

# Aperture Antennas

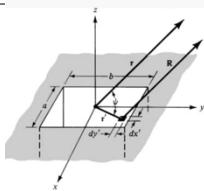
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Ref. Balanis Chpt. 12, Kraus Ch15

## Aperture Antennas

- Most common at microwave frequencies
- Can be flushed-mounted
- We will analyze radiation characteristics at far field
  - Rectangular aperture
  - Circular aperture

### Find Radiation Pattern in 2-D E-plane ( $\phi=0$ ), H-plane ( $\phi=90^\circ$ )



Ref. [2005] by Constantine A. Balanis  
is reprinted

Chapter 12  
Aperture Antennas

## Far field is the F of the near field

- Fourier Transform for 1-D

$$W(k_x) = \int_{-\infty}^{\infty} w(x) e^{j k_x x} dx$$

$$w(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(k_x) e^{-j k_x x} dk_x$$

- For two-dimensions, x and y;

$$U(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y) e^{j k_x x + j k_y y} dx dy$$

$$u(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(k_x, k_y) e^{-j k_x x - j k_y y} dk_x dk_y$$

## Properties of Fourier Transform

$$\mathcal{F}_t \frac{ds(t)}{dt} = j\omega \mathcal{F}_t s(t)$$

$$\mathcal{F}_x \frac{\partial u(x, y)}{\partial x} = -jk_x \mathcal{F}_x u(x, y)$$

$$\mathcal{F}_x \frac{\partial u^2(x, y)}{\partial x^2} = (-jk_x)^2 \mathcal{F}_x u(x, y)$$

$$\mathcal{F}_{yx} \frac{\partial u^2(x, y)}{\partial x^2} = -k_x^2 \mathcal{F}_{yx} u(x, y)$$

$$\nabla^2 \mathbf{E} + k_o^2 \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \mathbf{E} + k_o^2 \mathbf{E} = 0$$

$$\left( \frac{\partial E_x(x, y, z)}{\partial x} + \frac{\partial E_y(x, y, z)}{\partial y} + \frac{\partial E_z(x, y, z)}{\partial z} \right) = 0$$

Taking the Fourier transform of the 2 equations above:

$$\frac{\partial^2}{\partial z^2} \mathbf{E}(k_x, k_y, z) + (k_o^2 - k_x^2 - k_y^2) \mathbf{E}(k_x, k_y, z) = 0$$

$$\left( k_x E_x(k_x, k_y, z) + k_y E_y(k_x, k_y, z) + j \frac{\partial E_z(k_x, k_y, z)}{\partial z} \right) = 0$$

Now, we define,  $k_z^2 = k_o^2 - k_x^2 - k_y^2$   
 And we obtain,  $\frac{\partial^2 \mathbf{E}(k_x, k_y, z)}{\partial z^2} + k_z^2 \mathbf{E}(k_x, k_y, z) = 0$

Which has a solution of  $\mathbf{E}(k_x, k_y, z) = \mathbf{f}(k_x, k_y) e^{-jk_z z}$   
 Then we take the inverse transform

$$\mathbf{E}(x, y, z) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathbf{f}(k_x, k_y) e^{-jk \cdot r} dk_x dk_y$$

If  $z=0$ , then, we are at the aperture

$$\mathbf{E}_a(x, y) = \mathbf{E}_{\text{tan}}(x, y, 0) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathbf{f}(k_x, k_y) e^{-jk_x x - jk_y y} dk_x dk_y$$

Which looks like:

$$u(x, y) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} U(k_x, k_y) e^{-jk_x x - jk_y y} dk_x dk_y$$

Which is the inverse of F...

