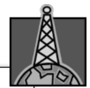

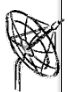

Electromagnetic waves

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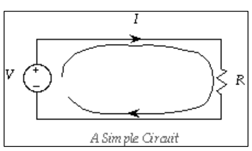
ECE UPR- Mayagüez, PR

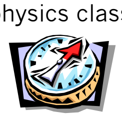
Outline

I. Faraday's Law & Origin of Electromagnetics
II. Transformer and Motional EMF
III. Displacement Current & Maxwell Equations
IV. Wave Incidence (normal, oblique)
  I. Lossy materials
  II. Multiple layers

Electricity => Magnetism

- In 1820 Oersted discovered that a steady current produces a magnetic field while teaching a physics class.





This is what Oersted discovered accidentally:

$$\oint_L \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot d\vec{S}$$

Would magnetism would produce electricity?

- Eleven(11) years later, and at the same time, (Mike) Faraday in London & (Joe) Henry in New York discovered that a time-varying magnetic field would produce an electric voltage!

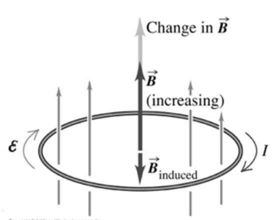
$$V_{emf} = -N\frac{d\Psi}{dt}$$

$$\oint_L E\cdot dl = -N\int_s \frac{\partial}{\partial t} B\cdot dS$$

Len's Law = (-)

- If N=1 (1 loop)

$$V_{emf} = -\frac{d\Psi}{dt}$$



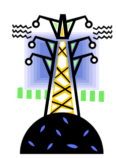
- The time change

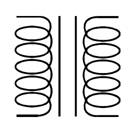
$$V_{emf} = \oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t} N\int_s \vec{B}\cdot d\vec{S}$$

can refer to B or S

Electromagnetics was born!

- This is Faraday's Law - the principle of motors, hydro-electric generators and transformers operation.



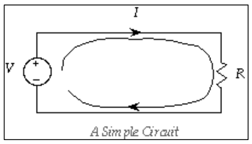


$$Faraday's\ Law \oint_L \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$$Ampere's\ Law \oint_L \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot d\vec{S}$$

Faraday's Law

- For N=1 and B=0



$$V_{emf} = -N\frac{d\Psi}{dt}$$

$$V_{emf} = \oint_L E \cdot dl = IR$$

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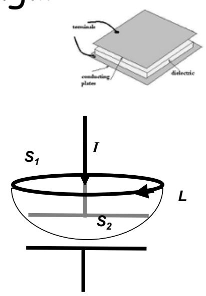
Maxwell noticed something was missing…

- And added J_d, the displacement current



$$\oint_L H \cdot dl = \int_{S_1} J \cdot dS = I_{enc} = I$$

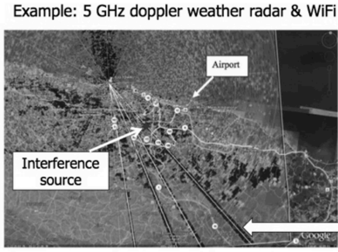
$$\oint_L H \cdot dl = \int_{S_2} J \cdot dS = 0$$

$$\oint_L H \cdot dl = \int_{S_2} J_d \cdot dS = \frac{d}{dt}\int_{S_2} D \cdot dS = \frac{dQ}{dt} = I$$

At low frequencies J>>J_d, but at radio frequencies both terms are comparable in magnitude.

Limitations of non-cooperative sharing

Example: 5 GHz doppler weather radar & WiFi LAN access point



Incident details:
- LAN access point < 250 mW
- In line of sight, 14 km from radar
- TX frequency 10MHz below radar band
- Transmitter leakage generated 6 dB interference/noise at the radar

Interference strobes

Field data from an incident in 2009, San Juan, Puerto Rico. Investigation results and figure from NTIA Technical Report TR-11-473

Spectrum access will be limited unless there is a mechanism to mitigate misconfiguration errors.

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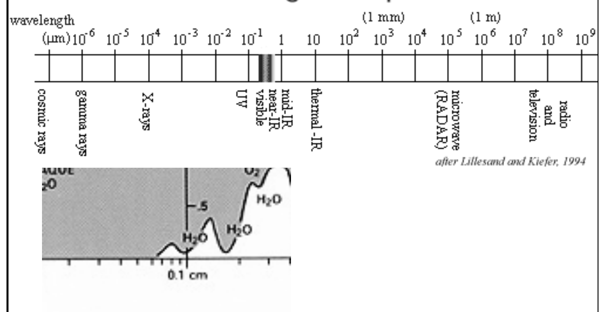
The Electromagnetic Spectrum



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Uniform plane em wave approximation

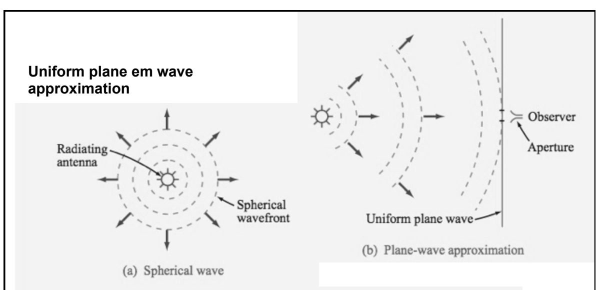

Figure 2-1: Waves radiated by an EM source, such as a lightbulb or an antenna, have spherical wavefronts, as in (a); to a distant observer, however, the wavefront across the observer's aperture appears approximately planar, as in (b).

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Maxwell Equations in General Form

Differential form	Integral Form	
$\nabla \cdot D = \rho_v$	$\oint_s D \cdot dS = \int_v \rho_v dv$	Gauss's Law for *E* field.
$\nabla \cdot B = 0$	$\oint_s B \cdot dS = 0$	Gauss's Law for *H* field. Nonexistence of monopole
$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_L E \cdot dl = -\frac{\partial}{\partial t}\int_s B \cdot dS$	Faraday's Law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_L H \cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$	Ampere's Circuit Law

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Would magnetism would produce electricity?

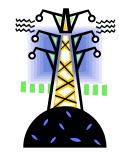
- Eleven years later, and at the same time, Mike Faraday in London and Joe Henry in New York discovered that a <u>time-varying</u> magnetic field would produce an electric current!

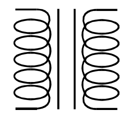
$$V_{emf} = -N\frac{d\Psi}{dt}$$

$$\oint_L E \cdot dl = -\frac{\partial}{\partial t}\int_s B \cdot dS$$

Electromagnetics was born!

- This is the principle of motors, hydro-electric generators and transformers operation.





This is what Oersted discovered accidentally:

$$\oint_L H \cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$$

***Mention some examples of em waves**

Special case

- Consider the case of a <u>lossless medium</u>

$$\sigma = 0$$

- with no charges, i.e. . $\rho_v = 0$

The wave equation can be derived from Maxwell equations as

$$\nabla^2 E + \omega^2 \mu \varepsilon_c E = 0$$

What is the solution for this differential equation?

- The equation of a wave!

Phasors for harmonic fields

Working with <u>harmonic fields</u> is easier, but requires knowledge of <u>phasor</u>.

- The phasor is multiplied by the time factor, $e^{j\omega t}$, and taken the real part.

$$\phi = \omega t + \theta$$

$$\mathrm{Re}\{re^{j\phi}\} = r\cos(\omega t + \phi)$$

$$\mathrm{Im}\{re^{j\phi}\} = r\sin(\omega t + \phi)$$

Maxwell Equations for Harmonic fields

Differential form*	
$\nabla \cdot \varepsilon E = \rho_v$	<u>Gauss's</u> Law for E field.
$\nabla \cdot \mu H = 0$	<u>Gauss's</u> Law for H field. No monopole
$\nabla \times E = -j\omega\mu H$	<u>Faraday's</u> Law
$\nabla \times H = J + j\omega\varepsilon E$	<u>Ampere's</u> Circuit Law

* (substituting $D = \varepsilon E$ and $H = \mu B$)

A wave

- Start taking the curl of Faraday's law

$$\nabla \times \nabla \times E_s = -j\omega\mu \nabla \times H_s$$

- Then apply the vectorial identity

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$$

- And you're left with

$$\nabla(\nabla \cdot E_s) - \nabla^2 E_s = -j\omega\mu(\sigma + j\omega\varepsilon)E_s$$
$$= -\gamma^2 E_s$$

A Wave

$$\nabla^2 E - \gamma^2 E = 0$$

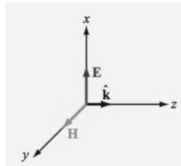
Let's look at a special case for simplicity without loosing generality:

