Single-phase Induction Machine

(From Guru et al Textbook)
Single-Phase Induction Motor

A motor that operates on a single-phase source is called a single-phase induction motor. A single-phase induction motor requires only one single-phase winding to keep the motor running. However, such a motor is not self-starting. Therefore, we must provide some external means to start a single-phase induction motor. Most single-phase induction motors are built in the fractional-horsepower range and are used in heating, cooling, and ventilating systems.

A properly designed direct-current (dc) series motor can be made to operate on both dc and alternating-current (ac) sources. For this reason, it is appropriately called a universal motor. These motors operate at relatively high speeds and are an integral part of such units as vacuum cleaners, food blenders, and portable electric tools such as saws and routers.

In Chapter 3 we made it clear that in order to make the motor self-starting there must at least be two phase windings placed in space quadrature and excited by a
According to this theory, a magnetic field that pulsates in time but is stationary in space can be resolved into two revolving magnetic fields that are equal in magnitude but revolve in opposite directions. Let us consider the standstill condition of the rotor again. The magnetic field produced by the motor pulsates up and down with time, and at any instant its magnitude may be given as

\[ B = B_m \cos \omega t \]

where \( B_m \) is the maximum flux density in the motor.

The flux density \( B \) can be resolved into two components \( B_1 \) and \( B_2 \) such that the magnitude of \( B_1 \) is equal to the magnitude of \( B_2 \). Thus, \( B_1 = B_2 = 0.5 B \). If we assume that \( B_1 \) rotates in the clockwise direction, the direction of rotation of \( B_2 \) is counterclockwise, as illustrated in Figure.

![Diagram showing resolution of a pulsating vector into two equal and oppositely revolving vectors.](image-url)

Resolution of a pulsating vector into two equal and oppositely revolving vectors.
We can look upon a single-phase induction motor as if it consists of two motors with a common stator winding but with rotors revolving in opposite directions. At standstill the two rotors develop equal torques in opposite directions, and the net torque developed is zero.

An equivalent circuit of a single-phase induction motor at rest.
One section of the rotor circuit is usually referred to as the **forward branch**, and the other is called the **backward branch**. When the motor rotates, say in the clockwise direction, the forward branch represents the effect of the revolving field in that direction. In this case, the backward branch corresponds to the rotor circuit associated with the counterclockwise revolving field. At standstill, both branches have the same impedance. The rotor circuit currents are also the same, and the same is true for the torques developed. Thus, when the rotor is at rest, the net torque developed by it is zero. We usually speak of torque developed by a branch. What we really mean is the torque developed by the rotor resistance in that particular branch.

Let us now assume that the rotor is rotating in the clockwise direction with a speed \( N_m \). The magnetic field revolving in the clockwise direction has a synchronous speed of \( N_s \) (\( N_s = 120f/P \)). The synchronous speed of the revolving field in the counterclockwise direction is then \(-N_s\). The per-unit slip in the forward (counterclockwise) direction is

\[
\begin{align*}
   s &= \frac{N_s - N_m}{N_s} = 1 - \frac{N_m}{N_s}
\end{align*}
\]

The per-unit slip in the backward (counterclockwise) direction is

\[
\begin{align*}
   s_b &= \frac{-N_s - N_m}{-N_s} = 1 + \frac{N_m}{N_s} = 2 - s
\end{align*}
\]

Note that at standstill, \( N_m = 0 \) and \( s = s_b = 1 \).
We can now incorporate the effect of slips in the forward and the backward rotor branches as we did for the three-phase induction motor. The modified equivalent circuit is given in Figure

An equivalent circuit of a single-phase induction motor at any slip $s$. 
At standstill, $s = s_b = 1$, the effective resistance in both branches of Figure 10.4 is the same. Thus, the torque developed by the two rotors is equal in magnitude but opposite in direction. That explains why there is no starting torque in a single-phase induction motor.

