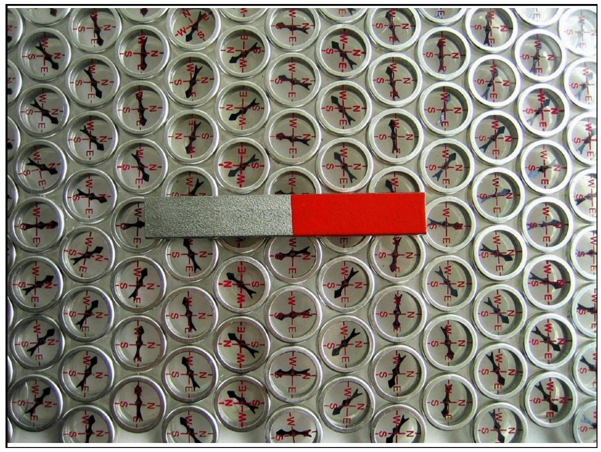
Magnetic Forces, Materials and Devices

INEL 4151 ***ch 8***

Dr. Sandra Cruz-Pol
Electrical and Computer Engineering Dept.
UPRM

http://www.treehugger.com/files/2008/10/spintronics-discover-could-lead-to-magnetic-batteries.php



Section 8.2

MAGNETIC FORCE

Forces due to Magnetic fields

$\boldsymbol{B}=\mu\boldsymbol{H}=$ magnetic field density
$\boldsymbol{H}=$magnetic field intensity

Force can be due to:

① ***B*** on moving charge, Q
② ***B*** on current
③ Two currents

B is defined as force per unit current element

① Forces on a Charge

Analogous to the electric force: $\vec{F}_e = Q\vec{E}$

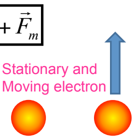
We have magnetic force: $\vec{F}_m = Q\vec{u}\times\vec{B}$

The total force is given by:

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

Note that $\boldsymbol{F}_m$ is perpendicular to both $\boldsymbol{u}$ and $\boldsymbol{B}$

Stationary and Moving electron



If the charge moving has a mass *m*, then:

$$\vec{F} = m\frac{d\vec{u}}{dt} = Q\left(\vec{E} + \vec{u}\times\vec{B}\right)$$

Usually $F_m < F_e$

② Forces on a current element

The current element can be expressed as: $I d\vec{l} = \frac{dQ}{dt}d\vec{l} = dQ\frac{d\vec{l}}{dt} = dQ\vec{u}$

So we can write:

$$\vec{F}_m = Q\vec{u}\times\vec{B} = I\vec{l}\times\vec{B}$$

For closed path Line *I*=[A] current element

$$\vec{F}_m = \oint_L I d\vec{l}\times\vec{B}$$

$$\vec{F}_m = \oint_S K d\vec{S}\times\vec{B}$$

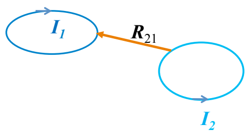
For Surface *K*=[A/m] current element

$$\vec{F}_m = \oint_v \vec{J} dv\times\vec{B}$$

For Volume *J*=[A/m^2] current element

③ Force between two currents

- Each current produces a field *B*, which exerts a force on the other element. Let's find the Force on 1, due to the field produced by 2.



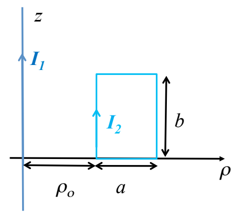
$$d\vec{F}_1 = I_1 d\vec{l}_1 \times \vec{B}_2$$

$$d\vec{B}_2 = \frac{\mu_o I_2 d\vec{l}_2 \times \hat{a}_{R_{21}}}{4\pi R_{21}^2}$$

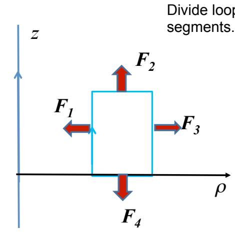
$$\vec{F}_1 = \frac{\mu_o I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\vec{l}_1 \times \left(d\vec{l}_2 \times \hat{a}_{R_{21}}\right)}{R_{21}^2}$$

P.E. 8.4 Find the force experienced by the loop due to the field produced by the line,

if $I_1=10A,\ I_2=5A,\ \rho_o=20cm,\ a=1cm,\ b=30cm$



Divide loop into 4 segments.



