Semantic Analysis
Typechecking in COOL

Lecture 7

Outline

- The role of semantic analysis in a compiler
  - A laundry list of tasks

- Scope

- Types

The Compiler So Far

- Lexical analysis
  - Detects inputs with illegal tokens
- Parsing
  - Detects inputs with ill-formed parse trees
- Semantic analysis
  - Last "front end" phase
  - Catches more errors

What's Wrong?

- Example 1

```
let y: Int in x + 3
```

- Example 2

```
let y: String ← "abc" in y + 3
```

Why a Separate Semantic Analysis?

- Parsing cannot catch some errors
- Some language constructs are not context-free
  - Example: All used variables must have been declared (i.e. scoping)
  - Example: A method must be invoked with arguments of proper type (i.e. typing)

What Does Semantic Analysis Do?

- Checks of many kinds . . . **coolc** checks:
  1. All identifiers are declared
  2. Types
  3. Inheritance relationships
  4. Classes defined only once
  5. Methods in a class defined only once
  6. Reserved identifiers are not misused

  And others . . .

- The requirements depend on the language

Scope

- Matching identifier declarations with uses
  - Important semantic analysis step in most languages
  - Including *COOL*!

Scope (Cont.)

- The scope of an identifier is the portion of a program in which that identifier is accessible

- The same identifier may refer to different things in different parts of the program
  - Different scopes for same name don't overlap

- An identifier may have restricted scope

Static vs. Dynamic Scope

- Most languages have static scope
  - Scope depends only on the program text, not run-time behavior
  - Cool has static scope

- A few languages are dynamically scoped
  - Lisp, SNOBOL
  - Lisp has changed to mostly static scoping
  - Scope depends on execution of the program

Static Scoping Example

```
let x: Int <- 0 in
  {
        x;
        let x: Int <- 1 in
              x;
        x;
  }
```

Static Scoping Example (Cont.)

```
let x: Int <- 0 in
   {
        x;
        let x: Int <- 1 in
              x;
        x;
   }
```

Uses of x refer to closest enclosing definition

Dynamic Scope

- A dynamically-scoped variable refers to the closest enclosing binding in the execution of the program
- Example

```
Class foo {
    a : Int ← 4;
    g(y : Int) : Int {y + a};
    f() : Int { let a ← 5 in g(2) }
```

  - When invoking f() the result will be 6
- More about dynamic scope later in the course

Scope in Cool

- Cool identifier bindings are introduced by
  - Class declarations (introduce class names)
  - Method definitions (introduce method names)
  - Let expressions (introduce object id's)
  - Formal parameters (introduce object id's)
  - Attribute definitions in a class (introduce object id's)
  - Case expressions (introduce object id's)

Implementing the Most-Closely Nested Rule

- Much of semantic analysis can be expressed as a recursive descent of an AST
  - Process an AST node *n*
  - Process the children of *n*
  - Finish processing the AST node *n*

Implementing . . . (Cont.)

- Example: the scope of let bindings is one subtree

  let x: Int ← 0 in e

- x can be used in subtree e

Symbol Tables

- Consider again: let x: Int ← 0 in e
- Idea:
  - Before processing e, add definition of x to current definitions, overriding any other definition of x
  - After processing e, remove definition of x and restore old definition of x
- A *symbol table* is a data structure that tracks the current bindings of identifiers

Scope in Cool (Cont.)

- Not all kinds of identifiers follow the most-closely nested rule

- For example, class definitions in Cool
  - Cannot be nested
  - Are *globally visible* throughout the program

- In other words, a class name can be used before it is defined

Example: Use Before Definition

```
Class Foo {
   . . . let y: Bar in . . .
};

Class Bar {
   . . .
};
```

More Scope in Cool

Attribute names are global within the class in which they are defined

```
Class Foo {
   f(): Int { a };
   a: Int ← 0;
}
```

More Scope (Cont.)

- Method and attribute names have complex rules
- A method need not be defined in the class in which it is used, but in some parent class
- Methods may also be redefined (overridden)

Class Definitions

- Class names can be used before being defined
- We can’t check this property
  - using a symbol table
  - or even in one pass
- Solution
  - Pass 1: Gather all class names
  - Pass 2: Do the checking
- Semantic analysis requires multiple passes
  - Probably more than two

Types

- What is a type?
  - The notion varies from language to language
- Consensus
  - A set of values
  - A set of operations on those values
- Classes are one instantiation of the modern notion of type

Why Do We Need Type Systems?

Consider the assembly language fragment

```
addi  $r1, $r2, $r3
```

What are the types of $r1, $r2, $r3?

Types and Operations

- Certain operations are legal for values of each type
  - It doesn't make sense to add a function pointer and an integer in C
  - It does make sense to add two integers
  - But both have the same assembly language implementation!

Type Systems

- A language's type system specifies which operations are valid for which types
- The goal of type checking is to ensure that operations are used with the correct types
  - Enforces intended interpretation of values, because nothing else will!
- Type systems provide a concise formalization of the semantic checking rules

What Can Types do For Us?