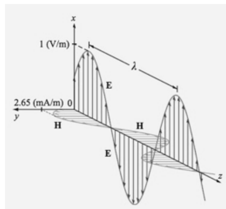
- **The electric field has only an *x*-component**
- **The field travels in *z* direction**

Then we have $E(z,t)$

whose general solution is

$$E(z) = E_o e^{-\gamma z} + E_o^{'} e^{+\gamma z}$$





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To change back to time domain

- From phasor

$$E_{xs}(z) = E_o e^{-\gamma z} = E_o e^{-z(\alpha + j\beta)}$$

- …to time domain

$$E(z,t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\hat{x}$$

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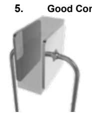
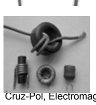
Several Cases of Media

1. **Free space** $(\sigma = 0,\ \varepsilon = \varepsilon_o,\ \mu = \mu_o)$
2. **Lossless dielectric** $(\sigma = 0,\ \varepsilon = \varepsilon_r\varepsilon_o,\ \mu = \mu_r\mu_o \text{ or } \sigma = 0)$
3. **Low-loss** $(\sigma \neq 0,\ \varepsilon = \varepsilon_r\varepsilon_o,\ \mu = \mu_r\mu_o \text{ or } \sigma << \omega\varepsilon)$
4. **Lossy dielectric** $(\sigma \neq 0,\ \varepsilon = \varepsilon_r\varepsilon_o,\ \mu = \mu_r\mu_o)$
5. **Good Conductor** $(\sigma \cong \infty,\ \varepsilon = \varepsilon_o,\ \mu = \mu_r\mu_o \text{ or } \sigma >> \omega\varepsilon)$



Permitivity: ε_o=8.854 x 10^{-12}[F/m]



Permeability: μ_o= 4π x 10^{-7} [H/m]

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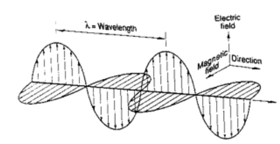
1. Free space

There are no losses, e.g.

$$E(z,t) = A\sin(\omega t - \beta z)\hat{x}$$

Let's define

- The phase of the wave
- The angular frequency
- Phase constant
- The phase velocity of the wave
- The period and wavelength
- How does it moves?



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3. Lossy Dielectrics (General Case)

$$E(z,t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\hat{x}$$

- In general, we had $\boxed{\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)}$ $\gamma = \alpha + j\beta$

$$-\operatorname{Re}\gamma^2 = \beta^2 - \alpha^2 = \omega^2\mu\varepsilon$$

$$\left|\gamma^2\right| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$$

- From this we obtain

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \quad \text{and} \quad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

- So , for a known material and frequency, we can find $\gamma = \alpha + j\beta$

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Summary

	Any medium	Lossless medium (σ=0)	Low-loss medium (ε"/ε' <.01)	Good conductor (ε"/ε' >100)	*Units*
α	$\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f\mu\sigma}$	[Np/m]
β	$\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f\mu\sigma}$	[rad/m]
η	$\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	[ohm]
u_c	ω/β	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\sqrt{\frac{4\pi f}{\mu\sigma}}$	[m/s]
λ	$2\pi/\beta = u_p/f$	$\frac{u_p}{f}$	$\frac{u_p}{f}$	$\frac{u_p}{f}$	[m]

**In free space; ε_o =8.85 10^{-12} F/m μ_o=4π 10^{-7} H/m η_o=120π Ω

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Table 2-1: Expressions for $\alpha, \beta, \eta_c, u_p$, and λ for various types of nonmagnetic media.†

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-Loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega \left[\frac{\mu_0 \varepsilon' \varepsilon_0}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right] \right]^{1/2}$	0	$\frac{\pi \varepsilon''}{\lambda_0 \sqrt{\varepsilon'}}$	$\frac{\pi \sqrt{2\varepsilon''}}{\lambda_0}$	(Np/m)
$\beta =$	$\omega \left[\frac{\mu_0 \varepsilon' \varepsilon_0}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} + 1 \right] \right]^{1/2}$	$\frac{2\pi \sqrt{\varepsilon'}}{\lambda_0}$	$\frac{2\pi \sqrt{\varepsilon'}}{\lambda_0}$	$\frac{\pi \sqrt{2\varepsilon''}}{\lambda_0}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu_0}{\varepsilon' \varepsilon_0}} \left(1 - j \frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\frac{\eta_0}{\sqrt{\varepsilon'}}$	$\frac{\eta_0}{\sqrt{\varepsilon'}}$	$\frac{(1+j)\eta_0}{\sqrt{2\varepsilon''}}$	(Ω)
$u_p =$	ω / β	$c/\sqrt{\varepsilon'}$	$c/\sqrt{\varepsilon'}$	$c\sqrt{2/\varepsilon''}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Notes: In practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' > 100$; $c = 3 \times 10^8$ m/s; $\eta_0 = 377\ \Omega$.

$$\varepsilon_c = \varepsilon' - j\frac{\sigma}{\omega \varepsilon_o}$$

(Relative) Complex Permittivity

$$k = \beta = \omega \sqrt{\mu_o \varepsilon_o \varepsilon_r}$$

**For lossless media,
The wavenumber, *k*, is equal to
The phase constant. This is
not so inside waveguides.**

The attenuation and phase constants can also be expressed as:

$$\alpha = -\omega \sqrt{\mu\ \varepsilon_o}\ \mathrm{Im}\left\{\sqrt{\varepsilon}\right\}$$
$$\beta = \omega \sqrt{\mu\ \varepsilon_o}\ Re\left\{\sqrt{\varepsilon}\right\}$$

Intrinsic Impedance, η

- If we divide *E* by *H*, we get units of ohms and the definition of the intrinsic impedance of a medium at a given frequency.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle \theta_\eta \qquad [\Omega]$$

$$E(z,t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\hat{x}$$
$$H(z,t) = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\hat{y}$$

***Not in-phase for a lossy medium**

Note…

$$E(z,t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\hat{x}$$
$$H(z,t) = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\hat{y}$$

- *E* and *H* are perpendicular to one another
- Travel is perpendicular to the direction of propagation
- The amplitude is related to the impedance
- And so is the phase
- H lags E

Loss Tangent

- If we divide the conduction current by the displacement current

$$\frac{|J_{cs}|}{|J_{ds}|} = \boxed{}$$

Relation between $tan\theta$ and ε_c

$$\nabla \times H = \sigma E + j\omega\varepsilon E = j\omega\varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]E$$

$$= j\omega\varepsilon_c E$$

The complex permittivity is

$$\varepsilon_c = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right] = \varepsilon' - j\varepsilon''$$

The loss tangent can be defined also as $\tan\theta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}$

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2. Lossless dielectric

$(\sigma = 0,\ \varepsilon = \varepsilon_r\varepsilon_o,\ \mu = \mu_r\mu_o$ or $\sigma = 0)$

- Substituting in the general equations:

$$\alpha = 0,\ \beta = \omega\sqrt{\mu\varepsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle 0^o$$

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Review: 1. Free Space

$(\sigma = 0,\ \varepsilon = \varepsilon_o,\ \mu = \mu_o)$

- Substituting in the general equations:

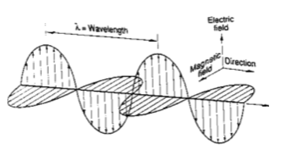
$$\alpha = 0,\ \beta = \omega\sqrt{\mu\varepsilon} = \omega / c$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c \qquad \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\mu_o}{\varepsilon_o}}\angle 0^o = 120\pi\ \Omega = 377\ \ \Omega$$

$$E(z,t) = E_o\cos(\omega t - \beta z)\hat{x} \quad V/m$$

$$H(z,t) = \frac{E_o}{\eta_o}\cos(\omega t - \beta z)\hat{y} \quad A/m$$



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4. Good Conductors

$(\sigma \cong \infty,\ \varepsilon = \varepsilon_o,\ \mu = \mu_r\mu_o)$

- Substituting in the general equations:

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \qquad \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$$

Is water a good conductor???

$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\hat{x} \quad [V/m]$$

$$H(z,t) = \frac{E_o}{\sqrt{\frac{\omega\mu}{\sigma}}_o} e^{-\alpha z}\cos(\omega t - \beta z - 45^o)\hat{y} \quad [A/m]$$

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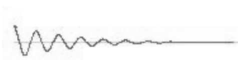
Skin depth, δ

- Is defined as the depth at which the electric amplitude is decreased to 37%

$$e^{-1} = 0.37 = (37\%)$$

$$e^{-\alpha z} = e^{-1} \text{ at } z = 1/\alpha = \delta$$



$$\boxed{\delta = 1/\alpha \quad [\text{m}]}$$

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Skin depth

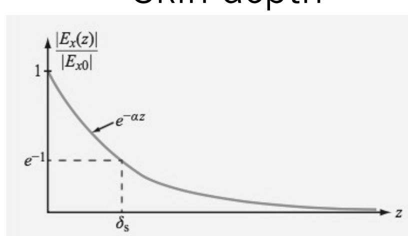

Figure 2-10: Attenuation of the magnitude of $E_x(z)$ with distance z. The skin depth δ_s is the value of z at which $|E_x(z)|/|E_{x0}| = e^{-1}$, or $z = \delta_s = 1/\alpha$.