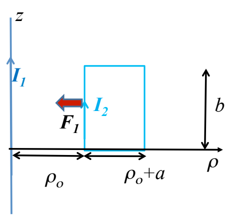
Apply superposition

$$\vec{F}_l = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

For segment #1, Force #1

Since I_1 is infinite long wire:

$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi\rho_o}$$



$$\vec{F}_1 = I_2 \int_{z=0}^{b} d\vec{l}_2 \times \vec{B}_1$$

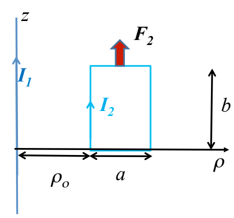
$$\vec{F}_1 = I_2 \int_{z=0}^{b} dz\, \hat{a}_z \times \vec{B}_1$$

$$\vec{F}_1 = I_2 \frac{b\mu_o I_1}{2\pi\rho_o}\left(-\hat{a}_\rho\right)$$

For segment #2,

The B field at segment #2 due to line current 1.

$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi(\rho)}$$



$$\vec{F}_2 = I_2 \int d\vec{l}_2 \times \vec{B}_1$$

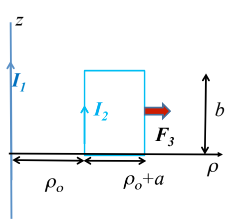
$$\vec{F}_2 = I_2 \int_{\rho=\rho_o}^{\rho_o+a} d\rho\hat{a}_\rho \times \frac{\mu_o I_1}{2\pi\rho}\hat{a}_\phi$$

$$\vec{F}_2 = \frac{\mu_o I_1 I_2}{2\pi} \ln\left(\frac{\rho_o + a}{\rho_o}\right)(\hat{a}_z)$$

For segment #3, Force #3

The field at segment 3:

$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi(\rho_o + a)}$$



$$\vec{F}_3 = I_2 \int d\vec{l}_2 \times \vec{B}_1$$

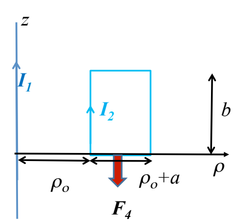
$$\vec{F}_3 = I_2 \int_{z=b}^{0} dz\, \hat{a}_z \times \frac{\mu_o I_1}{2\pi(\rho_o + a)}\hat{a}_\phi$$

$$\vec{F}_3 = \frac{\mu_o I_1 I_2 b}{2\pi(\rho_o + a)}(\hat{a}_\rho)$$

For segment #4,

The B field at segment #4 due to current 1.

$$\vec{B}_1 = \frac{\mu_o I_1 \hat{a}_\phi}{2\pi\rho}$$

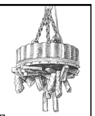


$$\vec{F}_4 = I_2 \int d\vec{l}_2 \times \vec{B}_1$$

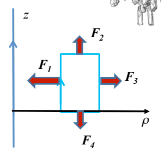
$$\vec{F}_4 = I_2 \int_{\rho=\rho_o+a}^{\rho_o} d\rho\hat{a}_\rho \times \frac{\mu_o I_1}{2\pi\rho}\hat{a}_\phi$$

$$\vec{F}_4 = -\frac{\mu_o I_1 I_2}{2\pi} \ln\frac{\rho_o + a}{\rho_o}(\hat{a}_z)$$

The total force en the loop is



I_1=10A, I_2=5A, ρ_o=20cm, a=1cm, b=30cm



- The sum of all four:

$$\vec{F}_l = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

- Note that 2 terms cancel out (F_2= -F_4):

$$\vec{F}_{loop} = -I_2 \frac{b\mu_o I_1}{2\pi}\left[\frac{1}{\rho_o} - \frac{1}{(\rho_o + a)}\right]\hat{a}_\rho$$

They Attract!

