$

$$\vec{F}_4 = -\frac{\mu_o I_1 I_2}{2\pi} \ln\left(\frac{\rho_o + a}{\rho_o}\right) (\hat{a}_z)$$

The total force on the loop is

$I_1=10A, I_2=5A, \rho_o=20cm, a=1cm, b=30cm$



- The sum of all four:

$$\vec{F}_l = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$
- Note that 2 terms cancel out ($F_2 = -F_4$):

$$\vec{F}_{loop} = -I_2 \frac{b\mu_o I_1}{2\pi} \left[\frac{1}{\rho_o} - \frac{1}{(\rho_o + a)} \right] \hat{a}_\rho$$

They Attract!

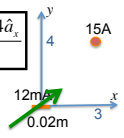
$$\vec{F}_{loop} = -5 \frac{(0.3)\mu_o 10}{2\pi} \left[\frac{1}{0.2} - \frac{1}{(0.2+0.01)} \right] \hat{a}_\rho = -7.14 [\mu N] \hat{a}_\rho$$

8.7 A current element of length 2 cm is located at the origin in free space and carries current 12 mA along \hat{a}_z . A filamentary current of 15A, I_1 is located along $x = 3, y = 4$.

Find the force on the current element due to the filament of 15A

$$\vec{B}_2 = \frac{\mu_o I_2 \hat{a}_\phi}{2\pi \rho}$$

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho = \hat{a}_z \times \left(\frac{-3\hat{a}_x - 4\hat{a}_y}{5} \right) = \frac{-3\hat{a}_y + 4\hat{a}_x}{5}$$

$$\vec{B}_2 = \frac{4\pi 10^{-7} (15)}{2\pi (5)} \left(\frac{-3\hat{a}_y + 4\hat{a}_x}{5} \right) = 6 \cdot 10^{-7} \left(\frac{-3\hat{a}_y + 4\hat{a}_x}{5} \right)$$


$$d\vec{F} = (I_1 d\vec{l}_1 \times \vec{B}_2)$$

Distance between line and element.
Element is at origin (0,0,0)
Line is at x=3, y=4
 $\vec{a}_\rho = (0,0,0) - (3,4,0)$
 $\rho = 5$

$$\vec{F} = \left(0.012(0.02)\hat{a}_z \times 6 \cdot 10^{-7} \left(\frac{-3\hat{a}_y + 4\hat{a}_x}{5} \right) \right)$$

$$\vec{F} = \left(0.012(0.02)6 \cdot 10^{-7} \left(\frac{-3\hat{a}_z}{5} \right) \right) = -86.4 \cdot 10^{-12} \hat{a}_z$$

Section 8.3-8.4

MAGNETIC TORQUE & MOMENT

Magnetic Torque and Moment

Inside a motor/generator we have many loops with currents, and the Magnetic fields from a magnet exert a torque on them.

The torque in [N m] is:

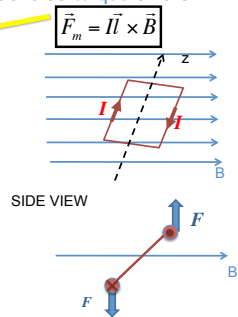
$$\vec{T} = \vec{r} \times \vec{F}$$

$$\vec{T}_m = \vec{I} \times \vec{B}$$

- Where \vec{m} is the magnetic dipole moment:

$$\vec{m} = IS \hat{a}_n$$
- Where S is the area of the loop $S = r l$
- and \hat{a}_n is its unit normal.

This applies if \vec{B} is uniform




Example: A current of 2mA on a 1000-turns solenoid with cross area of 10 x 30cm has a $B=400mT$. Find the torque:

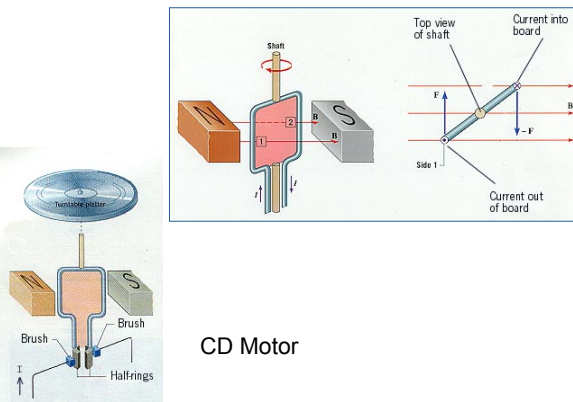
$$\vec{T} = \vec{m} \times \vec{B}$$

$$\vec{m} = IS \hat{a}_n$$

$$\vec{T} = N(ISB) = (1000)(0.002)(.010)(.030)\hat{a}_n \times 0.4\hat{a}_B$$

$$= 240 \mu Nm (\hat{a}_n \times \hat{a}_B)$$


Torque on a Current Loop in a Magnetic Field



CD Motor

Magnetic Torque and Moment

The Magnetic torque can also be expressed as:

The torque in [N m] is:

$$\vec{T} = \vec{m} \times \vec{B}$$

$$= Q_m \vec{l} \times \vec{B} \quad \vec{m} = I(S) \hat{a}_n$$

Section 8.5, 8.6

MAGNETIZATION

Magnetization

(similar to Polarization for E)

- Atoms have e- orbiting and spinning
 - Each have a magnetic dipole associated to it
- Most materials have **random orientation** of their magnetic dipoles if **NO external B-field** is applied.
- When a **B field is applied**, they try to **align** in the same direction.
- The **total magnetization** [A/m]

$$\vec{M} = \lim_{\Delta v \rightarrow 0} \frac{\sum_{k=1}^N \vec{m}_k}{\Delta v}$$

Magnetization

- The magnetization current density [A/m²]
- The total magnetic density is:
- Magnetic susceptibility is:
- The relative permeability is:

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{M} = \chi_m \vec{H} \quad \vec{B} = \mu_0 (1 + \chi_m) \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

$$\mu_r = (1 + \chi_m) = \frac{\mu}{\mu_0}$$

Permeability unit is [H/m].

Classification of Materials according to magnetism

Magnetic
 $\mu_r \neq 1$

Non-magnetic
 $\mu_r = 1$
Ex. air, free space, most materials in their natural state.


Diamagnetic
 $\mu_r \leq 1$
Electronic motions of spin and orbit cancel out.
lead, copper, Si, diamonds, superconductors
($\mu_r = \beta = 0$).
Are weakly affected by B Fields.

Paramagnetic
 $\mu_r \geq 1$
(air, platinum, tungsten, platinum)
Temperature dependent.
Not many uses except in masers

Ferromagnetic
 $\mu_r \gg 1$
Iron, Ni, Co, steel, alloys
Loose properties if heated above Curie T (770C)
Nonlinear:
 μ_r varies

$\chi_m = \mu_r - 1$ at 20°C

- Pure iron 150- 200,000
- Steel 50-100
- Iron oxide 720
- Iron Amonium alum 66
- Uranium 40
- Platinum 26
- Aluminum 2.2
- Magnesium 1.2
- Sodium 0.72
- Oxygen gas 0.19
- Amomia = -0.26



MagLev magnetic Levitation

- Use **diamagnetic** materials, which repel and are repelled by strong H fields.
- Superconductors** are diamagnetic.
- Floating one magnet over another**
- Regular train **187 mph**
- Maglev train **312 mph**

<http://www.youtube.com/watch?v=alwbrZ4knpg>

P.E. 8.7 B-field in a magnetic material.

$\vec{B} = 10e^{-y}\hat{z} \text{ mWb/m}^2$

In a region with $\mu_r=4.6$

- Find H , M and susceptibility

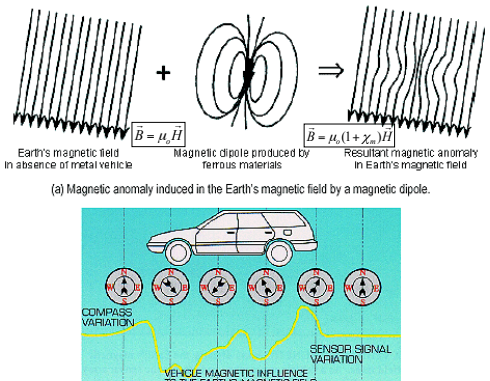
$$\chi_m + 1 = \mu_r$$

$$\chi_m = 3.6$$

$$H = \frac{\vec{B}}{\mu}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{H} = 1730e^{-y}\hat{z} [\text{A/m}]$$

$$\vec{M} = 6230e^{-y}\hat{z} \text{ A/m}$$


(a) Magnetic anomaly induced in the Earth's magnetic field by a magnetic dipole.

(b) Perturbation of Earth's magnetic field by a ferrous metal vehicle (Source: Nu-Metrics, Vanderbilt, PA).