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Short Cut …

- You can use Maxwell's or use

$$\vec{H} = \frac{1}{\eta}\hat{k} \times \vec{E}$$

$$\vec{E} = -\eta\,\hat{k} \times \vec{H}$$

where k is the direction of propagation of the wave, i.e., the direction in which the EM wave is traveling (a unitary vector).

Exercises: Wave Propagation in Lossless materials

- A wave in a nonmagnetic material is given by

$$\vec{H} = \hat{z}50\cos(10^9 t - 5y)\,[\text{mA/m}]$$

Find:

(a) direction of wave propagation,

(b) wavelength in the material

(c) phase velocity

(d) Relative permittivity of material

(e) Electric field phasor

Exercises: Wave Propagation in Lossless materials

- A wave in a nonmagnetic material is given by

$$\vec{H} = \hat{z}50e^{-2y}\cos(10^9 t - 5y)\,[\text{mA/m}]$$

Find:

(a) direction of wave propagation,

(b) wavelength in the material

(c) phase velocity

(d) Relative permittivity of material

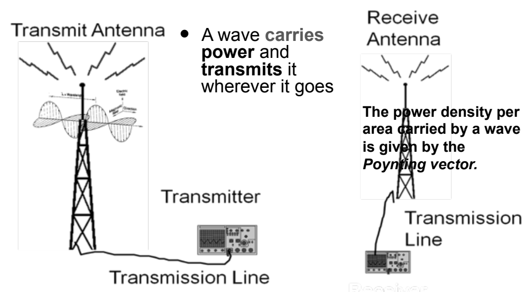
(e) Electric field phasor

Radio Wave Propagation

Power in a wave

- A wave **carries power** and **transmits** it wherever it goes

The power density per area carried by a wave is given by the *Poynting vector.*



Poynting Vector Derivation…

$$\underbrace{\oint_S (E \times H)\cdot dS}_{\text{Total power across surface of volume}} = -\underbrace{\frac{\partial}{\partial t}\int_v \left(\frac{\varepsilon}{2}E^2 + \frac{\mu}{2}H^2\right)dv}_{\text{Rate of change of stored energy in E or H}} - \underbrace{\int_v \sigma E^2 dv}_{\text{Ohmic losses due to conduction current}}$$

- Which means that the total power coming out of a volume is either due to the electric or magnetic field energy variations or is lost as ohmic losses.

Power: Poynting Vector

- Waves carry energy and information
- Poynting says that the net power flowing out of a given volume is = to the decrease in time in energy stored minus the conduction losses.

$$\vec{\mathcal{S}} = \vec{\mathcal{P}} = \vec{E} \times \vec{H} \quad [\text{W/m}^2]$$

Represents the instantaneous power vector associated to the electromagnetic wave.

Time Average Power

- The Poynting vector *averaged in time* is

$$\vec{\mathcal{S}}_{ave} = \frac{1}{T}\int_0^T \vec{\mathcal{S}}\,dt = \frac{1}{T}\int_0^T \left(\vec{E}\times\vec{H}\right)dt = \frac{1}{2}\mathrm{Re}\left\{\vec{E}_s \times \vec{H}_s^*\right\}$$

- For the general case wave:

$$E_s = E_o e^{-\alpha z} e^{-j\beta z}\hat{x} \quad [V/m]$$
$$H_s = \frac{E_o}{\eta} e^{-\alpha z} e^{-j\beta z}\hat{y} \quad [A/m]$$

$$\boxed{\vec{\mathcal{S}}_{ave} = \frac{|E_o|^2}{2|\eta|} e^{-2\alpha z}\cos\theta_\eta \hat{z}} \quad [\mathrm{W/m^2}]$$

For general lossy media

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Total Power in W

The <u>total power</u> through a surface *S* is

$$P_{ave} = \int_S \vec{\mathcal{P}}_{ave}\cdot dS \quad [W]$$

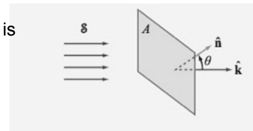

Figure 2-11: EM power flow through an apert

- Note that the units now are in <u>Watts</u>
- Note that the dot product indicates that the **surface area needs to be <u>perpendicular</u>** to the Poynting vector so that all the power will go thru. (give example of receiver antenna)

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Exercises: Power

1. At microwave frequencies, the power density considered safe for human exposure is 1 mW/cm^2. A radar radiates a wave with an electric field amplitude E that decays with distance as $E(R)$=3000/R [V/m], where R is the distance in meters. What is the radius of the unsafe region?

- Answer: 34.6 m

2. A 5GHz wave traveling in a nonmagnetic medium with ε_r=9 is characterized by $\vec{E} = \hat{y}3\cos(\omega t+\beta x) - \hat{z}2\cos(\omega t + \beta x)$[V/m] Determine the direction of wave travel and the average power density carried by the wave

- Answer: $\vec{\mathcal{S}}_{ave} = -\hat{x}0.05$ [W/m^2]

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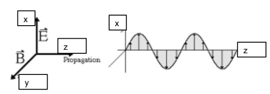
TEM wave



<u>T</u>ransverse <u>E</u>lectro<u>M</u>agnetic = plane wave

- **There are <u>no</u> fields <u>parallel</u> to the direction of propagation,**
- **only perpendicular (transverse).**
- **If have an electric field $E_x(z)$**
  - **...then must have a corresponding magnetic field $H_x(z)$**
- The direction of propagation is $\hat{a}_E \times \hat{a}_H = \hat{a}_k$

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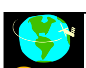


Polarization:

Why do we care??

- Antenna applications –
  - Antenna can only TX or RX a polarization it is designed to support. <u>Straight wires</u>, <u>square waveguides</u>, and similar rectangular systems support **linear waves** (polarized in one direction, often) <u>Circular waveguides</u>, <u>helical</u> or <u>flat spiral antennas</u> produce **circular or elliptical waves.**
- Remote Sensing and Radar Applications –
  - Many targets will <u>reflect</u> or <u>absorb</u> EM waves differently for different polarizations. Using multiple polarizations can give different information and improve results. Rain attenuation effect.
- Absorption applications –
  - Human body, for instance, will absorb waves with *E* oriented from <u>head to toe</u> better than <u>side-to-side</u>, esp. in grounded cases. Also, the frequency at which maximum absorption occurs is different for these two polarizations. This has ramifications in safety guidelines and studies.

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Polarization of a wave

IEEE Definition:

The trace of the tip of the E-field vector as a function of time seen from behind.

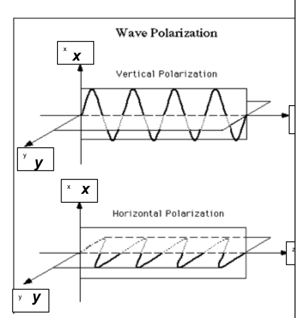
Basic types:

- *Vertical, E_x*

$$E_{xs}(z) = E_{ox}e^{-j\beta z}$$
$$E_x(z) = E_{ox}\cos(\omega t - \beta z)\hat{x}$$

- Horizontal, E_y

$$E_{ys}(z) = E_y = E_{oy}\,e^{-j\beta z + \delta}$$
$$E_y(z) = E_{oy}\cos(\omega t - \beta z + \delta)\hat{y}$$



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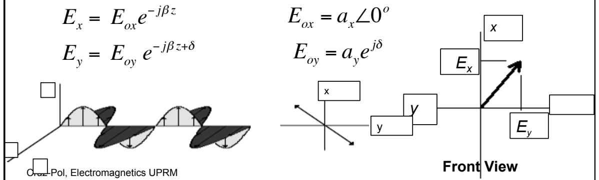
Polarization

- In general, plane wave has 2 components; in ***x*** & ***y***

$$E(z) = \hat{x}E_x + \hat{y}E_y$$

- *And y-component might be out of phase wrt to x-component, δ is the phase difference between x and y.*

$$E_x = E_{ox}e^{-j\beta z} \qquad E_{ox} = a_x\angle 0^o$$
$$E_y = E_{oy}\,e^{-j\beta z+\delta} \qquad E_{oy} = a_y e^{j\delta}$$



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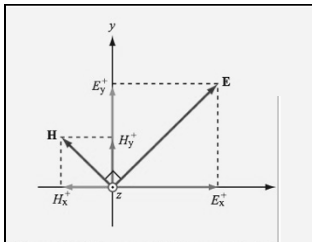

Figure 2-4: The wave (**E**,**H**) is equivalent to th[e sum] of two waves, one with fields (E_x^+, H_y^+) and anothe[r with] (E_y^+, H_x^+), with both traveling in the $+z$ direction.

