

# A COMPUTATIONAL ENVIRONMENT FOR THE ANALYSIS OF DISCRETE TIME DISCRETE FREQUENCY TIME-FREQUENCY SIGNALS

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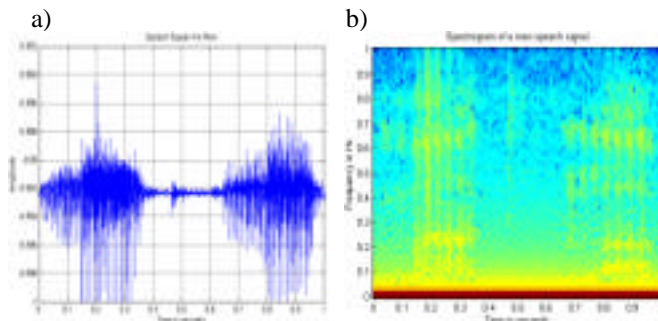
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## ABSTRACT

There are several engineering and scientific areas where the development of meaningful applications require the analysis and design of signals whose spectral characteristics change with respect to time. The tools used to treat these signals are known as “time-frequency tools”. This on going work presents the development of a computational environment that makes a uniform characterization of time-frequency signal analysis tools through a MATLAB environment to study the spectral content of time-varying signals on a specific applications.

## 1. INTRODUCTION

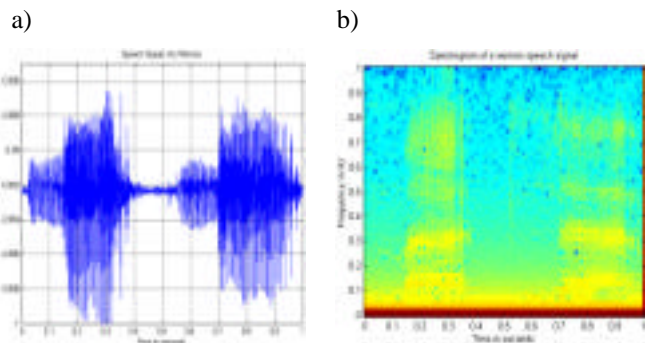
The analysis of signals whose spectral characteristics change with time (see Figure 1a) is very useful in many engineering and scientific applications. With time-frequency signal analysis tools one can study and analyze these signals and identify the temporal localization of the signal’s spectral components (Figure 1b). In particular, the values of the time-frequency representation of the signal provide an indication of the specific times at which the spectral components of the signal are observed.



**Figure 1.** Graphical representation of a time-frequency signal. a) Speech signal of a man: MAT-LAB, b) The spectrogram associated with this speech signal

This is of special importance since the frequency contents of the majority of the signals encountered in our everyday life change over time; for example, biomedical signals, power signals, speech signals (Figure 1-2), stock indexes time series, and seismic signals.

This fact is fostering the implementation of time-frequency signal analysis tools in many important scientific and engineering applications, such SAR (Synthetic Aperture Radar), spread spectrum signal detection and the analysis of FM signals such as chirp signals (Figure 4).



**Figure 2.** Graphical representation of a time-frequency signal. a) Speech signal of a woman: MAT-LAB, b) The spectrogram associated with this speech signal. \*Note the difference between a speech signal of a man (Figure 1) which is at a lower pitch than the speech signal of a woman whose pitch is higher.

This paper discusses the implementation of time-frequency signal analysis tools through a single computational framework. This framework makes a unified characterization of these tools, which is very practical, since one can study the effect of each tool in a specific application. This characterization then translates into an effective aid for automated procedures developed for time-frequency signal analysis, in general.

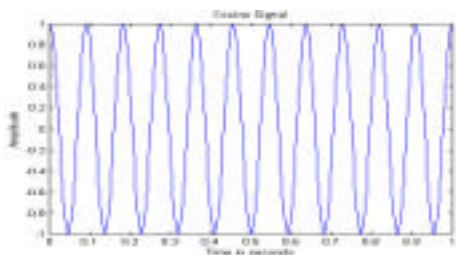
The environment was developed with the use of the computational and visualization software package MATLAB. It concentrates on some of the most commonly known time-frequency signal analysis tools, such as the ambiguity function (AF), the Wigner Distribution (WD), the short-time Fourier transform (STFT) and the discrete wavelet transform (DWT). The AF is a time-frequency tool that is broadly used in radar and sonar applications. The WD is widely used for signal detection and parameter estimation. The STFT is the same Fourier transform with a window function that can be moved through the time axis and is widely used in applications such as speech recognition. A relatively new emerging tool is the

discrete wavelet transform that is frequently used in applications such as transient signal analysis and image compression.

