Genetic Algorithms and a Variational Method for Image Denoising

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Abstract

Variational methods for image denoising consist of minimizing a functional which incorporates both the data and some penalty term. Choosing the penalty term to involve the total variation of the image has the advantage of cleaning speckles without smoothing out the edges. Our goal is to investigate the use of genetic algorithms to minimize the functional.

1. INTRODUCTION

The variational method for image processing proceeds by minimizing a functional, thereafter referred to as energy, which depends on the image and its space derivatives (gradient). The one we consider is the sum of two terms : one represents the deviation from a data image $\boldsymbol{z}$, which may be marred by noise, blur or speckles, and the other incorporates the variation of the function (in the mathematical sense). The last term penalizes oscillations and irregularities, but does not remove jump discontinuities altogether. Such discon-tinuities are considered necessary to preserve the information content (sharpness) of the image.

The energy we minimize is :

$$\tfrac{1}{2}\| \boldsymbol{Au} - \boldsymbol{z} \|^2 + \boldsymbol{a} \int | \nabla \boldsymbol{u} |$$

where $\boldsymbol{z}$ is the data image, and the second integral term represents the variation of the candidate image $\boldsymbol{u}$. When there is blur, $\boldsymbol{A}$ represents the action of blurring operator, and $\boldsymbol{z}$ is the blurred and noisy image. Here we will assume no blur and take $\boldsymbol{A}$ to be the identity. This is the "purely denoising" case. If $\boldsymbol{u}$ is smooth, the second term clearly measures its average oscillation over the domain; but this integral term may remain bounded even if $\boldsymbol{u}$ has jump discontinuities (as one expects of an image). For example, if $\boldsymbol{u}$ is the characteristic function of the unit disc, then the second integral term will be exactly $2\boldsymbol{\pi}$, which is the amount of "falloff" of **u** integrated over the boundary of the unit disc.

Iterative methods for carrying out the minimization have already been implemented [5]. The convergence rate to the denoised image seems to depend on its smoothness. Figure 1 illustrates the use of these methods in the one-dimensional case. Note that for a critical value of the coefficient $\boldsymbol{a}$, the minimizer of the energy develops the sharp edge at the same location as the original image.

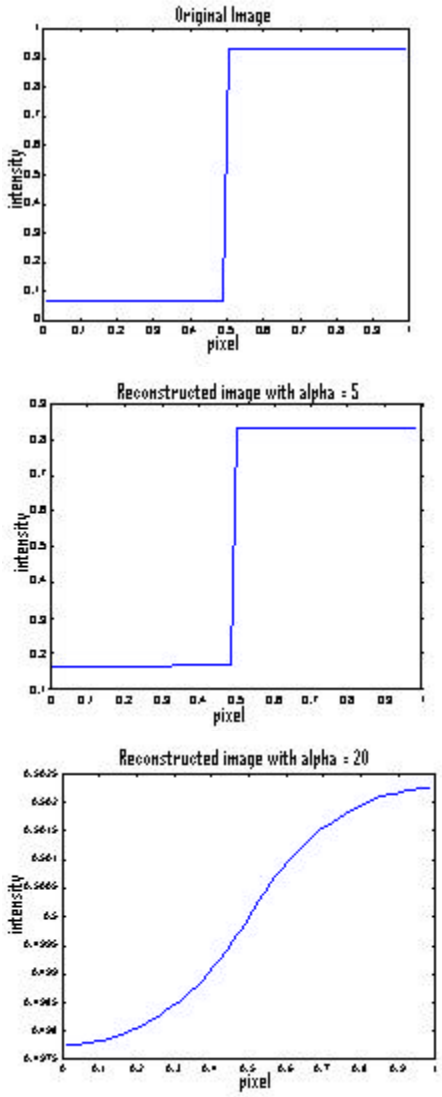

Figure 1 : *Varying penalty term* Reconstruction of a single step image in the absence of noise by an iterative method.