- Can detect certain kinds of errors
- Memory errors:
  - Reading from an invalid pointer, etc.
- Violation of abstraction boundaries:

```
class FileSystem {
   open(x : String) : File {
          …
   }
…
}
```

```
class Client {
   f(fs : FileSystem) {
       File fdesc <- fs.open("foo")
       …
   } -- f cannot see inside fdesc !
}
```

Type Checking Overview

- Three kinds of languages:
  - *Statically typed*: All or almost all checking of types is done as part of compilation (C, Java, Cool)
  - *Dynamically typed*: Almost all checking of types is done as part of program execution (Scheme)
  - *Untyped*: No type checking (machine code)

The Type Wars

- Competing views on static vs. dynamic typing
- Static typing proponents say:
  - Static checking catches many programming errors at compile time
  - Avoids overhead of runtime type checks
- Dynamic typing proponents say:
  - Static type systems are restrictive
  - Rapid prototyping easier in a dynamic type system

The Type Wars (Cont.)

- In practice, most code is written in statically typed languages with an “escape” mechanism
  - Unsafe casts in C, Java

- It’s debatable whether this compromise represents the best or worst of both worlds

Type Checking in Cool

Outline

- Type concepts in COOL
- Notation for type rules
  - Logical rules of inference
- COOL type rules
- General properties of type systems

Cool Types

- The types are:
  - Class names
  - SELF_TYPE
  - Note: there are no base types (as in Java int, …)

- The user declares types for all identifiers

- The compiler infers types for expressions
  - Infers a type for *every* expression

Type Checking and Type Inference

- Type Checking is the process of verifying fully typed programs

- Type Inference is the process of filling in missing type information

- The two are different, but are often used interchangeably

Rules of Inference

- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions (for the lexer)
  - Context-free grammars (for the parser)

- The appropriate formalism for type checking is logical rules of inference

Why Rules of Inference?

- Inference rules have the form

  If Hypothesis is true, then Conclusion is true

- Type checking computes via reasoning

  If E_1 and E_2 have certain types, then E_3 has a certain type

- Rules of inference are a compact notation for "If-Then" statements

From English to an Inference Rule

- The notation is easy to read (with practice)

- Start with a simplified system and gradually add features

- Building blocks
  - Symbol $\wedge$ is "and"
  - Symbol $\Rightarrow$ is "if-then"
  - $x{:}T$ is "x has type T"

From English to an Inference Rule (2)

If e_1 has type Int and e_2 has type Int,
then $e_1 + e_2$ has type Int

(e_1 has type Int $\wedge$ e_2 has type Int) $\Rightarrow$
$e_1 + e_2$ has type Int

$(e_1\text{: Int} \wedge e_2\text{: Int}) \Rightarrow e_1 + e_2\text{: Int}$

From English to an Inference Rule (3)

The statement

$$(e_1\text{: Int} \wedge e_2\text{: Int}) \Rightarrow e_1 + e_2\text{: Int}$$

is a special case of

$$(\text{Hypothesis}_1 \wedge \ldots \wedge \text{Hypothesis}_n) \Rightarrow \text{Conclusion}$$

This is an inference rule

Notation for Inference Rules

- By tradition inference rules are written

$$\frac{\text{` Hypothesis}_1 \quad \ldots \quad \text{` Hypothesis}_n}{\text{` Conclusion}}$$

- Cool type rules have hypotheses and conclusions of the form:

$$\text{`}\ e : T$$

- ` means “it is provable that . . .”

Two Rules

$$\frac{i \text{ is an integer}}{\text{`}\ i : \text{Int}} \quad [\text{Int}]$$

$$\frac{\begin{array}{c}\text{`}\ e_1 : \text{Int} \\ \text{`}\ e_2 : \text{Int}\end{array}}{\text{`}\ e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

Two Rules (Cont.)

- These rules give templates describing how to type integers and + expressions

- By filling in the templates, we can produce complete typings for expressions

Example: 1 + 2

$$\dfrac{\dfrac{1 \text{ is an integer}}{` \ 1 : \text{Int}} \qquad\qquad \dfrac{2 \text{ is an integer}}{` \ 2 : \text{Int}}}{` \ 1 + 2 : \text{Int}}$$

Soundness

- A type system is <u>sound</u> if
  - Whenever ` e : T
  - Then e evaluates to a value of type T

- We only want sound rules
  - But some sound rules are better than others:

i is an integer

` i : Object

Type Checking Proofs

- Type checking proves facts $e : T$
  - Proof is on the structure of the AST
  - Proof has the shape of the AST
  - One type rule is used for each kind of AST node
- In the type rule used for a node e:
  - Hypotheses are the proofs of types of e's subexpressions
  - Conclusion is the proof of type of e
- Types are computed in a bottom-up pass over the AST

Rules for Constants

$$\frac{}{` \; \text{false} : \text{Bool}} \quad [\text{Bool}]$$

$$\frac{s \text{ is a string constant}}{` \; s : \text{String}} \quad [\text{String}]$$

Rule for New

new T produces an object of type T

- Ignore SELF_TYPE for now . . .