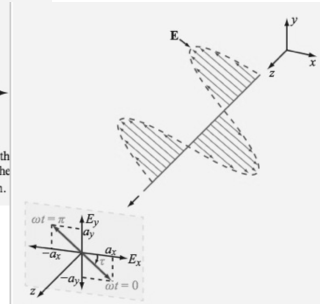

Figure 2-5: Linearly polarized wave traveling in the $+z$ direction.

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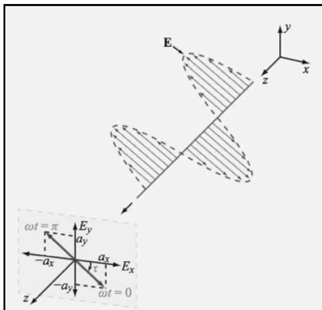

Figure 2-5: Linearly polarized wave traveling in the $+z$ direction.

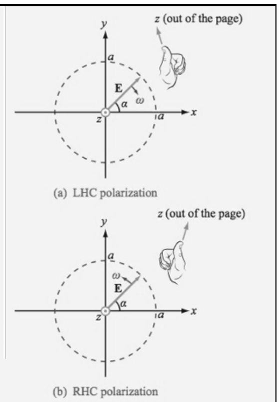

Figure 2-6: Circularly polarized plane waves propagating in the $+z$ direction (out of the page).

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Several Cases

- Linear polarization: $\delta=\delta_y-\delta_x = 0^o$ or $\pm 180^o n$
- Circular polarization: $\delta_y-\delta_x = \pm 90^o$ and $a_x = a_y$ *RHC is -90°*
- Elliptical polarization: $\delta_y-\delta_x = \pm 90^o$ and $E_{ox} \neq E_{oy}$, or $\delta \neq 0^o$ or $\neq 180^o n$ even if $E_{ox}=E_{oy}$
- Unpolarized- (Natural radiation)

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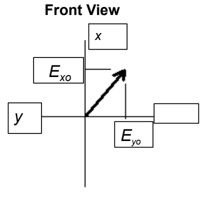
Linear polarization

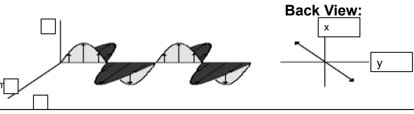
- $\delta = 0$

$$E_x = E_{xo}e^{-j\beta z}$$
$$E_y = E_{yo}\,e^{-j\beta z}$$



- @z=0 in time domain

$$E_x = a_x\cos(\omega t)$$
$$E_y = a_y\cos(\omega t)$$



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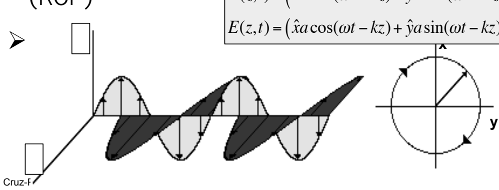
$$\cos(\xi \pm 90^o) = \mp\sin(\xi)$$

Circular polarization

- *Both components have same amplitude $a_x = a_y$,*
- $\delta = \delta_y - \delta_x = -90^o$ = Right circular polarized (RCP)
-

$$E_x = a_x\cos(\omega t)$$
$$E_y = a_y\cos(\omega t - 90^o)$$
in phasor:
$$E(z) = \left(\hat{x}a + \hat{y}ae^{-j90}\right)e^{-jkz} = a\left(\hat{x} - j\hat{y}\right)e^{-jkz}$$

$$E(z,t) = \left(\hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz - 90^o)\right)$$
$$E(z,t) = \left(\hat{x}a\cos(\omega t - kz) + \hat{y}a\sin(\omega t - kz)\right)$$



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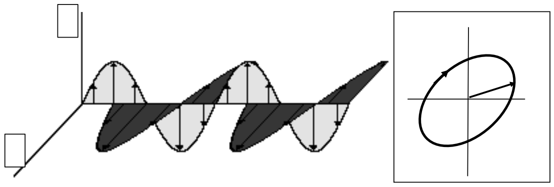
Elliptical polarization



- *X and Y components have different amplitudes* $a_x \neq a_y$, *and* $\delta = \pm 90^o$
- *Or* $\delta \neq \pm 90^o$ *and* $E_{ox} = E_{oy}$,

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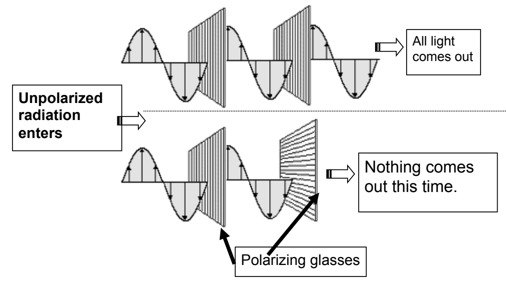
Polarization example



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Polarization Parameters

$$\tan 2\psi = (\tan 2\alpha_o)\cos\delta$$
$$\sin 2\chi = (\sin 2\alpha_o)\sin\delta$$

- Rotation angle, $-\frac{\pi}{2} \leq \psi \leq \frac{\pi}{2}$
- Ellipticity angle, $-\frac{\pi}{4} \leq \chi \leq \frac{\pi}{4}$
- *tan* of Axial ratio $\tan\alpha_o = \frac{a_y}{a_x}$

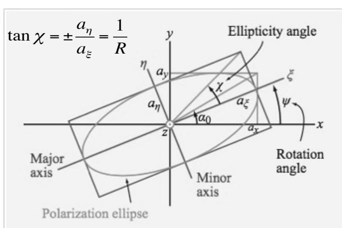

Figure 2-8: Polarization ellipse in the *x–y* plane, with the wave traveling in the *z* direction (out of the page).

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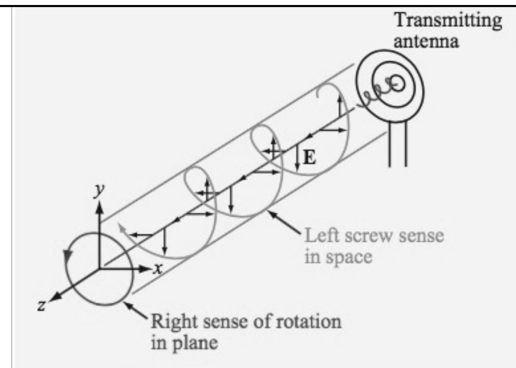

Figure 2-7: Right-hand circularly polarized wave radiated by a helical antenna.

Polarization for **em** waves

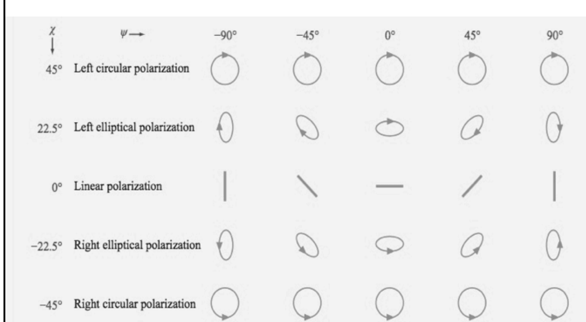

Figure 2-9: Polarization states for various combinations of the polarization angles (ψ, χ) for a wave traveling out of the page.†

$$\sin(\xi \pm 90^o) = \pm\cos(\xi) \qquad \sin(\xi \pm 180^o) = -\sin(\xi)$$
$$\cos(\xi \pm 90^o) = \mp\sin(\xi) \qquad \cos(\xi \pm 180^o) = -\cos(\xi)$$

- Determine the polarization state of a plane wave with electric field:

a. $E(z,t) = \hat{x}3\cos(\omega t - \beta z + 30^o) - \hat{y}4\sin(\omega t - \beta z + 45^o)$

b. $E(z,t) = \hat{x}3\cos(\omega t - \beta z + 45^o) + \hat{y}8\sin(\omega t - \beta z + 45^o)$

c. $E(z,t) = \hat{x}4\cos(\omega t - \beta z - 45^o) - \hat{y}4\sin(\omega t - \beta z + 45^o)$

d. $E_s(y) = 14(\hat{x} - j\hat{z})e^{-j\beta y}$

a. Elliptic
b. -90, RHEP
c. LP<135
d. -90, RHCP

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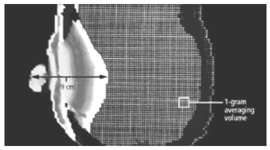
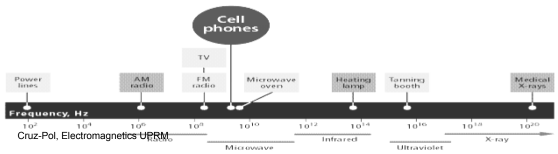
Cell phone & brain

- Computer model for Cell phone Radiation inside the Human Brain





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Decibel Scale

- In many applications need comparison of two powers, a power ratio, e.g. reflected power, attenuated power, gain,…
- The decibel (dB) scale is logarithmic

$$G = \frac{P_{out}}{P_{in}}$$

$$G[dB] = 10\log\left(\frac{P_{out}}{P_{in}}\right)$$

- Note that for voltages, the log is multiplied by 20 instead of 10.