How to make traffic light go Green when driving a bike or motorcycle

- Stop directly on top of induction loop on the street
- Attach **neodymium magnets** to the vehicle
- Move on top of loop
- Push crossing button
- Video detectors

<http://www.wikihow.com/Trigger-Green-Traffic-Lights>

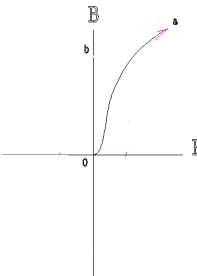
<http://www.labreform.org/education/loops.html>

B-H or Magnetization curve

- When an H-field is applied to ferromagnetic material, it's **B increases until saturation**.

$$\mu = \frac{B}{H}$$

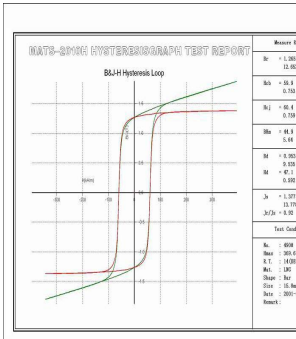
But when H is decreased, B doesn't follow the same curve.



Hysteresis Loop

Hysteresis Loop

- Some ferrites, have almost **rectangular B-H curves**, ideal for **digital** computers for storing information.
- The **area** of the loop gives the **energy loss** per volume during one cycle in the form of heat.
- Tall-narrow loops** are desirable for electric **generators, motors, transformers** to minimize the hysteresis losses.





MATS-5000H HYSTERESIS LOOP TEST REPORT		Measure Result
B&H Hysteresis Loop		B _{max} = 0.003 T
		H _{max} = 0.002 A/m
		B ₁₀ = 0.001 T
		H ₁₀ = 0.0005 A/m
		B ₅₀ = 0.0005 T
		H ₅₀ = 0.00025 A/m
		B ₉₀ = 0.00025 T
		H ₉₀ = 0.000125 A/m
		B ₉₅ = 0.000125 T
		H ₉₅ = 0.0000625 A/m
		B ₉₉ = 0.0000625 T
		H ₉₉ = 0.00003125 A/m
		B _{99.5} = 0.00003125 T
		H _{99.5} = 0.000015625 A/m
		B _{99.9} = 0.000015625 T
		H _{99.9} = 0.0000078125 A/m
		B _{99.95} = 0.0000078125 T
		H _{99.95} = 0.00000390625 A/m
		B _{99.99} = 0.00000390625 T
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		B _{99.999999999999999999999999} = 0.000000000000000000222044604925049677430555729140625 T
		H _{99.999999999999999999999999} = 0.000000000000000000111022302462524838717527859375 A/m
		B _{99.9999999999999999999999995} = 0.000000000000000000111022302462524838717527859375 T
		H _{99.9999999999999999999999995} = 0.0000000000000000000555111512312624193587892896875 A/m
		B _{99.9999999999999999999999999} = 0.0000000000000000000555111512312624193587892896875 T
		H _{99.9999999999999999999999999} = 0.0000000000000000000277555756156312209693946448375 A/m
		B _{99.99999999999999999999999995} = 0.0000000000000000000277555756156312209693946448375 T
		H _{99.99999999999999999999999995} = 0.00000000000000000001387778780781561104846932241875 A/m
		B _{99.99999999999999999999999999} = 0.00000000000000000001387778780781561104846932241875 T
		H _{99.99999999999999999999999999} = 0.0000000000000000000069388939039078055242346612209375 A/m
		B _{99.999999999999999999999999995} = 0.0000000000000000000069388939039078055242346612209375 T
		H _{99.999999999999999999999999995} = 0.000000000000000000003469446951953902762117330611046875 A/m
		B _{99.999999999999999999999999999} = 0.000000000000000000003469446951953902762117330611046875 T
		H _{99.999999999999999999999999999} = 0.00000000000000000000173472347597695138105866530234375 A/m
		B _{99.9999999999999999999999999995} = 0.00000000000000000000173472347597695138105866530234375 T
		H _{99.9999999999999999999999999995} = 0.000000000000000000000867361737988475690529332651171875 A/m
		B _{99.9999999999999999999999999999} = 0.000000000000000000000867361737988475690529332651171875 T
		H _{99.9999999999999999999999999999} = 0.0000000000000000000004336808689942377952646632555859375 A/m
		B _{99.99999999999999999999999999995} = 0.0000000000000000000004336808689942377952646632555859375 T
		H _{99.99999999999999999999999999995} = 0.00000000000000000000021684043449711889762333162779296875 A/m
		B _{99.99999999999999999999999999999} = 0.00000000000000000000021684043449711889762333162779296875 T
		H _{99.99999999999999999999999999999} = 0.00000000000000000000010842021724855944881166658138896875 A/m
		B _{99.999999999999999999999999999995} = 0.00000000000000000000010842021724855944881166658138896875 T
		H _{99.999999999999999999999999999995} = 0.0000000000000000000000542101086242772244083332906944375 A/m
		B _{99.999999999999999999999999999999} = 0.0000000000000000000000542101086242772244083332906944375 T
		H _{99.999999999999999999999999999999} = 0.000000000000000000000027105054312138612204166658138896875 A/m
		B _{99.9999999999999999999999999999995} = 0.000000000000000000000027105054312138612204166658138896875 T
		H _{99.9999999999999999999999999999995} = 0.00000000000000000000001355252715606930610202083332906944375 A/m