This is the Fourier transform for 2 dimensions, so:

$$U(k_x, k_y) = \iint_{-\infty}^{\infty} u(x, y) e^{jk_x x + jk_y y} dx dy$$

$\mathbf{f}_i(k_x, k_y) = \iint_{S_a} E_a(x, y) e^{jk_x x + jk_y y} dx dy$

It can be shown that,

$$\mathbf{E}(r) \approx \frac{j k_o \cos \theta}{2\pi r} e^{-jk_o r} \mathbf{f}_i(k_o a \cos \theta \cos \phi, k_o b \sin \theta \sin \phi)$$

Therefore, if we know the field distribution at the aperture, we can used these equations to find  $\mathbf{E}(r)$  @ff  
 =>First, we'll look at the case when the illumination at the rectangular aperture it's uniform.

**Uniformly illuminated rectangular aperture**

$$\mathbf{E}_a(x, y) = E_o \mathbf{x} \quad \text{for } |x| \leq a \quad |y| \leq b \\ = 0 \quad \text{elsewhere}$$

$\mathbf{f}_i = E_o \mathbf{x} \iint_{-a-b}^{a-b} e^{jk_x x + jk_y y} dx dy$

$$= 4ab E_o \mathbf{x} \frac{\sin k_o a \sin k_o b}{k_o a \quad k_o b} \\ = 4ab E_o \mathbf{x} \frac{\sin(k_o a \sin \theta \cos \phi) \sin(k_o b \sin \theta \sin \phi)}{k_o a \sin \theta \cos \phi \quad k_o b \sin \theta \sin \phi} \\ = 4ab E_o \mathbf{x} \frac{\sin u \sin v}{u \quad v}$$

$$\mathbf{E}(r) = \frac{j k_o 4ab E_o}{2\pi r} e^{-jk_o r} \frac{\sin u \sin v}{u \quad v} (\hat{\theta} \sin \phi - \hat{\phi} \cos \phi \cos \theta)$$

\*Note: in Balanis book, the aperture is  $a \times b$ , so no 4 factor on the eq. above.

**TE<sub>10</sub> illuminated rectangular aperture**

$$\mathbf{E}_a(x, y) = E_o \cos\left(\frac{\pi x'}{a}\right) \hat{\mathbf{y}} \quad \text{for } -a/2 \leq x' \leq a/2 \\ = 0 \quad \text{elsewhere}$$

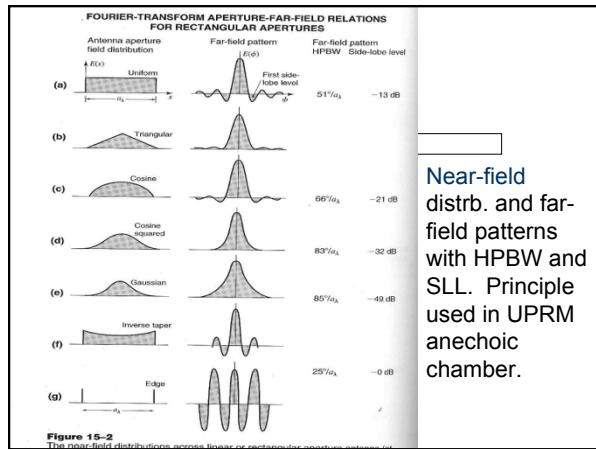
$$\mathbf{f}_i = E_o \hat{\mathbf{y}} \iint_{-a-b}^{a-b} \cos\left(\frac{\pi x'}{a}\right) e^{jk_x x + jk_y y} dx dy$$

$$X = \frac{u}{2} \quad u = k_o a \sin \theta \cos \phi \\ Y = \frac{v}{2} \quad v = k_o b \sin \theta \sin \phi$$

$$\mathbf{E}(r) = \frac{-jk_o ab E_o e^{-jk_o r}}{4r} \frac{\cos X \sin Y}{X^2 - \left(\frac{\pi}{2}\right)^2} (\hat{\theta} \sin \phi - \hat{\phi} \cos \phi \cos \theta)$$