On the other hand, let us now assume that the rotor is rotating with a slip $s$ such that $s < 1$. The effective rotor resistance, $R_2/s$, in the forward branch is greater than that in the backward branch, $R_2/(2 - s)$. Thus, the torque developed by the forward branch is higher than that developed by the backward branch. The resultant torque is in the forward direction, and it tends to maintain the rotation in that direction. Thus, once a single-phase induction motor is made to rotate in any direction by applying an external torque, it continues rotating in that direction as long as the load torque is less than the maximum net torque developed by it.
EXAMPLE

A 115-V, 60-Hz, 4-pole, single-phase induction motor is rotating in the clockwise direction at a speed of 1710 rpm. Determine its per-unit slip (a) in the direction of rotation and (b) in the opposite direction. If the rotor resistance at standstill is 12.5 Ω, determine the effective rotor resistance in each branch.

\[ N_s = \frac{120 \times 60}{4} = 1800 \text{ rpm} \]

(a) Slip in the forward direction is

\[ s = \frac{1800 - 1710}{1800} = 0.05 \quad \text{or} \quad 5\% \]

(b) Slip in the backward direction is

\[ s_b = 2 - 0.05 = 1.95 \quad \text{or} \quad 195\% \]

The effective rotor resistances are

Forward branch: \[ \frac{0.5R_2}{s} = \frac{0.5 \times 12.5}{0.05} = 125 \text{ Ω} \]

Backward branch: \[ \frac{0.5R_2}{s_b} = \frac{0.5 \times 12.5}{1.95} = 3.205 \text{ Ω} \]
Analysis of a Single-Phase Induction Motor

From the equivalent circuit of a single-phase induction motor (Figure 10.4), we obtain

\[
\hat{Z}_f = R_f + jX_f = 0.5 \frac{jX_m[R_2/s + jX_2]}{R_2/s + j(X_2 + X_m)}
\]

as the effective impedance of the forward branch and

\[
\hat{Z}_b = R_b + jX_b = 0.5 \frac{jX_m[R_2/(2 - s) + jX_2]}{R_2/(2 - s) + j(X_2 + X_m)}
\]
as the effective impedance of the backward branch. The simplified equivalent circuit in terms of $\hat{Z}_f$ and $\hat{Z}_b$ is given in Figure 10.5.

If $\hat{Z}_1 = R_1 + jX_1$ is the impedance of the stator winding, the input impedance is

$$\hat{Z}_{in} = \hat{Z}_1 + \hat{Z}_f + \hat{Z}_b$$

The stator winding current is

$$\bar{I}_1 = \frac{\bar{V}_1}{\hat{Z}_{in}}$$

The power input is

$$P_{in} = \text{Re}[\bar{V}_1 \bar{I}_1^*] = V_1 I_1 \cos \theta$$

where $\theta$ is the power-factor angle by which the current $I_1$ lags the applied voltage $\bar{V}_1$. The stator copper loss is

$$P_{sc} = I_1^2 R_1$$
When we subtract the stator copper loss from the total power input, we are left with the air-gap power. However, the air-gap power is distributed between

Simplified equivalent circuit of a single-phase induction motor.
the two air-gap powers: one due to the forward revolving field and the other due to the backward revolving field. In order to determine the air-gap power associated with each revolving field, we have to determine the rotor currents in both branches. If $\tilde{I}_{2f}$ is the rotor current in the forward branch, then

$$\tilde{I}_{2f} = \tilde{I}_1 \frac{jX_m}{\frac{R_2}{s} + j(X_2 + X_m)}$$

Similarly, the rotor current in the backward branch $\tilde{I}_{2b}$ is

$$\tilde{I}_{2b} = \tilde{I}_1 \frac{jX_m}{\frac{R_2}{(2 - s)} + j(X_2 + X_m)}$$

Hence, the air-gap powers due to the forward and backward revolving fields are

$$P_{agf} = \tilde{I}_{2f}^2 R_2 \frac{0.5}{s}$$

$$P_{agb} = \tilde{I}_{2b}^2 R_2 \frac{0.5}{2 - s}$$
Since $R_f$ and $R_b$ are the equivalent resistances in the forward and backward branches of the rotor circuit, the power transferred to the rotor must also be consumed by these resistances. In other words, we can also compute the air-gap powers as

$$P_{agf} = I_1^2 R_f$$

for the forward branch and

$$P_{agb} = I_1^2 R_b$$

for the backward branch.

