Genetic Algorithms and a Variational Method for Image Denoising

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Abstract

Variational methods for image denoising consist of minimizing a functional which incorporates both the data and some penalty term. Choosing the penalty term to involve the total variation of the image has the advantage of cleaning speckles without smoothing out the edges. Our goal is to investigate the use of genetic algorithms to minimize the functional.

1. INTRODUCTION

The variational method for image processing proceeds by minimizing a functional, thereafter referred to as energy, which depends on the image and its space derivatives (gradient). The one we consider is the sum of two terms : one represents the deviation from a data image $\boldsymbol{z}$, which may be marred by noise, blur or speckles, and the other incorporates the variation of the function (in the mathematical sense). The last term penalizes oscillations and irregularities, but does not remove jump discontinuities altogether. Such discontinuities are considered necessary to preserve the information content (sharpness) of the image.

The energy we minimize is :

$$^{1}/_{2} \,\| \boldsymbol{Au} - \boldsymbol{z} \|^{2} + \alpha \int | \quad \boldsymbol{u} |$$

where $\boldsymbol{z}$ is the data image, and the second integral term represents the variation of the candidate image $\boldsymbol{u}$. When there is blur, $\boldsymbol{A}$ represents the action of blurring operator, and $\boldsymbol{z}$ is the blurred and noisy image. Here we will assume no blur and take $\boldsymbol{A}$ to be the identity. This is the "purely denoising" case. If $\boldsymbol{u}$ is smooth, the second term clearly measures its average oscillation over the domain; but this integral term may remain bounded even if $\boldsymbol{u}$ has jump discontinuities (as one expects of an image). For example, if $\boldsymbol{u}$ is the characteristic function of the unit disc, then the second integral term will be exactly 2π, which is the amount of "falloff" of u integrated over the boundary of the unit disc.

Iterative methods for carrying out the minimization have already been implemented [4]. The convergence rate to the denoised image seems to depend on its smoothness. Figure 1 illustrates the use of these methods in the one-dimensional case. Note that for a critical value of the coefficient α, the minimizer of the energy develops the sharp edge at the same location as the original image.

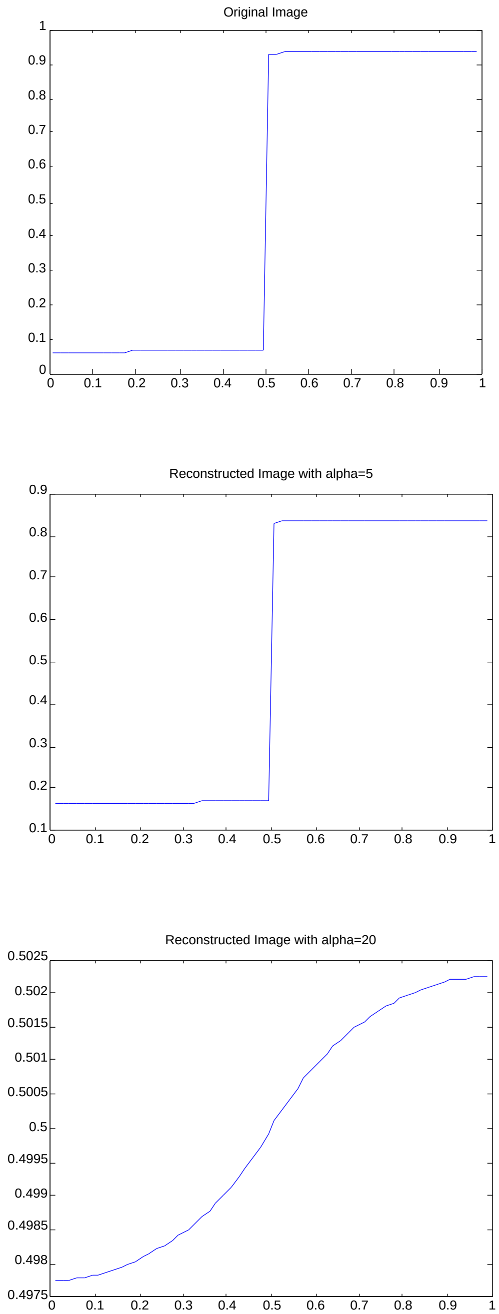

Figure 1. Reconstruction of a single step image in the absence of noise by an iterative method

2. GENETIC ALGORITHMS

The genetic algorithm is actually not a single algorithm, but rather, a class of methods which purport to minimize a function defined on a domain by:

1) Discretizing the domain so that each value of the state variable is coded by a vector of fixed length N (chromosome).

2) Randomly selecting an initial population of chromosomes by sampling from the domain using a uniform distribution.

3) Letting this initial population evolve over a given number of generations. The best fit member of the last generation is taken as the solution. The evolution rules, which vary, may be as follows:

3a) Selection: based on their fitness, randomly select couples for mating.

3b) Each couple will produce one (or, alternatively, two) offsprings by randomly generating a “crossover” rule.

3c) The offsprings now form the new population, which is of the same size as the previous population.

3d) An intermediate (and desirable) step between 3b and 3c is to mutate randomly some of the offsprings.

3. CURRENT PROGRESS

As a beginning , we are confining the image to be one-dimensional, so that N, the size of the state variable, is the size of the discretization of the interval. Since the energy is a convex functional , a preliminary step consist in testing the genetic algorithm on the simplest convex problem, namely, minimize x_ over an interval. Our control parameters include all the probabilities governing the random events, and also how to generate the crossover rule. While we do not expect the genetic algorithm to outperform the direct (iterative) method, it will be of interest to see how well it approximates the solution, and how the execution time grows with the size of the problem. Another advantage of the genetic algorithm compared with direct methods is that it is much easier to code. We are using MATLAB in our current project (see Figure 2).

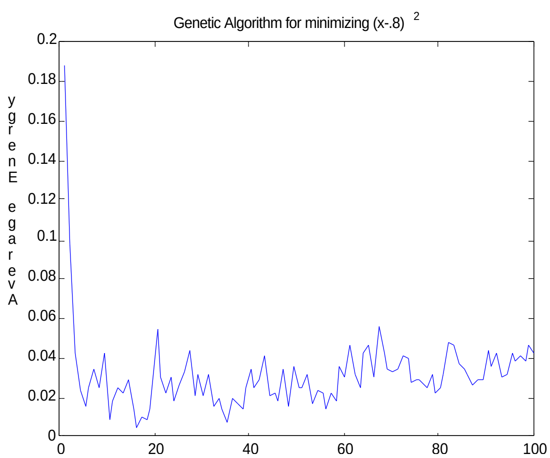

Figure 2. Preliminary results for genetic algorithm

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