Section 8.7 (not covered)

MAGNETIC B.C.

Magnetic B.C. [Boundary Conditions]

- use Gauss Law 
- &
- Ampere's Circuit law 

$\oint B \cdot dS = 0$

$\oint H \cdot dl = I$

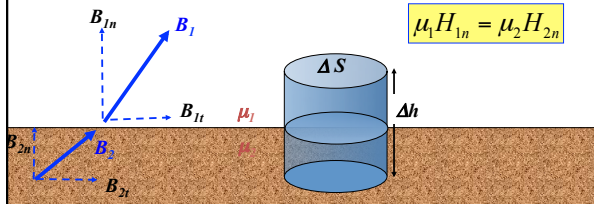
B.C.: Two magnetic media

- Consider the figure below:

$\oint_S B \cdot dS = 0$
 $B_{1n} \Delta S - B_{2n} \Delta S = 0$

$B_{1n} = B_{2n}$
 is continuous.

$\mu_1 H_{1n} = \mu_2 H_{2n}$



B.C.: Two magnetic media

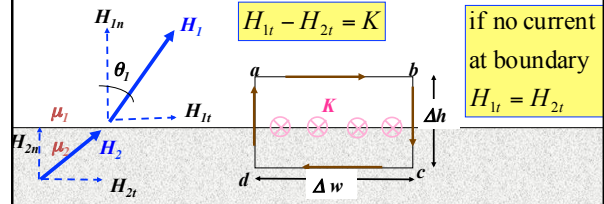
- Consider the figure below:

$\oint_l H \cdot dl = I = K \cdot \Delta w$
 $= H_{1t} \Delta w + H_{1n} \frac{\Delta h}{2} + H_{2n} \frac{\Delta h}{2} - H_{2t} \Delta w - H_{2n} \frac{\Delta h}{2} - H_{1n} \frac{\Delta h}{2}$

$H_{1t} - H_{2t} = K$


if no current
at boundary
 $H_{1t} = H_{2t}$

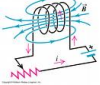
$\mu_1 H_{1n} = \mu_2 H_{2n}$



Section 8.8

INDUCTORS AND INDUCTANCE





$\Psi = \int_S \vec{B} \cdot d\vec{S} \text{ [Wb]}$


Inductors

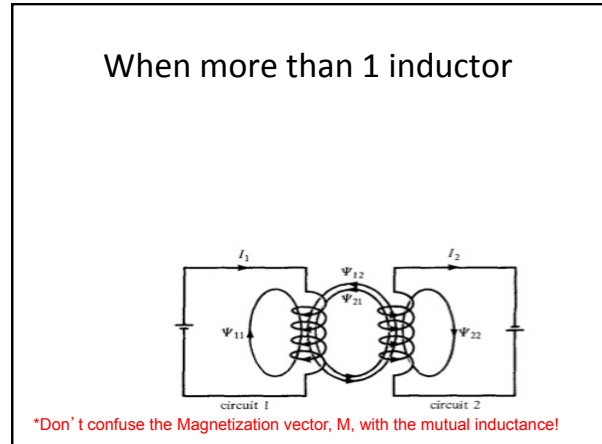
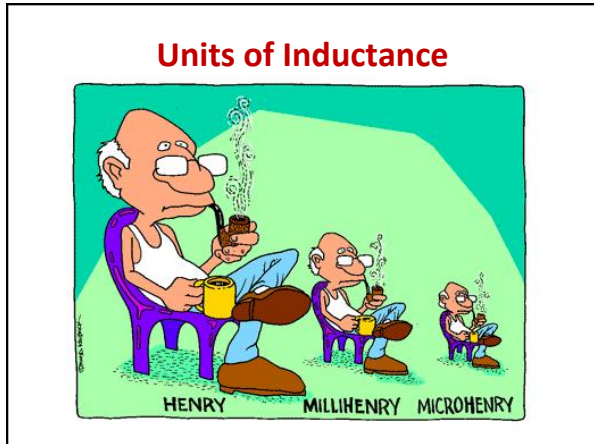
- So we can define the **inductance** as:

$\lambda = LI$
- If flux passes thru N turns, the total Flux Linkage is $\lambda = N\Psi$
- This is proportional to the current I
- then:

$L = \frac{N\Psi}{I}$
- The energy in Joules:

$W_m = \frac{1}{2} LI^2$





Self -inductance

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Psi_1}{I_1}$$

$$L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2 \Psi_2}{I_2}$$

- The total energy in the magnetic field is the sum of the energies:

$$W_m = W_1 + W_2 + W_{12}$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2$$
- The **positive** is taken if currents I_1 and I_2 flow such that the magnetic fields of the two circuits strengthen each other.

See table 8.3 in textbook with formulas for inductance of common elements like coaxial cable, two-wire line, etc.

P.E. 8.10 Solenoid

A long solenoid with 2 x 2 cm cross section has iron core (permeability is 1000x) and 4000 turns per meter. It carries current of 0.5A, Find:

- Self inductance per meter, L'

$$L' = \frac{L}{l} = N \Psi / I l$$

$$\Psi = BS$$

$$B = \mu H = \frac{\mu N I}{l} = \mu I n$$

Note L is independent of current

$$= 1000 \mu_o (4000)^2 (2cm)^2$$

$$= 8.042H / m$$

Energy stored

It can be shown that

- Units=Joules
- Energy stored in Magnetic Field is:

$$W_m = \frac{1}{2} \int \mu H^2 dv$$

Section 8.10

MAGNETIC CIRCUITS

Magnetic Circuits

- Magnetomotive force
In units of ampere-turns
 $\mathcal{F} = NI = \oint H \cdot dl$
- Reluctance
 $\mathcal{R} = \frac{l}{\mu S}$
- Like $V=IR$
 $\mathcal{F} = \Psi \mathcal{R}$
- Ex: magnetic relays, motors, generators, transformers, toroids
- Table 8.4 presents **analogy** between magnetic and electric circuits

Magnetic Circuits

Figure 8.24 Analogy between (a) an electric circuit, and (b) a magnetic circuit.

Magnetic Circuits

Figure 3. Magnetic nucleus with air gap (a) and an equivalent magnetic circuit (b).

ELECTRIC CIRCUIT RULES APPLY!

(a) $I = \frac{V}{(R_1 + R_2)}$
 (b) $\phi = \frac{\mathcal{F}}{(R_1 + R_2)}$

$\mathcal{R} = \frac{l}{\mu S}$ $\mathcal{F} = \Psi \mathcal{R}$ $\Psi = \Psi_1 + \Psi_2$

Ex. Find current in coil needed to produced magnetic field density of 1.5T in air gap.
 Assume $\mu_r=50$ and all cross sectional area is 10 cm^2

$\mathcal{R} = \frac{l}{\mu S}$ $\mathcal{R}_1 = \frac{0.30}{50\mu_0 10/10^4} = \mathcal{R}_2$

$\mathcal{R}_3 = \frac{0.09}{50\mu_0 10/10^4}$ $\mathcal{R}_{air} = \frac{0.01}{1\mu_0 10/10^4}$

$\mathcal{F} = NI = 400I = \Psi \mathcal{R}_T$

$\Psi = \Psi_a = B_a S$

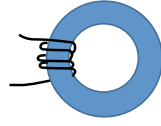
$I = 44.16A$

8.42 A cobalt ring ($\mu_r=600$) has mean radius of 30cm.

- If a coil wound on the ring carries 12A, calculate the N required to establish an average magnetic flux density of 1.5 Teslas in the ring.

$$\mathcal{F} = NI = \oint H \cdot dl$$

$$NI = Hl$$



radius of 30cm

$$N = \frac{Bl}{\mu_r \mu_o I} = \frac{1.5 \times 2\pi(0.3)}{600\mu_o 12} = 313 \text{ turns}$$