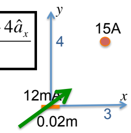
$$\vec{F}_{loop} = -5\frac{(0.3)\mu_o 10}{2\pi}\left[\frac{1}{0.2} - \frac{1}{(0.2+0.01)}\right]\hat{a}_\rho = -7.14\ [\mu N]\,\hat{a}_\rho$$

8.7 A current element of length 2 cm is located at the origin in free space and carries current 12 mA along $\mathbf{a}_x$. A filamentary current of 15$\mathbf{a}_z$ A is located along $x = 3, y = 4$.

Find the force on the current element due to the filament of 15A

$$\vec{B}_2 = \frac{\mu_o I_2 \hat{a}_\phi}{2\pi\rho} \qquad \hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho = \hat{a}_z \times \left(\frac{-3\hat{a}_x - 4\hat{a}_y}{5}\right) = \frac{-3\hat{a}_y + 4\hat{a}_x}{5}$$

$$\vec{B}_2 = \frac{4\pi 10^{-7}(15)}{2\pi(5)}\left(\frac{-3\hat{a}_y + 4\hat{a}_x}{5}\right) = 6\cdot 10^{-7}\left(\frac{-3\hat{a}_y + 4\hat{a}_x}{5}\right)$$



$$d\vec{F} = \left(I_1 d\vec{l}_1 \times \vec{B}_2\right)$$

Distance between line and element.
Element is at origin (0,0,0)
Line is at x=3, y=4
$\mathbf{a}_\rho$=(0,0,0)-(3,4,0)
ρ =5

$$\vec{F} = \left(0.012(0.02)\hat{a}_x \times 6\cdot 10^{-7}\left(\frac{-3\hat{a}_y + 4\hat{a}_x}{5}\right)\right)$$

$$\vec{F} = \left(0.012(0.02)6\cdot 10^{-7}\left(\frac{-3\hat{a}_z}{5}\right)\right) = -86.4\cdot 10^{-12}\hat{a}_z$$

Section 8.3-8.4

MAGNETIC TORQUE & MOMENT

Magnetic *Torque and Moment*

Inside a motor/generator we have many loops with currents, and the Magnetic fields from a magnet exert a torque on them.

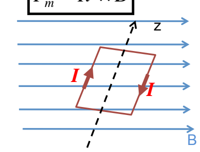
The torque in [N m]is: $\vec{F}_m = I\vec{l} \times \vec{B}$

$$\vec{T} = \vec{r} \times \vec{F}$$

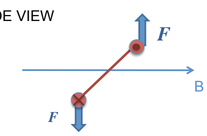
- Where m is the *magnetic dipole moment*:

$$\vec{m} = IS\,\hat{a}_n$$

- Where S is the area of the loop $S=r\ l$
- and $\mathbf{a}_n$ is its unit normal. This applies if $\mathbf{B}$ is uniform



SIDE VIEW

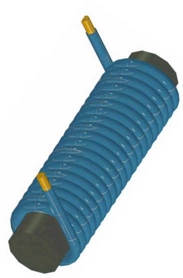


Example: A current of 2mA on a 1000-turns solenoid with cross area of 10 x 30cm has a B=400mT. Find the torque:

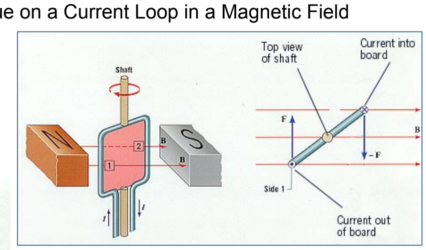
$$\vec{T} = \vec{m} \times \vec{B} \qquad \vec{m} = IS\,\hat{a}_n$$

