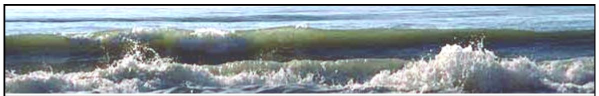

Wave Incidence at Oblique angles

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Quiz VIERNES

- Uno de los problemas asignados en el prontuario
- Enviar contestaciones a preguntas en Word por email

Wave Incidence

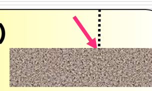
I. Normal incidence

Wave arrives at 90º from the surface

- Standing waves

II. Oblique incidence (lossless)

Wave arrives at an angle

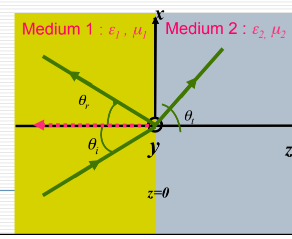


- Snell's Law and Critical angle
- Two types: Parallel or Perpendicular
- Brewster angle

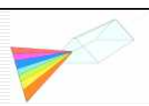
Oblique incidence

First, we need to define:

- Expression for uniform plane wave traveling in any direction
- The Normal , a_n
- Plane of incidence
- Angle of incidence



Oblique incidence



- Uniform plane wave in general form

$$E(\vec r,t)=E_o\cos(\vec k\cdot\vec r-\omega t)=\mathrm{Re}[E_o e^{j(\vec k\cdot\vec r-\omega t)}]$$

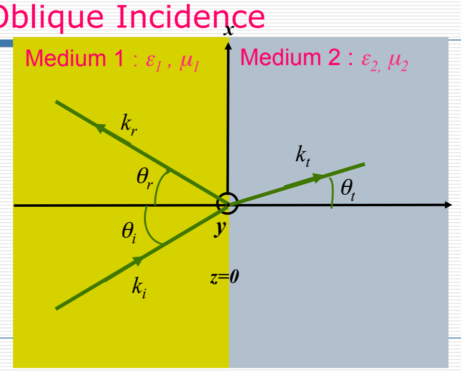
$\vec r = x\hat a_x + y\hat a_y + z\hat a_z$ the radius or position vector

$\vec k = k_x\hat a_x + k_y\hat a_y + k_z\hat a_z$ the wave number or propagation vector

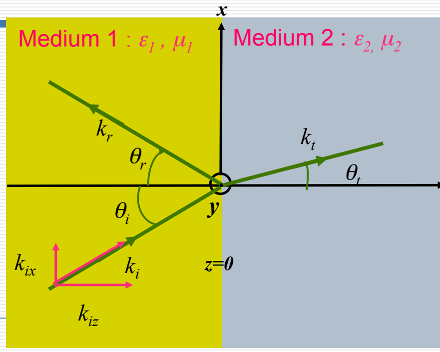
$$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon$$

- For lossless unbounded media, $k = \beta$

Oblique Incidence

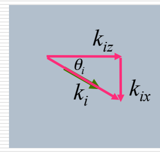


Look at the direction of travel



Expression for Waves

$$E_i = E_{io}\cos(k_{ix}x + k_{iz}z - \omega t)$$
$$E_r = E_{ro}\cos(k_{rx}x + k_{rz}z - \omega t)$$
$$E_t = E_{to}\cos(k_{tx}x + k_{tz}z - \omega t)$$



where $|k_i| = \sqrt{k_{ix}^2 + k_{iz}^2} = \beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$

$$k_{ix} = \beta_1 \sin\theta_i$$
$$k_{iz} = \beta_1 \cos\theta_i$$

Tangential E must be Continuous

$$\vec{E}_i(z=0) + \vec{E}_r(z=0) = \vec{E}_t(z=0)$$

$\omega_i = \omega_r = \omega_t = \omega$ ← From this we know that frequency is a property of the wave. So is color.

$k_{ix} = k_{rx} = k_{tx} = k_x$

$k_{iy} = k_{ry} = k_{ty} = k_y$

So 700nm is not always red!!

Give example of bathing suit.