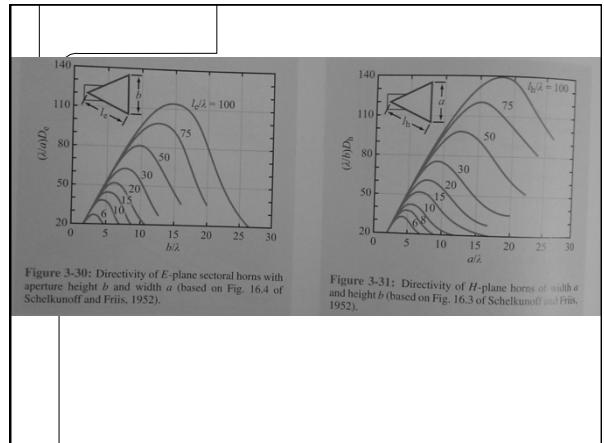
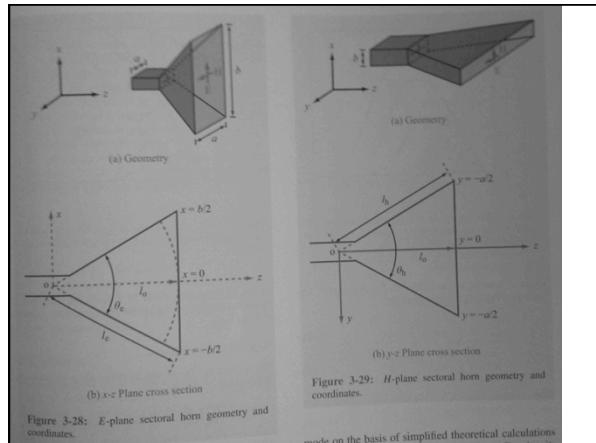
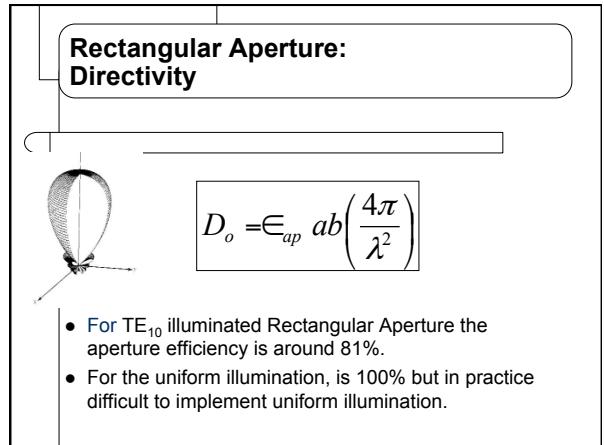
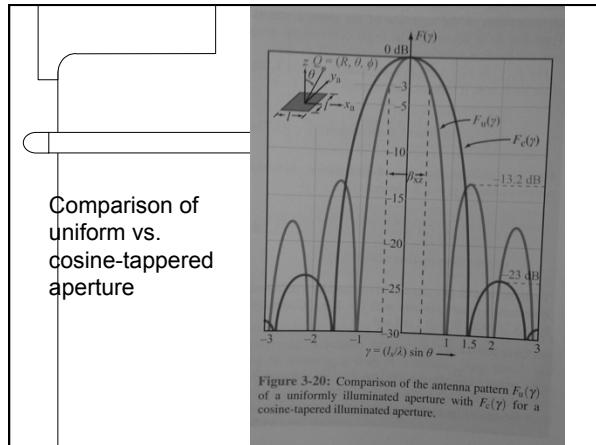
**How does this pattern looks...**