2. GENETIC ALGORITHMS

The genetic algorithm (GA) is actually not a single algorithm, but rather, a class of methods which purport to minimize a function defined on a domain by:

1) Discretizing the domain so that each value of the state variable is coded by a vector of fixed length N (chromosome).

2) Randomly selecting an initial population of chromosomes by sampling from the domain using a uniform distribution.

3) Letting this initial population evolve over a given number of generations. The best fit member of the last generation is taken as the solution. The evolution rules, which vary, may be as follows:

3a) Selection: based on their fitness, randomly select couples for mating.

3b) Each couple will produce one (or, alternatively, two) offsprings by randomly generating a “crossover” rule.

3c) The offsprings now form the new population, which is of the same size as the previous population.

3d) An intermediate (and desirable) step between 3b and 3c is to mutate randomly some of the offsprings.

See Figure 2 for best understanding of a Genetic Algorithm.

3. CURRENT PROGRESS

As a beginning , we are confining the image to be one-dimensional, so that N, the size of the state variable, is the size of the discretization of the interval. Since the

energy is a convex functional , a preliminary step consist in testing the genetic algorithm on the simplest convex problem, namely, minimize x^2 over an interval. Our control parameters include all the probabilities governing the random events, and also how to generate the crossover rule. While we do not expect the genetic algorithm to outperform the direct (iterative) method, it will be of interest to see how well it approximates the solution, and how the execution time grows with the size of the problem. Another advantage of the genetic algorithm compared with direct methods is that it is much easier to code. We are using MATLAB in our current project: see Fig 3 .

For the genetic algorithm some tests have been made with the scalar problem for coding and improving purposes. After this step we started implementing the one-dimensional case using the first term of the functional to be minimized. The first term of the functional is discretized as :

$$\Sigma \, \Delta x \, (\mathbf{u} - \mathbf{z})^2$$

where Δx is the width of each pixelWe tested the dependency of the final solution **u** with respect to the following parameters. The number of binary bits coding the intensity of each pixel, the number of generations and the size of the population. Figures 4-6 show those dependencies. As Figure 4 shows, increasing the number of bits improves the approximation. Figure 5 shows that increasing the number of generations in the range 5 to 20 also improves the approximation. For the case of the size of population selected Figure 6 we found that is not necessary increase this number indefinitely to find a good solution, in our test a population of 40 was enough to find good approximations considering the effect of the other parameters.

FUTURE WORK

Preliminary experiments with larger number of pixels seem to indicate a degradation of performance . Our next task is to check whether this is true and if so, why. It is imperative to address this question before going on to the two-dimensional case. After that the complete functional,including the penalty term must be implemented. Other improvements must be to add more statistics to the code in order to track the performance of the algorithm. Also we have to do some refinement of the fitness selection procedure.

ACKNOWLEDGEMENT

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REFERENCES

[1] Bogees,G and Mukheeth,A., *The Application of Genetic Algorithims to the scheduling of Engineering Units*. Dec31,1997

[2] N. Gershenfeld, *The Nature of Mathematical Modeling*, Cambridge University Press, 1999.

[3] D. Goldberg , *Genetic Algorithms in Search, Optimization and Maching learning*, Addison Wesley, 1989.

[4] P.V. Negron - Marrero, *A genetic algorithm for computing singular minimizers of the calculus of variations*, submitted.

[5] C.R Vogel and M. Oman, *Iterative methods for total variation denoising*, SIAM J Sci Comp 17:227-28, 1996.