The net air-gap power is

$$P_{ag} = P_{agf} - P_{agb}$$

The mechanical power developed by the motor is

$$P_d = (1 - s)P_{ag} = T_d \omega_m = T_d (1 - s) \omega_s$$

Hence, the torque developed by the single-phase motor is

$$T_d = \frac{P_{ag}}{\omega_s}$$

The power available at the shaft is

$$P_o = P_d - P_{rot}$$
\[ P_o = P_d - P_{rot} \]

where \( P_r \) is the rotational loss of the motor. In this case, the rotational loss consists of the friction and windage loss, the core loss, and the stray-load loss.

The load (shaft) torque of the motor is

\[ T_s = \frac{P_o}{\omega_m} \]

Finally, the motor efficiency is the ratio of the power available at the shaft \( P_o \) to the total power input \( P_{in} \).

We could also have computed the torque developed by the forward and the backward revolving fields as

\[ T_{df} = \frac{P_{agf}}{\omega_s} \]

\[ T_{db} = \frac{P_{agb}}{\omega_s} \]

The net torque developed by the motor is

\[ T_d = T_{df} - T_{db} \]
Speed-torque characteristic of a single-phase induction motor
Types of Single-Phase Induction Motors

Each single-phase induction motor derives its name from the method used to make it self-starting. Some of the motors discussed in this section are split-phase motor, capacitor-start motor, capacitor-start capacitor-run motor, and permanent split-capacitor motor. Another induction motor discussed later in this chapter is called the shaded-pole motor.

For an induction motor to be self-starting, it must have at least two phase windings in space quadrature and must be excited by a two-phase source, as detailed in Chapter 3. The currents in the two phase windings are 90° electrical out of phase with each other. The placement of the two phase windings in space quadrature in a single-phase motor is no problem. However, the artificial creation of a second phase requires some basic understanding of resistive, inductive, and capacitive networks. Let us now examine how the second phase is created in each induction motor.
Split-Phase Motor

This is one of the most widely used induction motors for mechanical applications in the fractional horsepower range. The motor employs two separate windings that are placed in space quadrature and are connected in parallel to a single-phase source. One winding, known as the **main winding**, has a low resistance and high inductance. This winding carries current and establishes the needed flux at the rated speed. The second winding, called the **auxiliary winding**, has a high resistance and low inductance. This winding is disconnected from the supply when the motor attains a speed of nearly 75% of its synchronous speed. A centrifugal switch is commonly used to disconnect the auxiliary winding from the source at a predetermined speed. The disconnection is necessary to avoid the excessive power loss in the auxiliary winding at full load.

At the time of starting, the two windings draw currents from the supply. The main-winding current lags the applied voltage by almost 90° owing to its high inductance (large number of turns) and low resistance (large wire size). The auxiliary-winding current is essentially in phase with the applied voltage owing to its high resistance (small wire size) and low inductance (few number of turns).

As you may suspect, the main-winding current does not lag exactly by 90°, nor is auxiliary-winding current precisely in phase with the applied voltage. In addition, the two phase-winding currents may also not be equal in magnitude. In a well-designed split-phase motor, the phase difference between the two currents may be as high as 60°. It is from this **phase-splitting** action that the **split-phase motor** derives its name.

Since the two phase-windings are wound in space quadrature and carry out-of-phase currents, they set up an unbalanced revolving field. It is this revolving field, albeit unbalanced, that enables the motor to start.

