Wave Incidence

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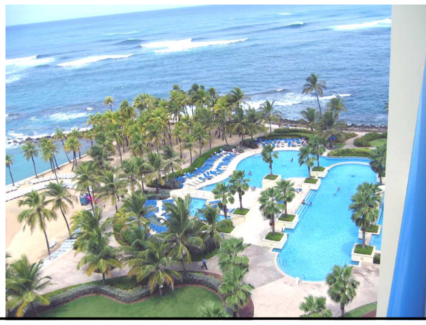
Ex. Light traveling in air encounters the water; another medium.



Wave incidence

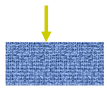
- For many applications, like fiber optics, it's necessary to know what happens to a wave when it meets a different medium.

How much is *transmitted*?

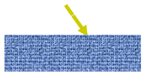
How much is *reflected* back?

We will look at…

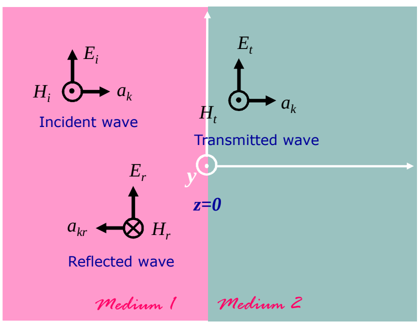
I.
Wave arrives at 0º from normal
Standing waves



II.
Wave arrives at another angle
Snell's Law and Critical angle
Parallel or Perpendicular
Brewster angle

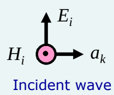


Reflection at Incidence



Now in terms of equations …

- Incident wave

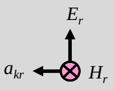


$$\vec{E}_{is}(z) = E_{io}e^{-\gamma_1 z}\hat{x}$$

$$\vec{H}_{is}(z) = H_{io}e^{-\gamma_1 z}\hat{y} = \frac{E_{io}}{\eta_1}e^{-\gamma_1 z}\hat{y}$$

Reflected wave

- It's traveling along $-z$ axis

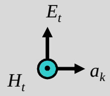


Reflected wave

$$\vec{E}_{rs}(z) = E_{ro} e^{\gamma_1 z} \hat{x}$$

$$\vec{H}_{rs}(z) = H_{ro} e^{\gamma_1 z}(-\hat{y}) = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z} \hat{y}$$

Transmitted wave



Transmitted wave

$$\vec{E}_{ts}(z) = E_{to} e^{-\gamma_2 z} \hat{x}$$

$$\vec{H}_{ts}(z) = H_{to} e^{-\gamma_2 z} \hat{y} = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z} \hat{y}$$

The total fields

- At medium 1 and medium 2

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \qquad \vec{E}_2 = \vec{E}_t$$

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r \qquad \vec{H}_2 = \vec{H}_t$$

- Tangential components must be continuous at the interface

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_t(0)$$

$$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

Define

- Reflection coefficient, Γ

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

- Transmission coefficient, τ

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Note:

- $1 + \Gamma = \tau$
- Both are dimensionless and may be complex
- $0 \le |\Gamma| \le 1$

PE 10.8

A 5GHz uniform plane wave $E_{is} = 10e^{-j\beta z}\ a_x$ in free space is incident normally on a large plane, lossless dielectric slab ($z>0$) having $\varepsilon = 4\varepsilon_o$ and $\mu = \mu_o$.

Find:

- the reflected wave E_{rs} and
- the transmitted wave E_{ts}.

Case 1:

- Medium 1 = perfect dielectric
  - $\sigma_1 = 0$
- Medium 2 = perfect conductor
  - $\sigma_2 = \infty$

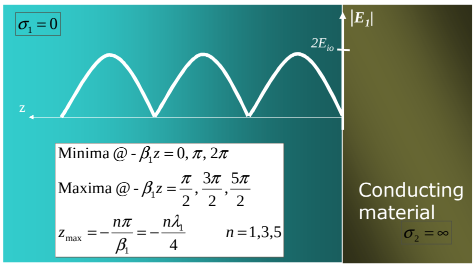
Halla impedancias int.
Reflección,
Transmisión
Y campos

$$\eta_2 = 0,$$
$$\Gamma = -1, \tau = 0$$
$$E_{1s} = -2jE_{io} \sin \beta_1 z\, \hat{x}$$
$$E_1 = 2E_{io} \sin \beta_1 z \sin \omega t\, \hat{x}$$

http://www.phy.ntnu.edu.tw/java/waveSuperposition/waveSuperposition.html

The EM field forms a Standing Wave on medium 1

$$E_1 = 2E_{io}\sin\beta_1 z \sin\omega t\,\hat{x}$$



Standing Wave Applets

- http://www.phy.ntnu.edu.tw/java/waveSuperposition/waveSuperposition.html
- http://www.ngsir.netfirms.com/englishhtm/StatWave.htm
- http://www.physics.smu.edu/~olness/www/03fall1320/applet/pipe-waves.html

Case 2:

- Medium 1 = perfect dielectric σ_1=0
- Medium 2 = perfect dielectric σ_2=0

If $\eta_2 > \eta_1$,
$\Gamma > 0$,
τ and Γ are real.

$$E_{1s} = E_{is} + E_{rs} = E_{oi}(e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z}) = E_{oi}e^{-j\beta_1 z}(1 + \Gamma e^{+2j\beta_1 z})$$