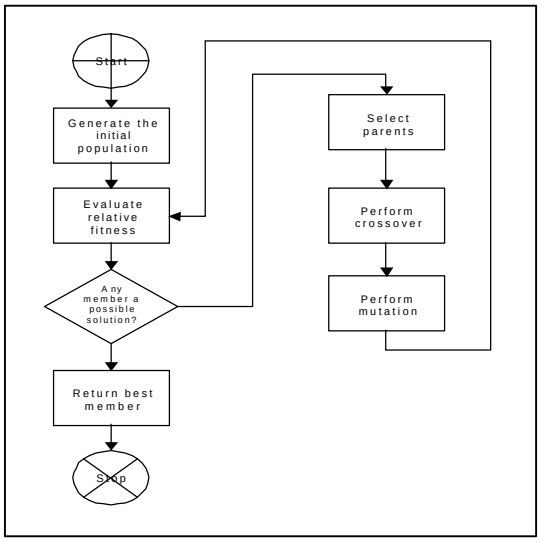

Figure 2: *General GA representation* in flow chart form.[1]

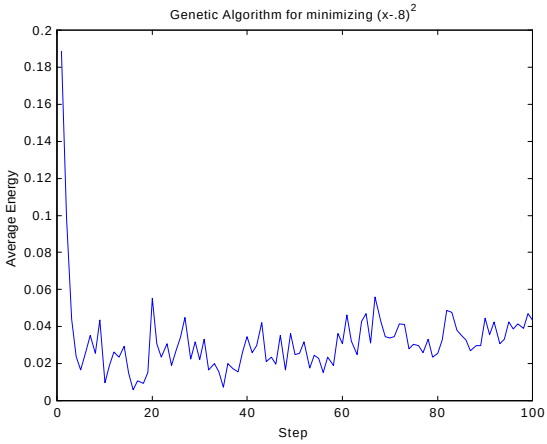

Figure 3: *Energy vs. Generations* Preliminary results for genetic algorithm

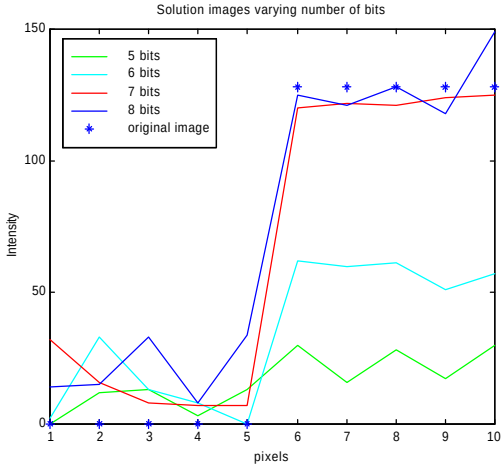

Figure 4: *Varying the number of bits* the solution approximates the given image **z**

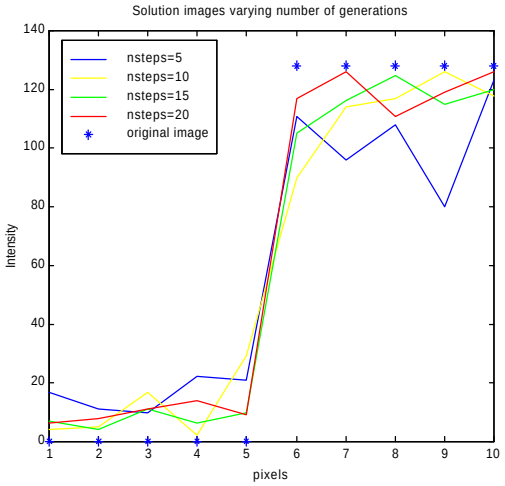

Figure 5: *Varying number of generations.*

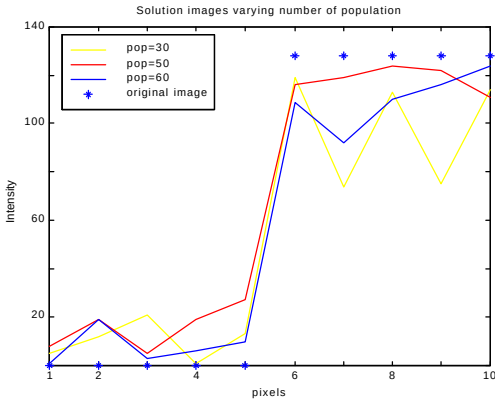

Figure 6: *Varying the amount of population per iteration.*