$k_{ix} = k_{rx}$

$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r$

Snell Law

- Equating, we get

$$k_{ix} = k_{tx}$$
$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

Also written as,

or

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

where, the index of refraction of a medium, n_i, is defined as the ratio of the phase velocity in free space (c) to the phase velocity in the medium. $n_1 = \sqrt{\varepsilon_{r1}}$

Critical angle, θ_c

...All is reflected

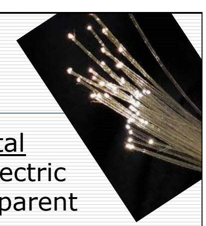
- When θ_t =90º, the refracted wave flows along the surface and no energy is transmitted into medium 2.
- The value of the angle of incidence corresponding to this is called critical angle, θ_c.
- If θ_i > θ_c, the incident wave is totally reflected.

$$\sin\theta_c = \frac{n_2}{n_1}\sin\theta_t \quad [\theta_t = 90^\circ]$$
$$= \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$$
(for $\mu_1 = \mu_2$)

Example

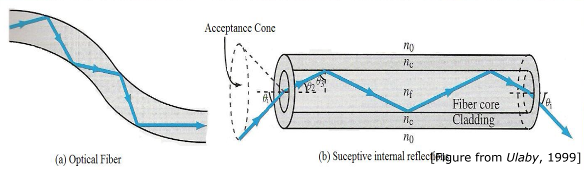
$$\sin\theta_c = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\sin 90^\circ$$
$$\sin 42^\circ = \sqrt{\frac{4}{9}}(1) = .67$$
$$\sin 50^\circ = .77 = .67(\sin ??^\circ)$$
$$\sin 40^\circ = .64 = .67(\sin 73^\circ)$$

Fiber optics



- Light can be guided with total reflections through thin dielectric rods made of glass or transparent plastic, known as optical fibers.
- The only power lost is due to reflections at the input and output ends and absorption by the fiber material (not perfect dielectric).

Optical fibers have cylindrical fiber core with index of refraction n_f, surrounded by another cylinder of lower, $n_c < n_f$, called a cladding.



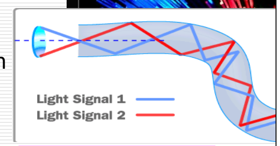
Waves can be guided along optical fibers as long as the reflection angles exceed the critical angle for total internal reflection.

$$\sin\theta_3 \ge \sin\theta_c = \frac{n_2}{n_1} \qquad \theta_2 + \theta_3 = 90^o$$

Acceptance angle

$$\sin\theta_a \le \frac{\sqrt{(n_f^2 - n_c^2)}}{n_0}$$

Exercise: optical fiber



- An optical fiber (in air) is made of fiber core with index of refraction of 1.52 and a cladding with an index of refraction of 1.49.
- Find the acceptance angle:

$$\sin\theta_a \le \frac{\sqrt{(n_f^2 - n_c^2)}}{n_0} = \frac{\sqrt{(1.52^2 - 1.49^2)}}{1}$$

Answer: 17.5 degrees

Parallel polarization

- It's defined as E is || to incidence plane

$$E_{is} = E_{io}(\cos\theta_i\hat{x} - \sin\theta_i\hat{z})e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

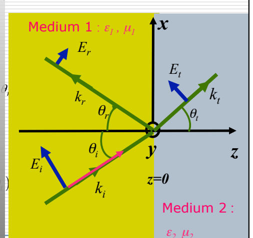
$$H_{is} = \frac{E_{io}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\hat{y}$$

$$E_{rs} = E_{ro}(\cos\theta_r\hat{x} + \sin\theta_r\hat{z})e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$H_{rs} = -\frac{E_{ro}}{\eta_1}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\hat{y}$$

$$E_{ts} = E_{to}(\cos\theta_t\hat{x} - \sin\theta_t\hat{z})e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$H_{ts} = \frac{E_{to}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\hat{y}$$



Equating for continuity, the tangent fields

Which components are tangent to the interface between two surfaces?