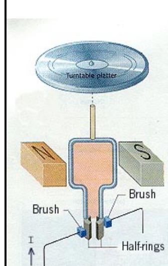
$$\vec{T} = N(ISB) = (1000)0.002(.010)(.030)\hat{a}_n \times 0.4\hat{a}_B$$

$$= 240\mu Nm\left(\hat{a}_n \times \hat{a}_B\right)$$



Torque on a Current Loop in a Magnetic Field



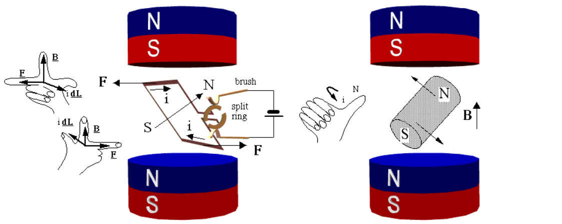


CD Motor

Magnetic *Torque and Moment*

The Magnetic torque can also be expressed as:

The torque in [N m]is: $\vec{T} = \vec{m} \times \vec{B} = Q_m \vec{l} \times \vec{B}$ $\quad \vec{m} = I(S)\hat{a}_n$

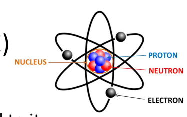


Section 8.5, 8.6

MAGNETIZATION

Magnetization

(similar to Polarization for E)

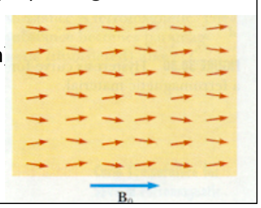


- Atoms have e- orbiting and spinning
  - Each have a magnetic dipole associated to it
- Most materials have **random orientation** of their magnetic dipoles if NO external B-field is applied.
- When a B field is applied, they try **to align**

in the same direction.

- The **total magnetization** [A/m]

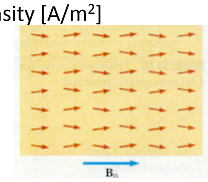
$$\vec{M} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} \vec{m}_k}{\Delta v}$$



Magnetization

- The magnetization current density [A/m²]

$$\vec{J}_b = \nabla \times \vec{M}$$



- The total magnetic density is:

$$\vec{B} = \mu_o(\vec{H} + \vec{M})$$

- Magnetic susceptibility is:

$$\vec{M} = \chi_m \vec{H} \qquad \vec{B} = \mu_o(1+\chi_m)\vec{H}$$

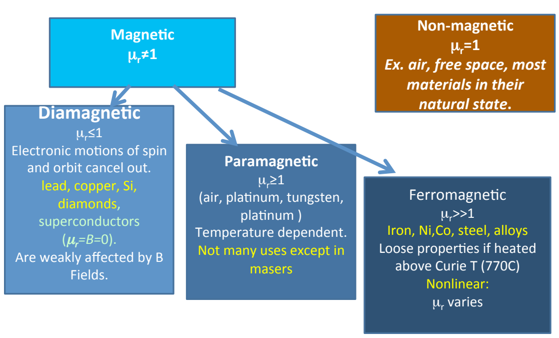
$$\vec{B} = \mu_o \mu_r \vec{H}$$

- The relative permeability is:

Permeability unit is [H/m].

$$\mu_r = (1+\chi_m) = \frac{\mu}{\mu_o}$$

Classification of Materials according to magnetism



$\chi_m = \mu_{r-1}$ at 20⁰C

- Pure iron 150- 200,000
- Steel 50-100
- Iron oxice 720
- Iron Amonium alum 66
- Uranium 40
- Platinum 26
- Aluminum 2.2
- Magnesium 1.2
- Sodium 0.72
- Oxygen gas 0.19
- Amomia = -0.26

MagLev magnetic Levitation

- Use diamagnetic materials, which repel and are repelled by strong H fields.
- Superconductors are diamagnetic.

