Dr Cruz-Pol

Solutions to assigned Antenna problems from Balanis

6.1) Three isotropic sources, with spacing d between them, are placed along the z-axis. The excitation coefficient of each outside element is unity while that of the center element is 2. for a spacing of $d = \lambda/4$ between the elements, find the
a) array factor
b) angles (in degrees) where the nulls of the pattern occur (between 0 and 180 degrees)
c) angles where the maxima of the pattern occur.
Solutions:

6.1a)

$$E_{tot} = E_1 + E_2 + E_3 = 2E_o \frac{e^{-jkr}}{r} + E_o \frac{e^{-jkr_1}}{r_1} E_o \frac{e^{-jkr_2}}{r_2}$$

$$r_1 \approx r - d\cos\theta$$

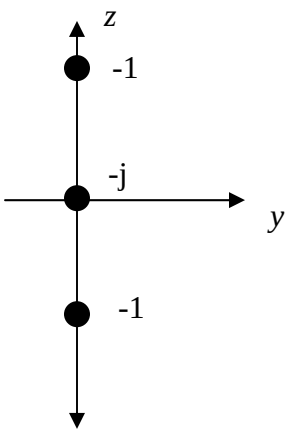
$$r_2 \approx r + d\cos\theta$$

$$AF(\theta) = \frac{1}{2}\left[1 + \cos(kd\cos\theta)\right] = \cos^2\left(\frac{kd}{2}\cos\theta\right)$$

[using $2\cos^2(A) = 1 + \cos(2A)$]

6.1b) the nulls are at AF=0 or $\cos^{-1}(2n)$, n= 1,-1, +3,-3,…, Therefore no nulls exist.
6.1c) AF= 1 or $\theta_m = 90^o$

6.3) a 3-element array of isotropic sources has the phase and magnitude relationships shown.



The spacing between the elements is $d = \lambda/2$.
a) Find the array factor

$$a)\, AF = 2\sin(\pi\cos\theta) + 1 = \frac{\sin\frac{3}{2}\left[\pi\cos\theta - \pi/2\right]}{\sin\frac{1}{2}\left[\pi\cos\theta - \pi/2\right]}$$

b) all the nulls $b)\, AF = 0\ at$:

$$\theta_{null} = 99.6^o, 146.44^o$$

6.10) Design an ordinary end-fire uniform linear array with only one maximum so that its directivity is 20dB (above isotropic). The spacing between the elements is $d = \lambda/4$, and its length is much greater than the spacing. Determine the:
(a) number of elements
b) Overall length of the array (in wavelengths)
c) Approximate half-power beamwidth (deg)
amplitude level (compared to the maximum of the major lobe) of the first minor lobe (in dB)
Solution: a) N=100, b) L =24.75λ, c) θ=21.6, d) SSL=-13.5dB, e) 90 degrees

6.14) Find the beamwidth and directivity of a 10-element uniform scanning array of isotropic sources laced along the z-axis. The spacing between the elements is $\lambda/4$ and the maximum is directed at 45^o from its axis.

Dr Cruz-Pol
Solution:
a) 30.2 degrees, b) D= 5.321

6.30) Design a broadside binomial array of six elements placed along the z-axis separated by a distance $d = \lambda/2$

a) Find the amplitude excitation coefficients
b) What is the progressive phase excitation between the elements?
c) Write the array factor.
d) Now assume that the elements are $\lambda/4$ dipoles oriented in the z-direction. Write the expression for the electric field vector in the far field.

Solution:

a_1= 10, a_2=5 , a_3=1, α=0

$$AF = 2\sum_{n=1}^{3} a_n \cos\left[(2n-1)\frac{\pi d}{\lambda}\cos\theta\right]$$

$$E = \hat{\theta}\, j\eta \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right) - \cos\left(\frac{\pi}{4}\right)}{\sin\theta}\right]\left\{10\cos\left(\frac{\pi}{2}\cos\theta\right) + 5\cos\left(\frac{3\pi}{2}\cos\theta\right) + \cos\left(\frac{5\pi}{2}\cos\theta\right)\right\}$$

6.40) Design a 5-element, -40dB SSL Dolph-Tschebyscheff array of isotropic elements. The elements are placed along the x-axis with a spacing of $d = \lambda/4$ between them. Find the

a) normalized amplitude coefficients
b) array factor
c) directivity
d) half-power beamwidth

Solution: a_3 = 16.429, a_2 = 49.503, a_1 = 34.074

$AF = 2.074 + 3.013\cos\left(\frac{\pi}{2}\cos\phi\right) + \cos(\pi\cos\phi)$, D_o=1.889= 2.76dB, HPBW= 54.9^o

6.46) In high-performance radar arrays low-sidelobes are very desirable. In a particular application it is desired to design a broadside linear array which maintains all the sidelobes at the same level of -30 dB. The number of elements must be 3 and the spacing between them must be $\lambda/4$.

a) state the design that will meet the specifications
b) what are the amplitude excitations of the elements?
c) What is the half-power beamwidth (in deg) of the main lobe?
d) What is the directivity (in dB) of the array?

Solution: Tschebyscheff with a_2 = 8.157, a_1 = 15.314 f = 1.144
HPBW= 72.4 , HPBW$_{Tsc}$= 82.8256, D_o=1.312= 1.1793 dB

6.50 Design a 10 x 8 (10 in the x direction and 8 in the y) element uniform planar array so that the main maximum is oriented along $\theta_o = 10_o, \phi_o = 90^o$. For a spacing of $d_x = d_y = \lambda/8$ between the elements, find the progressive phase shift in both directions, directivity of the array.