$$u = k_o a \sin \theta \cos \phi \\ v = k_o b \sin \theta \sin \phi$$



**Near-field distrb. and far-field patterns with HPBW and SLL. Principle used in UPRM anechoic chamber.**

Amplitude distribution <sup>a</sup>	Relative directivity <sup>b</sup>	Sidelobe level (dB) <sup>c</sup>	Half-power beamwidth (radians) <sup>d</sup>
Cosine: $E_1(x_1) = \cos^2(\pi x_1 / l_a)$	1.00	13.2	$0.88\lambda / l_a$
$\pi = 0^\circ$	0.81	23	$1.20\lambda / l_a$
$\pi = 2$	0.67	45	$1.45\lambda / l_a$
$\pi = 3$	0.58	40	$1.66\lambda / l_a$
$\pi = 4$	0.52	48	$1.94\lambda / l_a$
Parabolic: $E_1(x_1) = 1 - (\Delta x_1)^2$	1.00	13.2	$0.88\lambda / l_a$
$\Delta = 1.0^\circ$	0.99	15.8	$0.92\lambda / l_a$
$\Delta = 0.8$	0.97	17.1	$0.97\lambda / l_a$
$\Delta = 0.5$	0.93	20.6	$1.15\lambda / l_a$
Triangular: $E_1(x_1) = 1 -  x_1 $	0.75	26.4	$1.28\lambda / l_a$



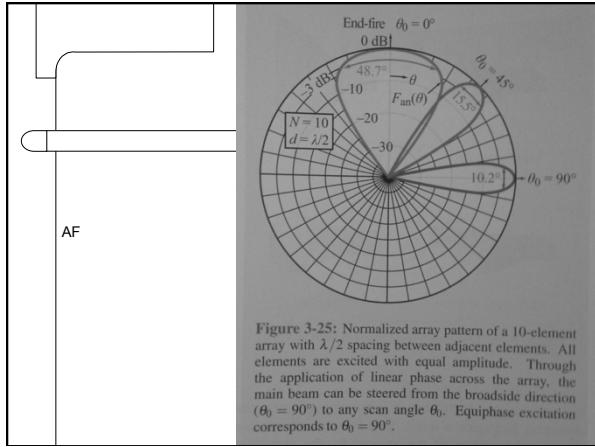


Figure 3-25: Normalized array pattern of a 10-element array with  $\lambda/2$  spacing between adjacent elements. All elements are excited with equal amplitude. Through the application of linear phase across the array, the main beam can be steered from the broadside direction ( $\theta_0 = 90^\circ$ ) to any scan angle  $\theta_0$ . Equiphase excitation corresponds to  $\theta_0 = 90^\circ$ .

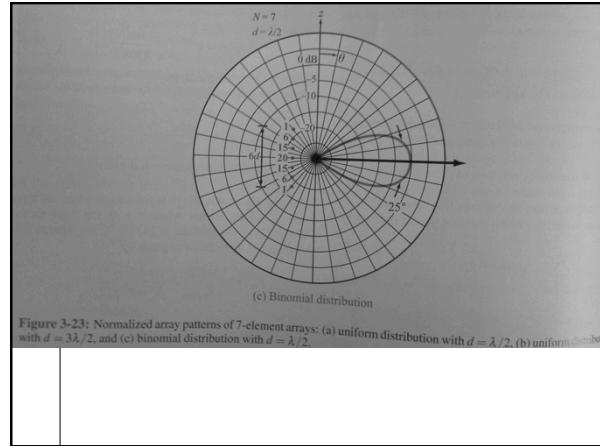


Figure 3-23: Normalized array patterns of 7-element arrays: (a) uniform distribution with  $d = \lambda/2$ , (b) uniform distribution with  $d = 3\lambda/2$ , and (c) binomial distribution with  $d = \lambda/2$ .