- Floating one magnet over another
- Regular train 187 mph
- Maglev train 312 mph

http://www.youtube.com/watch?v=aIwbrZ4knpg

P.E. 8.7 B-field in a magnetic material. $\vec{B} = 10e^{-y}\hat{z} \quad mWb/m^2$

In a region with μ_r=4.6

- Find H, M and susceptibility

$$\chi_m + 1 = \mu_r$$

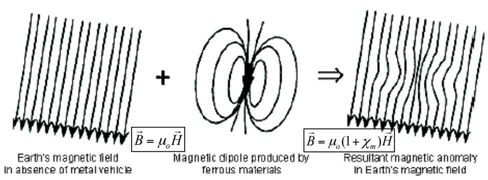
$$\chi_m = 3.6$$

$$H = \frac{\vec{B}}{\mu}$$

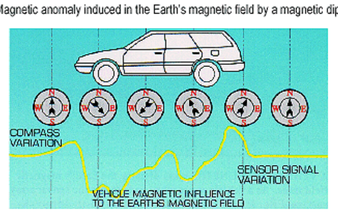
$$\vec{M} = \chi_m \vec{H}$$

$$\vec{H} = 1730e^{-y}\hat{z}[\text{A/m}]$$

$$\vec{M} = 6230e^{-y}\hat{z} \ \ A/m$$



(a) Magnetic anomaly induced in the Earth's magnetic field by a magnetic dipole.



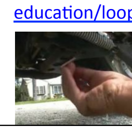
(b) Perturbation of Earth's magnetic field by a ferrous metal vehicle (Source: Nu-Metrics, Vanderbilt, PA).

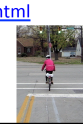
How to make traffic light go Green when driving a bike or motorcycle

- Stop directly on top of induction loop on the street
- Attach neodymium magnets to the vehicle
- Move on top of loop
- Push crossing button
- Video detectors

- http://www.wikihow.com/Trigger-Green-Traffic-Lights

- http://www.labreform.org/education/loops.html



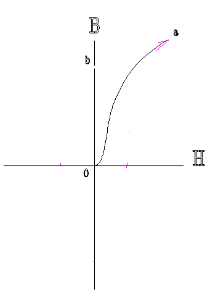


B-H or Magnetization curve

- When an H-field is applied to ferromagnetic material, it's *B* increases until saturation.

$$\mu = \frac{B}{H}$$

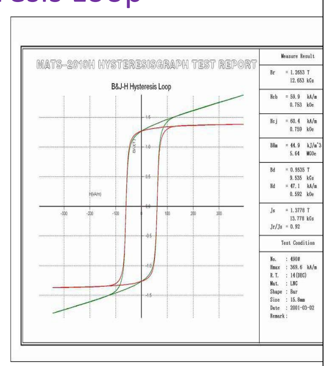
But when *H* is decreased, *B* doesn't follow the same curve.



Hysteresis Loop

Hysteresis Loop

- Some ferrites, have almost rectangular B-H curves, ideal for **digital** computers for storing information.
- The **area** of the loop gives the energy loss per volume during one cycle in the form of heat.
- **Tall-narrow loops** are desirable for electric generators, motors, transformers to minimize the hysteresis losses.



Section 8.7 (not covered)

MAGNETIC B.C.

Magnetic B.C. [Boundary Conditions]

- use Gauss Law

&

- Ampere's Circuit law

$$\oint B \cdot dS = 0$$

$$\oint H \cdot dl = I$$

B.C.: Two magnetic media

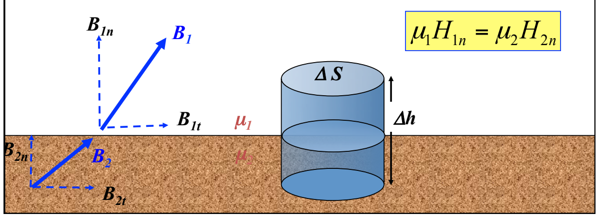
- Consider the figure below:

$$\oint_S B \cdot dS = 0$$

$$B_{1n}\Delta S - B_{2n}\Delta S = 0$$

$B_{1n} = B_{2n}$ is continuous.