In section 2 we will explain the mathematical representation of the AF, the WD, the STFT and the DWT. In section 3 we present the computational environment and explain in detail its internal structure. Section 4 then presents and discusses some results in applications such as SAR. Finally, in section 5, some conclusions are discussed.

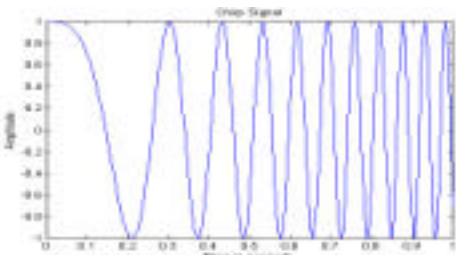
## 2. TIME-FREQUENCY TOOLS

The Fourier transform has been the most common tool to study a signal's frequency properties. It establishes in conjunction with the inverse Fourier transform a one-to-one relationship between the time domain and the frequency domain or spectrum space of the signal, which constitute two alternative ways of looking at a signal. Although the Fourier transform (Figure 5) allows a passage from one domain to the other, it does not allow for a simultaneous combination of the two domains.



**Figure 3.** Cosine: a single tone signal, its spectral characteristics does not vary with time.

This presents a problem if we are interested in studying the frequency components of signals which are transient, or their spectral content vary as a function of time (Figure 4), i.e., speech signals (Figures 1-2).

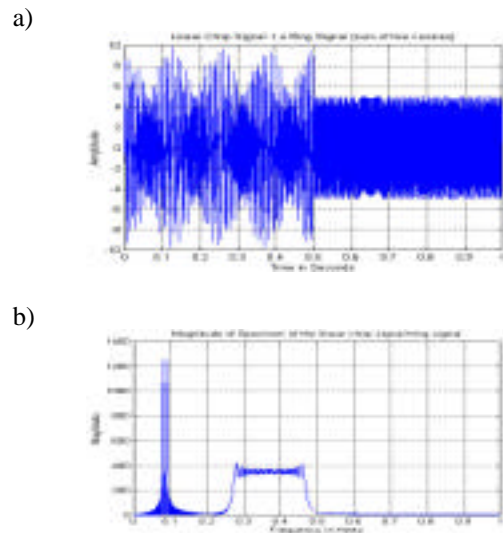


**Figure 4.** Linear chirp signal (a band of frequencies), its spectral characteristics do vary with time.

Time-frequency tools have existed for a while and have been used successfully in diverse areas such as sonar, radar, audio, acoustics, biomedical engineering and seismic geology.

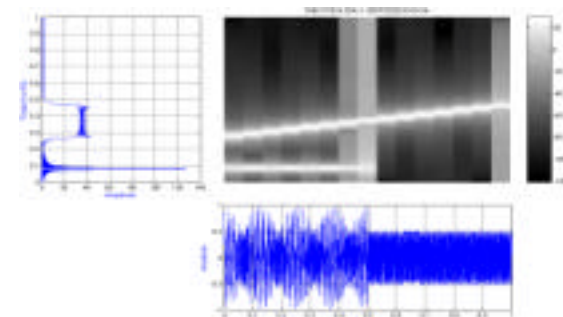
The tools already exist individually. It then emerges that there is an imperative need of an environment that can create a uniform characterization of these tools for different engineering applications, since it would be

very helpful in identifying similarities and differences between the various tools (Figure 6).



**Figure 5.** A time-frequency signal: linear chirp signal + ring signal (sum of two cosines)..a) Its graphical representation in time. b) Its Fourier transform. We can identify the single tone of the ring signal (left) and the spectrum of the chirp signal changing (right). \*Note that with only this information it is difficult to determine which signal precedes the other, and the particular spectral characteristics of each signal.

Within time-frequency tools, the ambiguity function, the short-time Fourier transform, the Wigner distribution and the discrete wavelet transform seem to possess properties that can be very significant for their application to important problems encountered in time-frequency signal analysis.