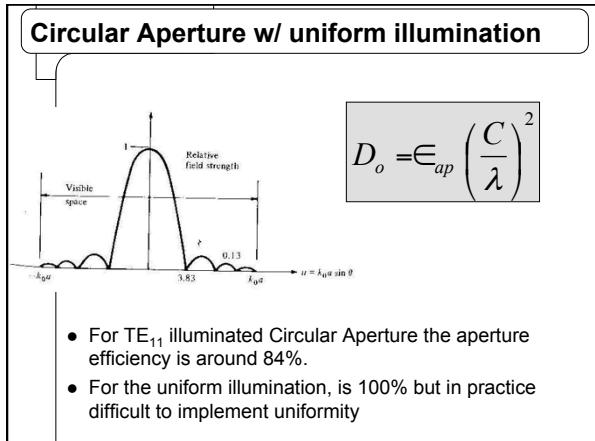
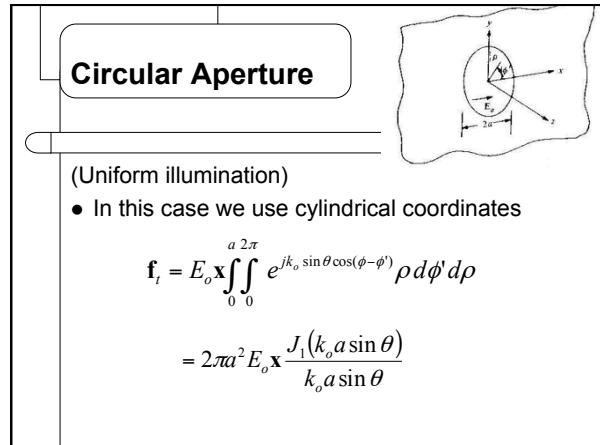
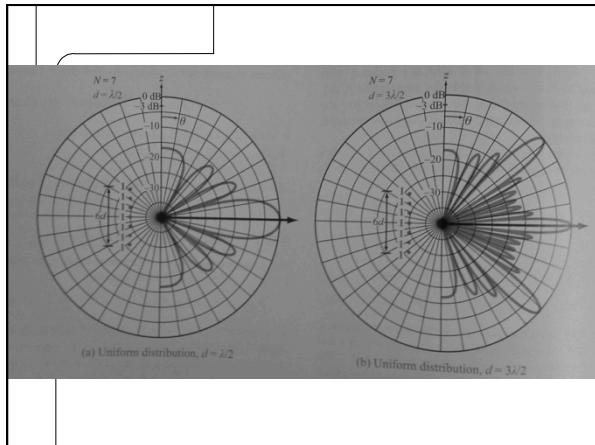


Table 15-1 Beamwidth and side-lobe level for rectangular and circular aperture distributions <sup>1</sup>			
Aperture field distribution	$E(x)$	Half-power beamwidth	Level of first side lobe, dB
Rectangular or linear apertures			
Tapered to $\frac{1}{3}$ at edge (~10 dB down)	$E(x) = 1 - 2x^2/3$	$\frac{59^\circ}{L_\lambda}$	-19
Tapered to zero at edge	$E(x) = 1 - x^2 \approx \cos(\pi x/2)$	$\frac{66^\circ}{L_\lambda}$	-21
Tapered to zero at edge	$E(x) = \cos(\pi x/2)$	$\frac{83^\circ}{L_\lambda}$	-32
<hr/>			
Circular apertures	$E(r)$		
Uniform		$\frac{58^\circ}{D_\lambda}$	-18
Tapered to $\frac{1}{3}$ at edge (~10 dB down)	$E(r) = 1 - 2r^2/3$	$\frac{66^\circ}{D_\lambda}$	-23
Tapered to zero at edge	$E(r) = 1 - r^2$	$\frac{73^\circ}{D_\lambda}$	-25
Tapered to zero at edge	$E(r) = (1 - r^2)^2$	$\frac{84^\circ}{D_\lambda}$	-31

<sup>1</sup> $D_h = D/\lambda$ ,  $D_o = D/\lambda$ . It is assumed that  $D_h \gg 1$  and  $D_o \gg 1$ . For a uniform rectangular or linear aperture  $HPBW \approx 51^\circ/D_h$  with first side lobe = -13 dB.