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$



B.C.: Two magnetic media

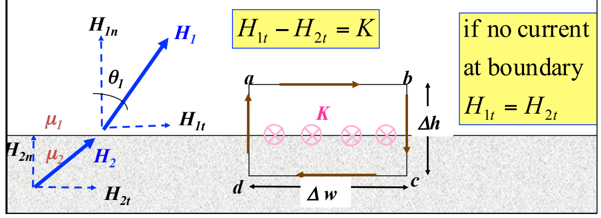
- Consider the figure below:

$$\oint_l H \cdot dl = I = K \cdot \Delta w$$

$$= H_{1t}\Delta w + H_{1n}\frac{\Delta h}{2} + H_{2n}\frac{\Delta h}{2} - H_{2t}\Delta w - H_{2n}\frac{\Delta h}{2} - H_{1n}\frac{\Delta h}{2}$$

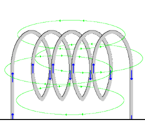
$$H_{1t} - H_{2t} = K$$

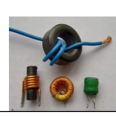
if no current at boundary $H_{1t} = H_{2t}$

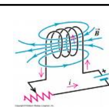


Section 8.8

INDUCTORS AND INDUCTANCE







$$\Psi = \int_S \vec{B} \cdot d\vec{S} \quad [\text{Wb}]$$

Inductors

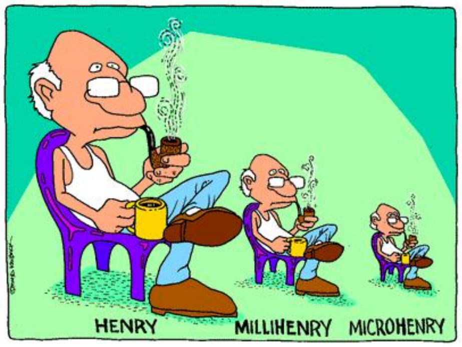
- If flux passes thru N turns, the total Flux Linkage is $\lambda = N\Psi$
- This is proportional to the current I

- So we can define the inductance as:

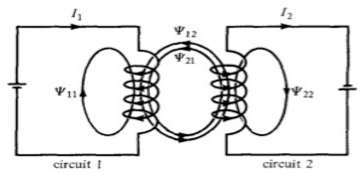
$$\lambda = LI$$

- then: $L = \dfrac{N\Psi}{I}$
- The energy in Joules: $W_m = \dfrac{1}{2}LI^2$

Units of Inductance

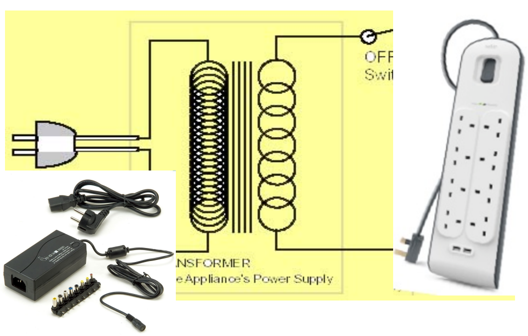


When more than 1 inductor



*Don't confuse the Magnetization vector, M, with the mutual inductance!

¿Qué son Cargas Fantasmas?



Self -inductance

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1\Psi_1}{I_1} \qquad L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2\Psi_2}{I_2}$$

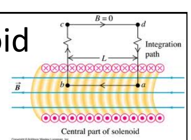
- The total energy in the magnetic field is the sum of the energies:

$$W_m = W_1 + W_2 + W_{12}$$
$$= \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 \pm M_{12} I_1 I_2$$

- The positive is taken if currents I_1 and I_2 flow such that the magnetic fields of the two circuits strengthen each other.

See table 8.3 in textbook with formulas for inductance of common elements like coaxial cable, two-wire line, etc.