**Figure 6.** Time-frequency (STFT-explained in section 2.3) representation of time-frequency signal in Figure 5.

For this reason, we considered it a practical decision to concentrate in these four tools. We proceed to describe these tools in more detail.

### 2.1 Ambiguity Function

The ambiguity function (AF) is a time-frequency tool that has as its objective to extract parameters such as frequency shift and time delay from a specific signal

(parameter estimation), and is frequently used for signal estimation and Doppler effects. The ambiguity function is defined as:

$$A_{x,y}[k, m] = \sum_{n=0}^{N-1} x[n] y^*[n+m] W_N^{kn} \quad (1)$$

where,  $W_N = e^{-j\frac{2\pi}{N}}$ ,  $j = \sqrt{-1}$ ,  $x$  is the transmitted signal,  $k$  is the frequency shift,  $y$  is the received signal, and  $m$  is the time delay. Also,  $*$  denotes complex conjugation.

## 2.2 Wigner Distribution

The Wigner Distribution (WD) is defined as follows [3]:

$$W_{x,y}[m, k] = \frac{1}{2N} \sum_{n=0}^{N-1} x[n] y^*[m-n] W_{2N}^{k(2n-m)} \quad (2)$$

where,  $W_N = e^{-j\frac{2\pi}{N}}$ ,  $j = \sqrt{-1}$ ,  $x$  is the transmitted signal,  $k$  is the frequency shift,  $y$  is the received signal, and  $m$  is the time delay. Also,  $*$  denotes complex conjugation.

## 2.3 Short-Time Fourier Transform

The short-time Fourier transform (STFT) is a time-frequency tool that consists of a Fourier transform with a sliding time window. The time localization of frequency components is obtained by suitably pre-windowing the input signal. The process is as follows. The input signal is multiplied by a function that has a much shorter duration than itself in order to get a short-time section of this input signal. The function that allows taking the short-time sections of the input signal is known as the analysis window. The STFT is defined as follows:

$$S_x[n, k] = \sum_{m=0}^{M-1} x[m] w[m-n] W_M^{km} \quad (3)$$

where,  $W_N = e^{-j\frac{2\pi}{N}}$ ,  $j = \sqrt{-1}$ ,  $x$  is the input signal,  $w$  is the analysis window,  $k$  is the frequency offset, and  $m$  is the time delay.

## 2.4 Discrete-Wavelet Transform

The DWT is a tool that has been used for the development of new signal processing applications. It has been used for signal and image coding, speech analysis, acoustic and seismic signal processing, stochastic signal processing, and fractal analysis, among others.

## 3. COMPUTATIONAL ENVIRONMENT

The main objective of the environment presented in this paper is the development of a single framework that combines traditional time-domain and frequency-domain concepts. The framework has been developed with the use of the scientific computation and visualization software package MATLAB. The environment concentrates on the use of time-frequency tools such as the Ambiguity Function (AF), the Wigner Distribution (WD), the Short-Time Fourier Transform (STFT), and the Discrete Wavelet Transform (DWT). The environment is divided into three major modules: Analysis and Synthesis module (Figure 7), Demonstration module and a Tutorial module (Figure 8). The Analysis and Synthesis Module provides the user with the essential tools for managing different types of signals as: \*.WAV, \*.MAT, \*.ASCII. It possesses capabilities of retrieval and storage of data. In the environment, special attention was given to the graphical visualization and data rendering capabilities, since signal analysis is one of our main concerns.

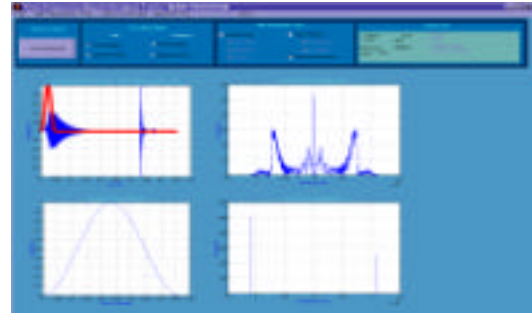
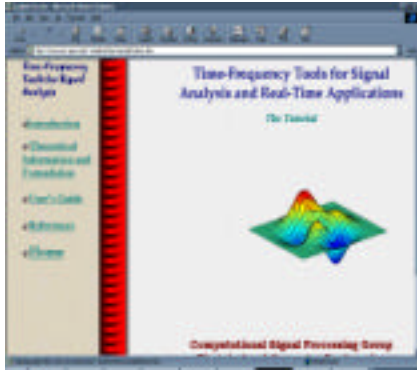


Figure 7. Computational Environment in MATLAB.