- y and x

At $z = 0$ (interface):

$$\hat{x}: E_{io}(\cos\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} + E_{ro}(\cos\theta_r)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} = E_{to}(\cos\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\hat{y}: \frac{E_{io}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} - \frac{E_{ro}}{\eta_1}e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} = \frac{E_{to}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\hat{x}: E_{io}\cos\theta_i + E_{ro}\cos\theta_r = E_{to}\cos\theta_t$$

$$\hat{y}: \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

Reflection and Transmission Coefficients: Parallel Incidence

- Reflection

$$\Gamma_{\|} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\tau_{\|} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\text{where} \quad \tau_{\|} = (1 + \Gamma_{\|})\frac{\cos\theta_i}{\cos\theta_t}$$

Reflection and Transmission Coefficients: Perpendicular Incidence

$$\Gamma_{\perp} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\tau_{\perp} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$1 + \Gamma_{\perp} = \tau_{\perp}$$

Java Animation

- http://www.amanogawa.com/archive/Oblique/Oblique-2.html

Exercise

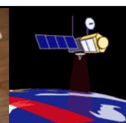

A plane wave in air with

$$E_i = \hat{y}10e^{-j(3x+4z)}[V/m]$$

Is incident upon planar surface of nonmagnetic dielectric material with ε_r=4, on z>0, Find

- The polarization of the incident wave
- The angle of incidence
- The time-domain expressions for the reflected electric and magnetic fields.
- The average power density carried by the wave in the dielectric medium.

Exercise

A plane wave in free sp

$$E_i = (10\hat{y} + 5\hat{z})\cos \quad]$$

Is incident upon planar ... tic lossless dielectric material w

- The polarization of the incident wave
- The angle of incidence and transmission
- The reflection and transmission coefficients.
- The E field in reflected and in dielectric
- Brewster angle

Answers:

$26.6^o, 12.9^o, \quad -0.295, 0.65,$

$(-2.946\hat{y} + 1.47\hat{z})\cos(\omega t + 2y + 4z),$

$(6.5\hat{y} + 3.2\hat{z})\cos(\omega t + 2y - 8.7z), 63.4^o$

Brewster angle, θ_B

- Is defined as the incidence angle at which the reflection coefficient is 0 (**all transmission**).

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_B}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_B} = 0$$

$$\eta_2 \cos\theta_t - \eta_1 \cos\theta_B = 0$$

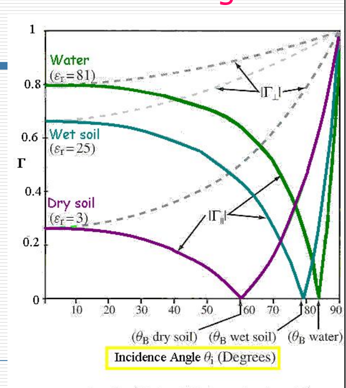
$$\sin\theta_{B\parallel} = \sqrt{\frac{1-(\varepsilon_1\mu_2/\varepsilon_2\mu_1)}{1-(\varepsilon_1/\varepsilon_2)^2}}$$

* θ_B is known as the polarizing angle

- The Brewster angle does not exist for perpendicular polarization for nonmagnetic materials.

Reflection vs. Incidence angle.

Reflection vs. incidence angle for different types of soil and parallel or perpendicular polarization.



Exercise (Brewster angle)

A wave in air is incident upon the flat boundary of a nonmagnetic soil medium with ε_r=4, at θ_i=50°.

- Find reflection and transmission coefficients for both incident polarizations, and the Brewster angle.
- Answers:

$$\Gamma_{\parallel} = -0.16, \quad \tau_{\parallel} = 0.58$$

$$\Gamma_{\perp} = -0.48, \quad \tau_{\perp} = 0.52$$

$$\theta_{B\parallel} = 63.4^o$$

Summary

Property	Normal Incidence	Perpendicular	Parallel
Reflection coefficient	$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Transmission coefficient	$\tau = \frac{2\eta_{2i}}{\eta_2 + \eta_{1t}}$	$\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\tau_\parallel = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Relation	$\tau = 1 + \Gamma$	$\tau_\perp = 1 + \Gamma_\perp$	$\tau_\parallel = (1 + \Gamma_\parallel)\frac{\cos\theta_i}{\cos\theta_t}$
Power Reflectivity	$R = \lvert\Gamma\rvert^2$	$R_\perp = \lvert\Gamma_\perp\rvert^2$	$R_\parallel = \lvert\Gamma_\parallel\rvert^2$
Power Transmissivity	$T = 1 - R$	$T_\perp = 1 - R_\perp$	$T_\parallel = 1 - R_\parallel$
Snell's Law:	$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i \quad \text{where} \quad n_2 = \sqrt{\mu_{r2}\varepsilon_{r2}}$		