P.E. 8.10 Solenoid



A long solenoid with 2 x 2 cm cross section has iron core (permeability is 1000x) and 4000 turns per meter. If carries current of 0.5A, Find:

- Self inductance per meter, *L'*

$$L' = \frac{L}{l} = N\Psi / Il$$

$$\Psi = BS$$

$$B = \mu H = \frac{\mu I N}{l} = \mu I n$$

Note *L* is Independent of current

$$= 1000\mu_o (4000)^2 (2cm)^2$$
$$= 8.042 H/m$$

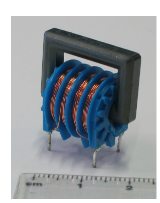
Energy stored

It can be shown that

- Energy stored in Magnetic Field is:

$$W_m = \frac{1}{2}\int \mu H^2 dv$$

- Units=Joules



Section 8.10

MAGNETIC CIRCUITS

Magnetic Circuits

- Magnetomotive force

  In units of ampere-turns

  $$\mathscr{F} = NI = \oint H \cdot dl$$

- Reluctance

  $$\mathscr{R} = \frac{l}{\mu S}$$

- Like V=IR

  $$\mathscr{F} = \Psi \mathscr{R}$$

- Ex: magnetic relays, motors, generators, transformers, toroids
- Table 8.4 presents analogy between magnetic and electric circuits

Magnetic Circuits

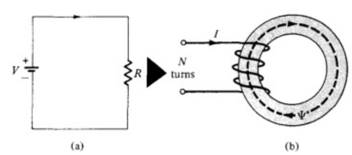


Figure 8.24 Analogy between **(a)** an electric circuit, and **(b)** a magnetic circuit.

Magnetic Circuits

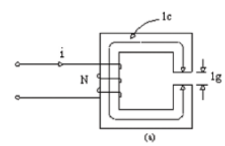
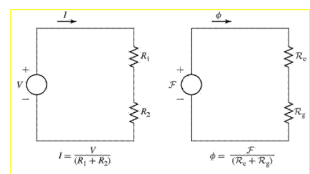
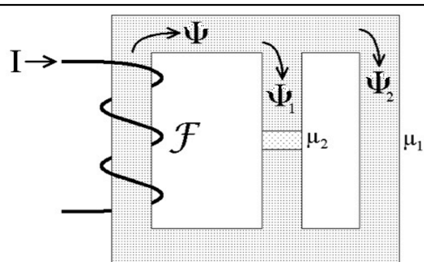


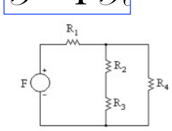
Figure 3. Magnetic nucleus with air gap (a) and an equivalent magnetic circuit (b).

ELECTRIC CIRCUIT RULES APPLY!



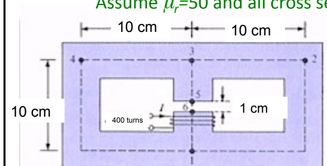


$$\mathscr{R} = \frac{l}{\mu S} \qquad \mathscr{F} = \Psi \mathscr{R} \qquad \Psi = \Psi_1 + \Psi_2$$



Ex. Find current in coil needed to produced magnetic field density of 1.5T in air gap.

Assume μ_r=50 and all cross sectional area is 10 cm^2

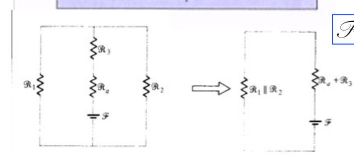


$$\mathscr{R} = \frac{l}{\mu S}$$

$$\mathscr{R}_1 = \frac{0.30}{50\mu_o 10/10^4} = \mathscr{R}_2$$

$$\mathscr{R}_3 = \frac{0.09}{50\mu_o 10/10^4} \qquad \mathscr{R}_{air} = \frac{0.01}{1\mu_o 10/10^4}$$

$$\mathscr{F} = NI = 400I = \Psi \mathscr{R}_T$$



$$\Psi = \Psi_a = B_a S$$

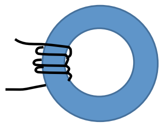
$$I = 44.16A$$

8.42 A cobalt ring (μ_r=600) has mean radius of 30cm.

- If a coil wound on the ring carries 12A, calculate the N required to establish an average magnetic flux density of 1.5 Teslas in the ring.

$$\mathscr{F} = NI = \oint H \cdot dl$$

$$NI = Hl$$



$$N = \frac{Bl}{\mu_r \mu_o I} = \frac{1.5 \times 2\pi(0.3)}{600\mu_o 12} = 313 \text{ vueltas}$$