This part allows the user to choose the type of plot, the color-map, the shading and rotation of the figure. The environment also allows individual use of the tools and comparison of tool application results. The Demonstration Module is a self-paced, step by step demonstration of the entire environment, including a user's guide which has sufficient information on how to use the environment effectively and the function of each menu and buttons within the environment. The tutorial module (Figure 8) serves as a teaching and reference tool to the user for the technical aspects related with the development of the environment.

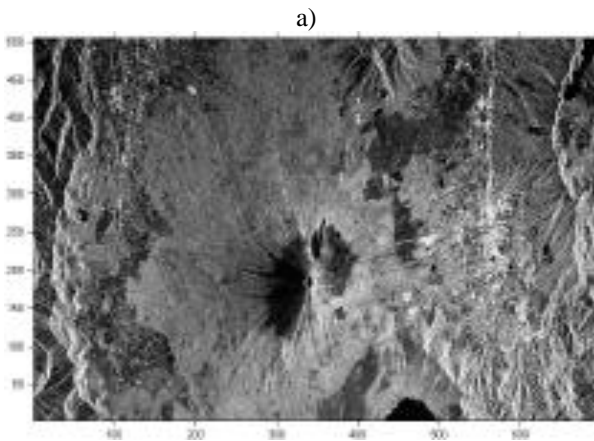


**Figure 8.** The tutorial in HTML format.

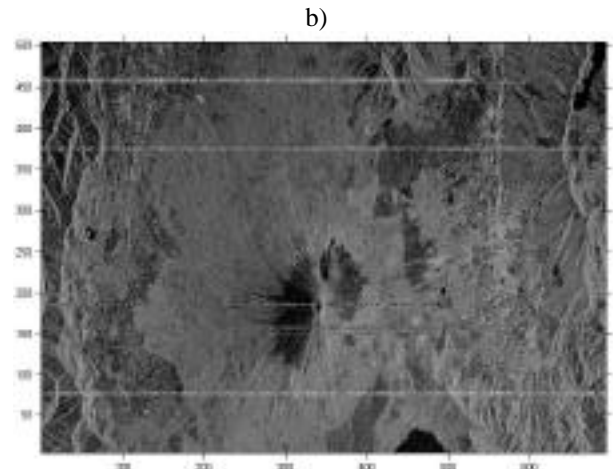
In the following section some results obtained in applications such SAR will be discussed.

#### 4. APPLICATION RESULTS

At the present time we have three of the four tools implemented in the computational environment, and we have tested these tools in various applications such as speech and chirp signal analysis. Now, we are using these tools in the analysis of SAR data. Of particular importance is the detection of radio-frequency (RF) interference in synthetic aperture radar (SAR) raw data as a preprocessing operation before the SAR image formation operation is performed. As an example of uncorrected RF interference, we present two SAR images below. The first image, shown in Figure 9, is produced from SAR raw data without RF interference. In Figure 10 the RF interference appears as horizontal bright lines.



a)



b)

**Figure 9.** a) SAR image of Mount Fuji located in Japan. b) SAR image degraded by simulated RF interference.

As next step we will use the STFT and the WD in the analysis of the raw SAR data before the image formation, as a pre-processing operation in order to detect the interference and develop filters for their removal.

#### 5. CONCLUSIONS

A computational environment for the analysis of discrete time discrete frequency time-frequency signals has been presented. The need of a unified characterization of time-frequency tools through a computational environment is clearly justified. It has been shown with this paper that with a computational environment, we can do signal analysis at different levels, from the much simpler level (speech) to a more complex level (analysis of SAR images). A computational environment for time-frequency signal analysis also gives us the advantage to compare similarities and differences between the different tools, without the necessity to run each function separately.

#### 6. REFERENCES

- [1] Auslander, L., Tolimieri R., "Computing Decimated Finite Cross-Ambiguity Functions". *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol 36, No 3, March 1988.
- [2] Rodríguez, D., Seguel J., "On the implementation of time-frequency tools for chirp signals". *SISCAP '94: International Symposium on Intelligent Systems in Communications and Power*, 1994.