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Power Ratios

G	G [dB]
10^x	10x dB
100	20 dB
4	6 dB
2	3 dB
1	0 dB
0.5	-3 dB
0.25	-6 dB
0.1	-10 dB
0.001	-30 dB

Attenuation rate, A

- Represents the rate of decrease of the magnitude of $P_{ave}(z)$ as a function of propagation distance

$$A = 10\log\left(\frac{P_{ave}(z)}{P_{ave}(0)}\right) = 10\log\left(e^{-2\alpha z}\right)$$

$$= -20\alpha z \log e = -8.68\alpha z = -\alpha_{dB} z \quad [\mathrm{dB}]$$

where

$$\alpha_{dB}[\mathrm{dB/m}] = 8.68\alpha[\mathrm{Np/m}]$$

Assigned problems ch 2 1-3,5,7,9,13,16,17,24,26,28, 32,36, 37,40,42, 43

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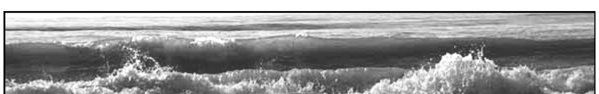

Reflection and Transmission

Wave incidence

- *Wave arrives at an angle*
  - Snell's Law and Critical angle
  - Parallel or Perpendicular
  - Brewster angle

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Incidence

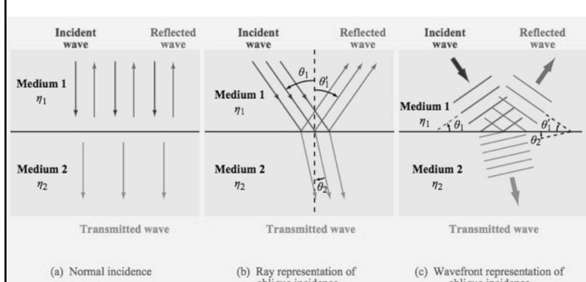

Figure 2-12: Ray representation of wave reflection and transmission at (a) normal incidence and (b) oblique incidence, and (c) wavefront representation of oblique incidence.

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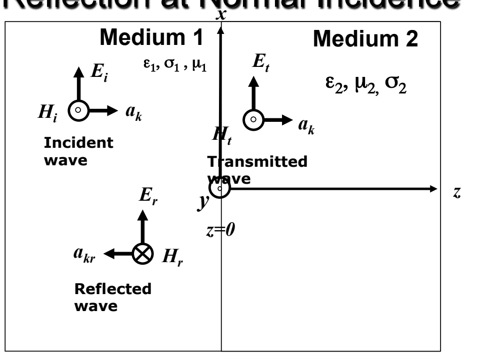
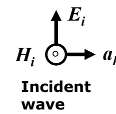
Reflection at Normal Incidence



Now in terms of equations …

- Incident wave

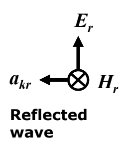


$$\vec{E}_{is}(z) = E_{io} e^{-\gamma_1 z}\hat{x}$$

$$\vec{H}_{is}(z) = H_{io} e^{-\gamma_1 z}\hat{y} = \frac{E_{io}}{\eta_1} e^{-\gamma_1 z}\hat{y}$$

Reflected wave

- It's traveling along $-z$ axis



$$\vec{E}_{rs}(z) = E_{ro} e^{\gamma_1 z}\hat{x}$$

$$\vec{H}_{rs}(z) = H_{ro} e^{\gamma_1 z}(-\hat{y}) = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z}\hat{y}$$

The total fields

- At medium 1 and medium 2

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \qquad \vec{E}_2 = \vec{E}_t$$

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r \qquad \vec{H}_2 = \vec{H}_t$$

- Tangential components must be continuous at the interface

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_t(0)$$

$$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

Normal Incidence

- Reflection coefficient, ρ

$$\rho = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\varepsilon_1'} - \sqrt{\varepsilon_2'}}{\sqrt{\varepsilon_1'} + \sqrt{\varepsilon_2'}}$$

- Transmission coefficient, τ

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Note:

- **$1 + \rho = \tau$**
- **Both are dimensionless and may be complex**
- **$0 \leq |\rho| \leq 1$**

Normal Incidence

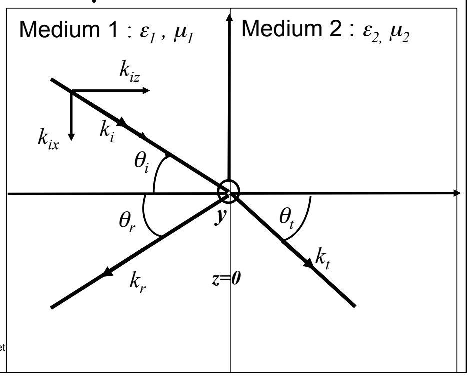
Table 2-3: Expressions for EM fields in lossless and lossy media under normal incidence.†

Lossless Media	Lossy Media
$\mathbf{E}^i = \hat{\mathbf{y}} E_0^i e^{jk_1 z}$, $\quad \mathbf{H}^i = \hat{\mathbf{x}} \frac{E_0^i}{\eta_1} e^{jk_1 z}$	$\mathbf{E}^i = \hat{\mathbf{y}} E_0^i e^{\gamma_1 z}$, $\quad \mathbf{H}^i = \hat{\mathbf{x}} \frac{E_0^i}{\eta_{c_1}} e^{\gamma_1 z}$
$\mathbf{E}^r = \hat{\mathbf{y}} \rho E_0^i e^{-jk_1 z}$, $\quad \mathbf{H}^r = -\hat{\mathbf{x}} \rho \frac{E_0^i}{\eta_1} e^{-jk_1 z}$	$\mathbf{E}^r = \hat{\mathbf{y}} \rho E_0^i e^{-\gamma_1 z}$, $\quad \mathbf{H}^r = -\hat{\mathbf{x}} \rho \frac{E_0^i}{\eta_{c_1}} e^{-\gamma_1 z}$
$\mathbf{E}^t = \hat{\mathbf{y}} \tau E_0^i e^{jk_2 z}$, $\quad \mathbf{H}^t = \hat{\mathbf{x}} \tau \frac{E_0^i}{\eta_2} e^{jk_2 z}$	$\mathbf{E}^t = \hat{\mathbf{y}} \tau E_0^i e^{\gamma_2 z}$, $\quad \mathbf{H}^t = \hat{\mathbf{x}} \tau \frac{E_0^i}{\eta_{c_2}} e^{\gamma_2 z}$
Phase matching: $(\mathbf{E}^i + \mathbf{E}^r)\big\|_{z=0} = \mathbf{E}^t\big\|_{z=0}$	Phase matching: $(\mathbf{E}^i + \mathbf{E}^r)\big\|_{z=0} = \mathbf{E}^t\big\|_{z=0}$
$(\mathbf{H}^i + \mathbf{H}^r)\big\|_{z=0} = \mathbf{H}^t\big\|_{z=0}$	$(\mathbf{H}^i + \mathbf{H}^r)\big\|_{z=0} = \mathbf{H}^t\big\|_{z=0}$
$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, $\quad \tau = 1 + \rho$	$\rho = \frac{\eta_{c_2} - \eta_{c_1}}{\eta_{c_2} + \eta_{c_1}}$, $\quad \tau = 1 + \rho$
$k_1 = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_1'}$, $\quad k_2 = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_2'}$	$\gamma_1 = \alpha_1 + j\beta_1$, $\quad \gamma_2 = \alpha_2 + j\beta_2$
$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_1'}}$, $\quad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_2'}}$	$\eta_{c_1} = \frac{\eta_0}{\sqrt{\varepsilon_1}}$, $\quad \eta_{c_2} = \frac{\eta_0}{\sqrt{\varepsilon_2}}$

Notes: $\varepsilon_1 = \varepsilon_1' - j\varepsilon_1''$; $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$.