The starting torque developed by a split-phase motor is typically 150% to 200% of the full-load torque. The starting current is about 6 to 8 times the full-load current. The schematic representation of a split-phase motor is given in Figure 10.7 along with its typical speed-torque characteristic. Note the drop in torque at the time the auxiliary winding is disconnected from the supply.
Split-Phase Starting Circuit

(a) Schematic representation
(b) Speed-torque characteristic of a split-phase motor.
Capacitor-Start Motor Circuit
Capacitor-Start Motor

In a capacitor-start motor a capacitor is included in series with the auxiliary winding. If the capacitor value is properly chosen, it is possible to design a capacitor-start motor such that the main-winding current lags the auxiliary-winding current by exactly 90°. Therefore, the starting torque developed by a capacitor motor can be as good as that of any polyphase motor.

Once again, the auxiliary winding and the capacitor are disconnected at about 75% of the synchronous speed. Therefore, at the rated speed the capacitor-start motor operates only on the main winding like a split-phase induction motor. The need for an external capacitor makes the capacitor-start motor somewhat more expensive than a split-phase motor. However, a capacitor-start motor is used when the starting torque requirements are 4 to 5 times the rated torque. Such a high starting torque is not within the realm of a split-phase motor. Since the capacitor is used only during starting, its duty cycle is very intermittent. Thus, an inexpensive and relatively small ac electrolytic-type capacitor can be used for all capacitor-start motors. A schematic representation of a capacitor-start motor and its speed-torque characteristic are given in Figure 10.8.
Capacitor-Start Capacitor-Run Motor Circuit

(a) Circuit diagram

(b) Graph showing percent of full-load torque vs. percent of speed for both capacitors, run capacitor, and load line.
Capacitor-Start Capacitor-Run Motor

Although the split-phase and capacitor-start motors are designed to satisfy the rated load requirements, they have low power factor at the rated speed. The lower the power factor, the higher the power input for the same power output. Thus, the efficiency of a single-phase motor is lower than that of a polyphase induction motor of the same size. For example, the efficiency of a capacitor-start or a split-phase single-phase motor is usually 50% to 60% in the fractional horsepower range. On the other hand, for the same application, a three-phase induction motor may have an efficiency of 70% to 80%.

The efficiency of a single-phase induction motor can be improved by employing another capacitor when the motor runs at the rated speed. This led to the development of a capacitor-start capacitor-run (CSCR) motor. Since this motor requires two capacitors, it is also known as the two-value capacitor motor. One capacitor is selected on the basis of starting torque requirements (the start capacitor), whereas the other capacitor is picked for the running performance (the run capacitor). The auxiliary winding stays in circuit at all times, but the centrifugal switch helps in switching from the start capacitor to the run capacitor at about 75% of the synchronous speed. The start capacitor is of the ac electrolytic type, whereas the run capacitor is of an ac oil type rated for continuous operation. Since both windings are active at the rated speed, the run capacitor can be selected to make the winding currents truly in quadrature with each other. Thus, a CSCR motor acts like a two-phase motor both at the time of starting and at its rated speed. Although the CSCR motor is more expensive because it uses two different capacitors, it has relatively high efficiency at full load compared with a split-phase or capacitor-start motor. A schematic representation of a CSCR motor and its speed-torque characteristic are given in Figure 10.9.
Permanent-Split Capacitor Motor

[Diagram of a Permanent-Split Capacitor Motor]

[Graph showing percent of full-load torque vs percent of speed]
Permanent Split-Capacitor Motor

A less expensive version of a CSCR motor is called a permanent split-capacitor (PSC) motor. A PSC motor uses the same capacitor for both starting and full load. Since the auxiliary winding and the capacitor stay in the circuit as long as the motor operates, there is no need for a centrifugal switch. For this reason, the motor length is smaller than for the other types discussed above. The capacitor is usually selected to obtain high efficiency at the rated load. Since the capacitor is not properly matched to develop optimal starting torque, the starting torque of a PSC motor is lower than that of a CSCR motor. PSC motors are, therefore, suitable for blower applications with minimal starting torque requirements. These motors are also good candidates for applications that require frequent starts. Other types of motors discussed above tend to overheat when started frequently, and this may badly affect the reliability of the entire system.