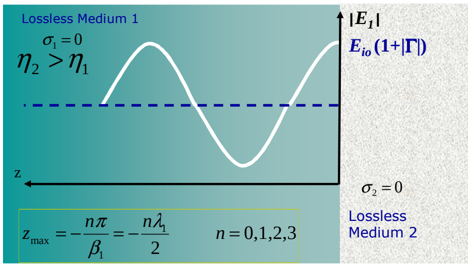
$$-2\beta_1 z_{max} = 0, 2\pi, 4\pi, 6\pi...$$
$$\text{or } -\beta_1 z_{max} = 0, \pi, 2\pi, 3\pi,...$$

$$z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2} \qquad n = 0,1,2,3$$

$$z_{min} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)\lambda_1}{4} \qquad n = 0,1,2,3$$

Standing waves due to reflection

$$E_1 = E_i + E_r = E_{oi}(e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z}) = E_{oi}e^{-j\beta_1 z}(1 + \Gamma e^{+2j\beta_1 z})$$



Case 3:

- Medium 1 = perfect dielectric σ_1=0
- Medium 2 = perfect dielectric σ_2=0

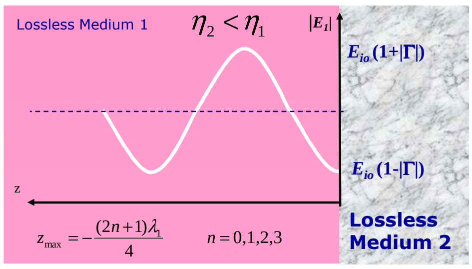
If $\eta_2 < \eta_1$,
$\Gamma < 0$, τ and Γ are real.

$$z_{max} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)\lambda_1}{4} \qquad n = 1,2,3$$

$$z_{min} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2} \qquad n = 0,1,2,3$$

Standing waves due to reflection

$$E_1 = E_i + E_r = E_{oi}(e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z}) = E_{oi}e^{-j\beta_1 z}(1 + \Gamma e^{+2j\beta_1 z})$$



Standing Wave Ratio, *s*

- Measures the amount of reflections, the more reflections, the larger the standing wave that is formed.
- The ratio of $|E_1|_{max}$ to $|E_1|_{min}$

$$s = \frac{|E_1|_{\max}}{|E_1|_{\min}} = \frac{|H_1|_{\max}}{|H_1|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Ideally (0dB) No reflections

or

$$|\Gamma| = \frac{s-1}{s+1}$$

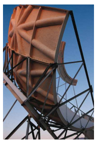
PE 10.9

- The plane wave $E=50 \sin(\omega t - 5x)\, \mathbf{a}_y$ V/m in a lossless medium ($\mu=4\mu_o$, $\varepsilon=\varepsilon_o$) encounters a lossy medium ($\mu=\mu_o$, $\varepsilon=4\varepsilon_o$, σ=0.1 mhos/m) normal to the *x*-axis at *x*=0. Find
- Γ
- τ
- s
- E_r
- E_t

Ex. Antenna Radome

A 10GHz aircraft radar uses a narrow-beam scanning antenna mounted on a gimbal behind a dielectric radome.

- Even though the radome shape is far from planar, it is approximately planar over the narrow extent of the radar beam.
- If the radome material is a lossless dielectric with μ_r=1 and ε_r=9, choose its thickness *d* such that the radome appears transparent to the radar beam.
- Mechanical integrity requires *d* to be greater that 2.3 cm.



Power Flow in Medium 1

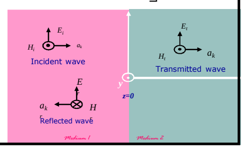
- The net <u>average power density</u> flowing in medium 1

$$P_{ave1}(z) = \frac{1}{2}\mathrm{Re}[E_1 \times H_1^*]$$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{x}E_{io}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right) \times \hat{y}\frac{E_{io}^*}{\eta_1}\left(e^{j\beta_1 z} + \Gamma^* e^{-j\beta_1 z}\right)\right]$$

$$= \hat{z}\frac{|E_{io}|^2}{2\eta_1}\left(1-|\Gamma|^2\right)$$

$$= P_{ave}^i + P_{ave}^r$$



Power Flow in wave

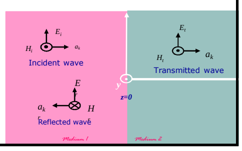
- The net average power density flowing in medium 2

$$P_{ave2}(z) = \frac{1}{2}\mathrm{Re}[E_2 \times H_2^*]$$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{x}\tau E_{io} e^{-j\beta_2 z} \times \hat{y}\tau^* \frac{E_{io}^*}{\eta_2} e^{j\beta_2 z}\right]$$

$$= \hat{z}|\tau|^2 \frac{|E_{io}|^2}{2\eta_2}$$

where $\hat{a}_k = \hat{z}$



Power in Media

$$P_{ave1}(z) = \hat{z}\frac{|E_o^i|^2}{2\eta_{c1}}\left(e^{-2\alpha_1 z} - |\Gamma|^2 e^{2\alpha_1 z}\right)$$

$$P_{ave2}(z) = \hat{z}|\tau|^2 \frac{|E_o^i|^2}{2} e^{-2\alpha_2 z}\,\mathrm{Re}\left(\frac{1}{\eta_{c2}^*}\right)$$

where

$$\Gamma = \frac{\sqrt{\varepsilon_{c1}} - \sqrt{\varepsilon_{c2}}}{\sqrt{\varepsilon_{c1}} + \sqrt{\varepsilon_{c2}}} \quad \text{and} \quad \varepsilon_{c2} = \varepsilon_2 - j\frac{\sigma_2}{\omega_2}$$

We will look at…

I. Normal incidence

Wave arrives at 90º from the surface

- Standing waves

II. Oblique incidence (   )

Wave arrives at an angle

- Snell's Law and Critical angle
- Parallel or Perpendicular
- Brewster angle