Oblique Incidence

- Normal, a_n
- Plane of incidence
- Angle of incidence



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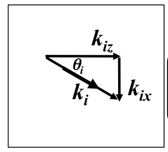
Expression for fields

$$E_i = E_{io}\cos(k_{ix}x + k_{iz}z - \omega t)$$
$$E_r = E_{ro}\cos(k_{rx}x + k_{rz}z - \omega t)$$
$$E_t = E_{to}\cos(k_{tx}x + k_{tz}z - \omega t)$$



where $|k_i| = \sqrt{k_{ix}^2 + k_{iz}^2} = \beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$

$$k_{ix} = \beta_1 \sin\theta_i$$
$$k_{iz} = \beta_1 \cos\theta_i$$

Tangential E must be Continuous

$$\vec{E}_i(z=0) + \vec{E}_r(z=0) = \vec{E}_t(z=0)$$

$$\omega_i = \omega_r = \omega_t = \omega$$
$$k_{ix} = k_{rx} = k_{tx} = k_x$$
$$k_{iy} = k_{ry} = k_{ty} = k_y$$

From this we know that frequency is a property of the wave. So is color.

So 700nm is not always red!!

$$k_{ix} = k_{rx}$$
$$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r$$

$$\theta_i = \theta_r$$

Snell Law

- Equating, we get $k_{ix} = k_{tx}$

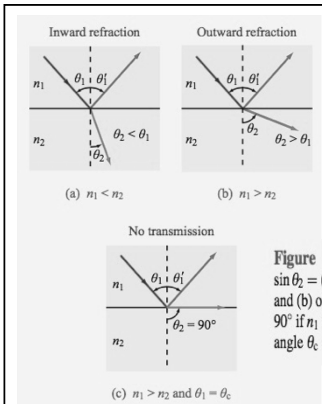
$$\boxed{\beta_1 \sin\theta_i = \beta_2 \sin\theta_t}$$

$$n_1 = \frac{c}{u_1} = \sqrt{\varepsilon_{r1}}$$

Also written as,

or $\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$

where, the index of refraction of a medium, n_i, is defined as the ratio of the phase velocity in free space (c) to the phase velocity in the medium.



When going to a denser medium, the refraction is inward.

Figure 2-15: Snell's laws state that $\theta_1' = \theta_1$ and $\sin\theta_2 = (n_1/n_2)\sin\theta_1$. Refraction is (a) inward if $n_1 < n_2$ and (b) outward if $n_1 > n_2$; and (c) the refraction angle is 90° if $n_1 > n_2$ and θ_1 is equal to or greater than the critical angle $\theta_c = \sin^{-1}(n_2/n_1)$.

Ulaby,

Critical angle, θ_c

...All is reflected

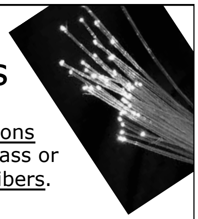
- When θ_t =90°, the refracted wave flows along the surface and no energy is transmitted into medium 2.
- The value of the angle of incidence corresponding to this is called critical angle, θ_c.
- If θ_i > θ_c, the incident wave is totally reflected.

$$\sin\theta_c = \frac{n_2}{n_1}\sin\theta_t \;\; [\theta_t = 90°]$$
$$= \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$$
(for $\mu_1 = \mu_2$)

$Example;\ \varepsilon_{r1} = 9;\ \varepsilon_{r2} = 4$

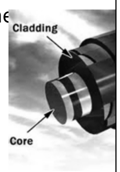
$$\sin\theta_c = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\sin 90°$$
$$\sin 42° = \sqrt{\frac{4}{9}}(1) = .67$$
$$\sin 40° = .64 = .67(\sin 73°)$$
$$\sin 50° = .77 = .67(\sin ??°)$$

Fiber optics

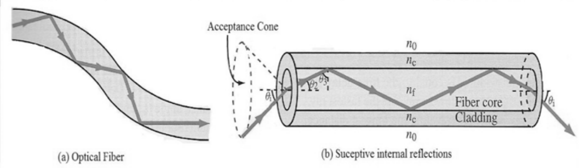


- Light can be guided with total reflections through thin dielectric rods made of glass or transparent plastic, known as optical fibers.

- The only power lost is due to reflections at the input and output ends and absorption by the fiber material (not perfect dielectric).



Optical fibers have cylindrical fiber core with index of refraction n_f, surrounded by another cylinder of lower, $n_c < n_f$, called a cladding.



[Figure from *Ulaby*, 1999]

Waves can be guided along optical fibers as long as the reflection angles exceed the critical angle for total internal reflection.

Use Snell and critical angle to derive:

- For total reflection:

$$\sin\theta_3 \ge \sin\theta_c = \frac{n_2}{n_1} \qquad \theta_2 + \theta_3 = 90^o$$

Acceptance angle

$$\sin\theta_a \le \frac{\sqrt{(n_f^2 - n_c^2)}}{n_0}$$

Perpendicular (H) or Parallel(V)

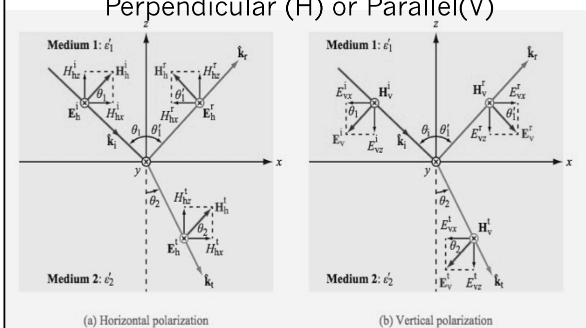


Figure 2-16: The plane of incidence is the plane containing the direction of wave travel, $\hat{k}_i$, and the surface normal to the boundary. In the present case the plane of incidence containing $\hat{k}_i$ and $\hat{z}$ coincides with the plane of the paper. A wave is (a) perpendicularly polarized (also called ***horizontally polarized***) when its electric field vector is perpendicular to the plane of incidence and (b) parallel polarized (also called ***vertically polarized***) when its electric field vector lies in the plane of incidence.

Parallel (V) polarization

- It's defined as E is || to incidence plane

$$E_{is} = E_{io}(\cos\theta_i\hat{x} - \sin\theta_i\hat{z})e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

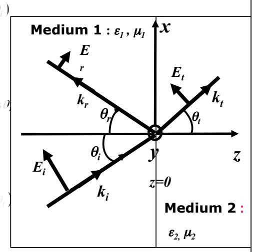
$$H_{is} = \frac{E_{io}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\hat{y}$$

$$E_{rs} = E_{ro}(\cos\theta_r\hat{x} + \sin\theta_r\hat{z})e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$H_{rs} = -\frac{E_{ro}}{\eta_1}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\hat{y}$$

$$E_{ts} = E_{to}(\cos\theta_t\hat{x} - \sin\theta_t\hat{z})e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$H_{ts} = \frac{E_{to}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\hat{y}$$



Equating for continuity, the tangent fields

Which components are tangent to the interface between two surfaces?

- y and x

At $z = 0$ (interface):

$$\hat{x}: E_{io}(\cos\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} + E_{ro}(\cos\theta_r)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} = E_{to}(\cos\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\hat{y}: \frac{E_{io}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} - \frac{E_{ro}}{\eta_1}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} = \frac{E_{to}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\boxed{\begin{aligned} &\hat{x}: E_{io}\cos\theta_i + E_{ro}\cos\theta_r = E_{to}\cos\theta_t \\ &\hat{y}: \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2} \end{aligned}}$$

Reflection and Transmission Coefficients: Parallel (V) Incidence

- Reflection

$$\rho_{\|} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\tau_{\|} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

where $\tau_{\|} = (1+\rho_{\|})\dfrac{\cos\theta_i}{\cos\theta_t}$

Reflection and Transmission Coefficients: Perpendicular(H) Incidence

$$\rho_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$1+\rho_\perp = \tau_\perp$$

Property	Normal Incidence	Perpendicular	Parallel
Reflection coefficient	$\rho = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$	$\boxed{\rho_{12} = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}}$	$\boxed{\rho_{12} = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1}}$
Transmission coefficient	$\tau = \frac{2\eta_{2i}}{\eta_2+\eta_{1t}}$	$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$	$\tau_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$
Relation	$\tau = 1+\rho$	$\boxed{\tau_{12} = 1+\rho_{12}}$	$\boxed{\tau_{12} = (1+\rho_{12})\frac{\cos\theta_1}{\cos\theta_2}}$
Power Reflectivity	$\Gamma = \lvert\rho\rvert^2$	$\Gamma_H = \lvert\rho_H\rvert^2$	$\Gamma_\parallel = \lvert\rho_\parallel\rvert^2$
Power Transmissivity	$T = 1-\Gamma$	$T_\perp = 1-\Gamma_\perp$	$T_\parallel = 1-\Gamma_\parallel$
Snell's Law: $\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i \quad \text{where} \quad n_2 = \sqrt{\mu_{r2}\varepsilon_{r2}}$			

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Table 2-5: Expressions for ρ, τ, Γ, and $\mathbb{T}$ for wave incidence from a lossless medium with intrinsic impedance η_1 onto a second lossless medium with intrinsic impedance η_2. Angles θ_1 and θ_2 are the angles of incidence and transmission, respectively.†

Property	Normal Incidence $\theta_1 = \theta_2 = 0$	Horizontal Polarization	Vertical Polarization
Reflection coefficient	$\rho = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$	$\rho_h = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$	$\rho_v = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1}$
Transmission coefficient	$\tau = \frac{2\eta_2}{\eta_2+\eta_1}$	$\tau_h = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$	$\tau_v = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1}$
Relation of ρ to τ	$\tau = 1+\rho$	$\tau_h = 1+\rho_h$	$\tau_v = (1+\rho_v)\frac{\cos\theta_1}{\cos\theta_2}$
Reflectivity	$\Gamma = \lvert\rho\rvert^2$	$\Gamma^h = \lvert\rho_h\rvert^2$	$\Gamma^v = \lvert\rho_v\rvert^2$
Transmissivity	$\mathbb{T} = \lvert\tau\rvert^2\left(\frac{\eta_1}{\eta_2}\right)$	$\mathbb{T}^h = \lvert\tau_h\rvert^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1}$	$\mathbb{T}^v = \lvert\tau_v\rvert^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1}$
Relation of Γ to $\mathbb{T}$	$\mathbb{T} = 1-\Gamma$	$\mathbb{T}^h = 1-\Gamma^h$	$\mathbb{T}^v = 1-\Gamma^v$
Notes: $\sin\theta_2 = \sqrt{\varepsilon_1'/\varepsilon_2'}\sin\theta_1$; $\eta_1 = \eta_0/\sqrt{\varepsilon_1'}$; $\eta_2 = \eta_0/\sqrt{\varepsilon_2'}$.			

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Brewster angle, θ_B

- Is defined as the incidence angle at which the reflection coefficient is 0 (total transmission).

$$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_B}{\eta_2\cos\theta_t + \eta_1\cos\theta_B} = 0$$

$$\eta_2\cos\theta_t - \eta_1\cos\theta_B = 0$$

$$\sin\theta_{B\parallel} = \sqrt{\frac{1-(\varepsilon_1\mu_2/\varepsilon_2\mu_1)}{1-(\varepsilon_1/\varepsilon_2)^2}}$$

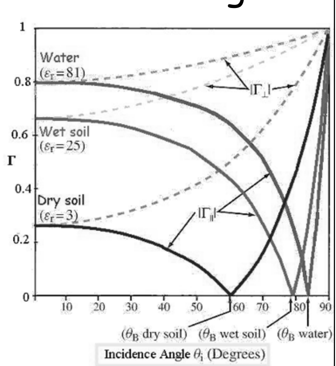
* θ_B is known as the polarizing angle

http://www.amanogawa.com/archive/Oblique/Oblique-2.html

lar

Reflection vs. Incidence angle.

Reflection vs. incidence angle for different types of soil and parallel or perpendicular polarization.



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Plots for $|\Gamma_\perp|$ and $|\Gamma_\parallel|$ as a function of θ_i

Table 2-4: Expressions for EM fields in lossless and lossy* media under normal incidence.†

Horizontal Polarization	Vertical Polarization
$\mathbf{E}_h^i = \hat{\mathbf{y}}E_{h0}^i e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{E}_v^i = (-\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1)E_{v0}^i e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{H}_h^i = (\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1)\frac{E_{h0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{H}_v^i = \hat{\mathbf{y}}\frac{E_{v0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{E}_h^r = \hat{\mathbf{y}}\rho_h E_{h0}^i e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{E}_v^r = (\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1)\rho_v E_{v0}^i e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{H}_h^r = (-\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1)\rho_h\frac{E_{h0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{H}_v^r = \hat{\mathbf{y}}\rho_v\frac{E_{v0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{E}_h^t = \hat{\mathbf{y}}\tau_h E_{h0}^i e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{E}_v^t = (-\hat{\mathbf{x}}\cos\theta_2 - \hat{\mathbf{z}}\sin\theta_2)\tau_v E_{v0}^i e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$
$\mathbf{H}_h^t = (\hat{\mathbf{x}}\cos\theta_2 + \hat{\mathbf{z}}\sin\theta_2)\tau_h\frac{E_{h0}^i}{\eta_2} e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{H}_v^t = \hat{\mathbf{y}}\tau_v\frac{E_{v0}^i}{\eta_1} e^{-jk_2(x\sin\theta_2 - z\cos\theta_2)}$
$\rho_h = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} = \frac{\cos\theta_1 - \sqrt{(\varepsilon_2'/\varepsilon_1') - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{(\varepsilon_2'/\varepsilon_1') - \sin^2\theta_1}}$	$\rho_v = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} = \frac{\left[\frac{\varepsilon_2'}{\varepsilon_1'} - \sin^2\theta_1\right]^{1/2} - \left(\frac{\varepsilon_2'}{\varepsilon_1'}\right)\cos\theta_1}{\left[\frac{\varepsilon_2'}{\varepsilon_1'} - \sin^2\theta_1\right]^{1/2} + \left(\frac{\varepsilon_2'}{\varepsilon_1'}\right)\cos\theta_1}$
$\tau_h = 1+\rho_h$	$\tau_v = (1+\rho_v)\frac{\cos\theta_1}{\cos\theta_2}$
$k_1 = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_1}$; $\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_1}}$; $\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_2}}$	
$k_2 = \frac{2\pi}{\lambda}\sqrt{\varepsilon_2}$; $k_2\sin\theta_2 = k_1\sin\theta_1$; $\cos\theta_2 = \left[1 - \left(\frac{k_1}{k_2}\sin\theta_1\right)^2\right]^{1/2}$	
*Notes: (1) Lossless medium: $\varepsilon = \varepsilon'$; (2) lossy medium: $\varepsilon = \varepsilon' - j\varepsilon''$ and jk should be replaced with $\gamma = j(2\pi\sqrt{\varepsilon' - j\varepsilon''})/\lambda_0$.	

Table 2-6: Oblique incidence in a lossless medium onto a lossy medium.†

Horizontal Polarization	Vertical Polarization
$\mathbf{E}_h^i = \hat{\mathbf{y}} E_{h0}^i e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{E}_v^i = (-\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1) E_{v0}^i e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{H}_h^i = (\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1) \frac{E_{h0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$	$\mathbf{H}_v^i = \hat{\mathbf{y}} \frac{E_{v0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 - z\cos\theta_1)}$
$\mathbf{E}_h^r = \hat{\mathbf{y}} \rho_h E_{h0}^i e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{E}_v^r = (\hat{\mathbf{x}}\cos\theta_1 - \hat{\mathbf{z}}\sin\theta_1) \rho_v E_{v0}^i e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{H}_h^r = (-\hat{\mathbf{x}}\cos\theta_1 + \hat{\mathbf{z}}\sin\theta_1) \rho_h \frac{E_{h0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$	$\mathbf{H}_v^r = \hat{\mathbf{y}} \rho_v \frac{E_{v0}^i}{\eta_1} e^{-jk_1(x\sin\theta_1 + z\cos\theta_1)}$
$\mathbf{E}_h^t = \hat{\mathbf{y}} \tau_h E_{h0}^i e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{E}_v^t = (-\hat{\mathbf{x}}\cos\theta_2 - \hat{\mathbf{z}}\sin\theta_2) \tau_v E_{v0}^i e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$
$\mathbf{H}_h^t = (\hat{\mathbf{x}}\cos\theta_2 + \hat{\mathbf{z}}\sin\theta_2) \tau_h \frac{E_{h0}^i}{\eta_2} e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$	$\mathbf{H}_v^t = \hat{\mathbf{y}} \tau_v \frac{E_{v0}^i}{\eta_1} e^{-\gamma_2(x\sin\theta_2 - z\cos\theta_2)}$
$\rho_h = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$	$\rho_v = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1}$
$\tau_h = 1 + \rho_h$	$\tau_v = (1+\rho_v)\frac{\cos\theta_1}{\cos\theta_2}$

Notes:

$k_1 = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_1'}$ $\quad \eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_1'}}$ $\quad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_2}}$

$\gamma_2 = \alpha_2 + j\beta_2 = j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon_2}$ $\quad \gamma_2 \sin\theta_2 = k_1 \sin\theta_1$ $\quad \varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$

$$\cos\theta_2 = \left[1 - \left(\frac{jk_1}{\gamma_2}\sin\theta_1\right)^2\right]^{1/2} = \left[1 - \left(\frac{\varepsilon_1'}{\varepsilon_2' - j\varepsilon_2''}\right)\sin^2\theta_1\right]^{1/2}$$

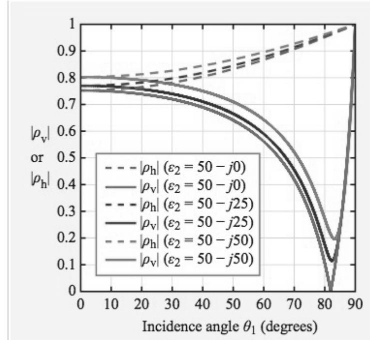

Figure 2-19: Angular plots of $|\rho_v|$ and $|\rho_h|$ for incidence in air upon a material with $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$.

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Dielectric Slab:
2 layers

- Medium 1: Air $\alpha_1 = 0$
- Medium 2: layer of thickness d, low-loss (ice, oil, snow) $\varepsilon_2 = \varepsilon_2' - j\varepsilon_2''$ $\quad \gamma_2 = \alpha_2 + j\beta_2$
- Medium 3: Lossy $\varepsilon_3 = \varepsilon_3' - j\varepsilon_3''$ $\quad \gamma_3 = \alpha_3 + j\beta_3$

Snell's Law Phase matching condition at interphase:

$$\gamma_1 \sin\theta_1 = \gamma_2 \sin\theta_2 = \gamma_3 \sin\theta_3$$

$$\boxed{\cos\theta_2 = \sqrt{\left[1 - \left(\frac{\beta_1}{\beta_2}\sin\theta_1\right)^2\right]} \; if\, 2\; low-loss} \qquad \cos\theta_3 = \sqrt{\left[1 - \left(\frac{\gamma_1}{\gamma_3}\sin\theta_1\right)^2\right]}$$

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Reflections at interfaces

- At the top boundary, ρ_{12},
- At the bottom boundary, ρ_{23}

For H polarization:

$$\rho_{12} = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} \qquad \rho_{23} = \frac{\eta_3\cos\theta_2 - \eta_2\cos\theta_3}{\eta_3\cos\theta_2 + \eta_2\cos\theta_3}$$

For V polarization:

$$\rho_{12} = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} \qquad \rho_{23} = \frac{\eta_3\cos\theta_2 - \eta_2\cos\theta_3}{\eta_3\cos\theta_2 + \eta_2\cos\theta_3}$$

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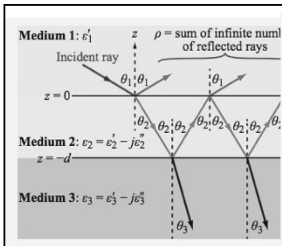

Figure 2-20: Multiple reflections in a two composite.

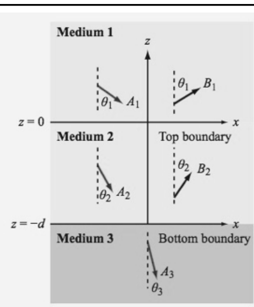

Figure 2-21: Electric field amplitudes of downward and upward propagating waves in media 1, 2 and 3.

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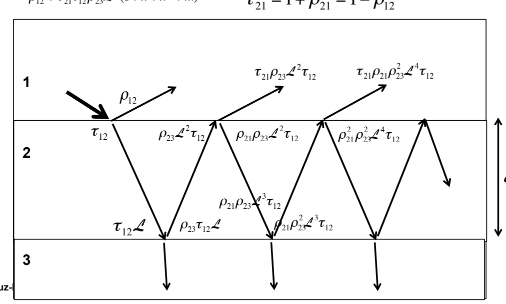
Multi-reflection Method

- Propagation factor: $\mathcal{L} = e^{-\gamma_2 d\cos\theta_2}$

$$\rho = \rho_{12} + \tau_{21}\rho_{23}\mathcal{L}^2\tau_{12} + \tau_{21}\rho_{21}\rho_{23}^2\mathcal{L}^4\tau_{12} + \ldots$$
$$= \rho_{12} + \tau_{21}\tau_{12}\rho_{23}\mathcal{L}^2(1 + x + x^2 + \ldots)$$

$$\tau_{12} = 1 + \rho_{12}$$
$$\tau_{21} = 1 + \rho_{21} = 1 - \rho_{12}$$

$$\boxed{\frac{1}{1-x} = 1 + x + x^2 + \ldots}$$



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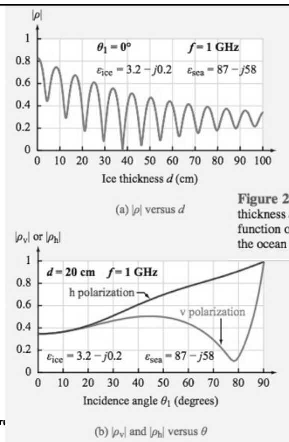

Figure 2-22: Plots of (a) $|\rho|$ as a function of ice-layer thickness d for normal incidence and (b) $|\rho_v|$ and $|\rho_h|$ as a function of incidence angle θ_1 for a 20 cm ice layer over the ocean surface.

Cont… for H Polarization

$$\rho = \rho_{12} + \tau_{21}\rho_{23}\mathcal{L}^2\tau_{12} + \tau_{21}\rho_{21}\rho_{23}^2\mathcal{L}^4\tau_{12} + \ldots$$
$$= \rho_{12} + \tau_{21}\tau_{12}\rho_{23}\mathcal{L}^2(1 + x + x^2 + \ldots)$$

$$\tau_{12} = 1 + \rho_{12}$$
$$\tau_{21} = 1 + \rho_{21} = 1 - \rho_{12}$$

Substituting the geometric series: $\frac{1}{1-x} = 1 + x + x^2 + \ldots$

$$\boxed{\rho = \rho_{12} + \frac{(1-\rho_{12})(1-\rho_{12})\rho_{23}\mathcal{L}^2}{1-\rho_{21}\rho_{23}\mathcal{L}^2}}$$

And then Substituting $\mathcal{L} = e^{-\gamma_2 d\cos\theta_2}$ **and** $\rho_{21} = -\rho_{12}$

$$\boxed{\rho = \frac{\rho_{12} + \rho_{23}\,e^{-2\gamma_2 d\cos\theta_2}}{1+\rho_{12}\rho_{23}e^{-2\gamma_2 d\cos\theta_2}}}$$

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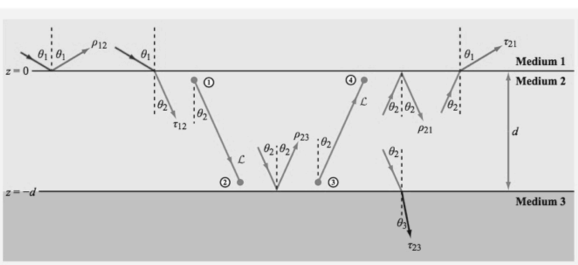

Figure 2-23: Reflection, transmission, and propagation mechanisms.

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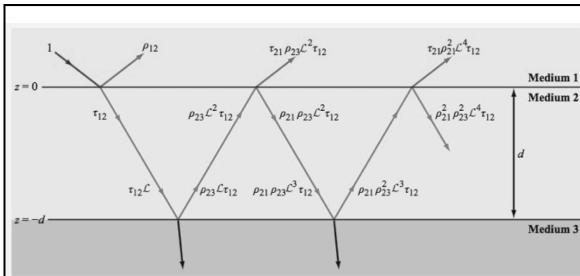

Figure 2-24: Multiple reflection process.

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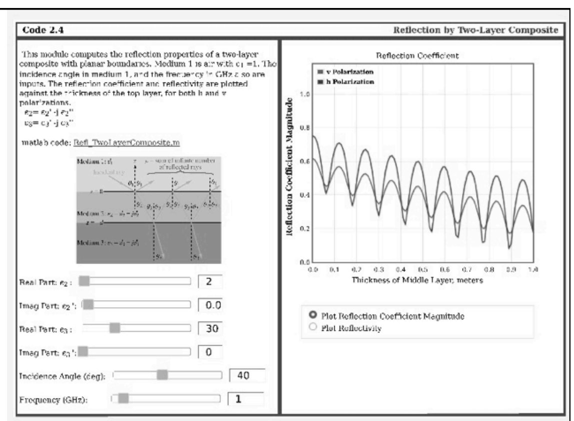

An example of one of the interactive modules available at the book website: mrs.eecs.umich.edu.

Antennas

Now let's review antenna theory

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