$$\frac{}{\vdash \text{new } T : T} \quad \text{[New]}$$

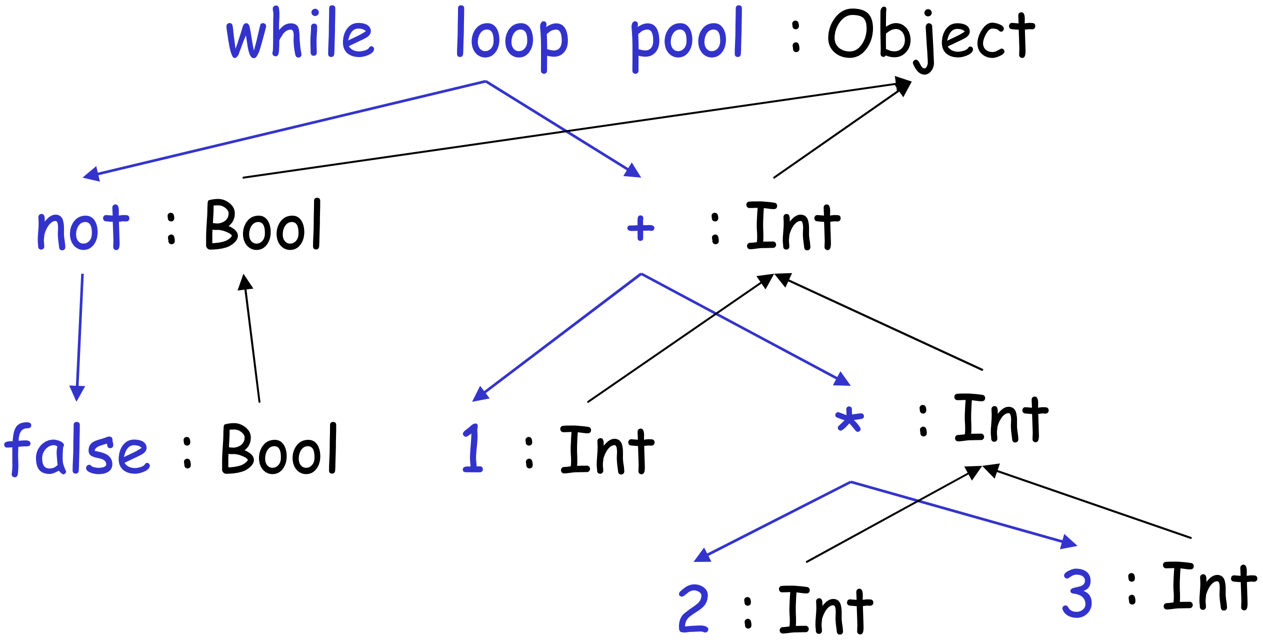
Two More Rules

$$\frac{\vdash e : \text{Bool}}{\vdash \text{not } e : \text{Bool}} \quad [\text{Not}]$$

$$\frac{\begin{array}{c}\vdash e_1 : \text{Bool} \\ \vdash e_2 : T\end{array}}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \quad [\text{Loop}]$$

Typing: Example

- Typing for while not false loop 1 + 2 * 3 pool



Typing Derivations

- The typing reasoning can be expressed as a tree:

$$\frac{\dfrac{\text{` false : Bool}}{\text{` not false : Bool}} \qquad \dfrac{\text{` 1 : Int} \qquad \dfrac{\text{` 2 : Int} \qquad \text{` 3 : Int}}{\text{` 2 * 3 : Int}}}{\text{` 1 + 2 * 3: Int}}}{\text{` while not false loop 1 + 2 * 3 : Object}}$$

- The root of the tree is the whole expression
- Each node is an instance of a typing rule
- Leaves are the rules with no hypotheses

A Problem

- What is the type of a variable reference?

$$\frac{x \text{ is an identifier}}{\text{\`{}}\ x : ?}\quad [\text{Var}]$$

- The local, structural rule does not carry enough information to give x a type.

A Solution: Put more information in the rules!

- A *type environment* gives types for *free* variables
  - A type environment is a function from ObjectIdentifiers to Types
  - A variable is free in an expression if:
    - It occurs in the expression
    - It is declared outside the expression
  - E.g. in the expression “x”, the variable “x” is free
  - E.g. in “let x : Int in x + y” only “y” is free

Type Environments

Let O be a function from ObjectIdentifiers to Types

The sentence O ` e : T

is read: Under the assumption that variables have the types given by O, it is provable that the expression e has the type T

Modified Rules

The type environment is added to the earlier rules:

$$\frac{i \text{ is an integer}}{O \text{ ` } i : \text{Int}} \quad \text{[Int]}$$

$$\frac{\begin{array}{c} O \text{ ` } e_1 : \text{Int} \\ O \text{ ` } e_2 : \text{Int} \end{array}}{O \text{ ` } e_1 + e_2 : \text{Int}} \quad \text{[Add]}$$

New Rules

And we can write new rules:

$$\frac{O(x) = T}{O \text{ ` } x : T} \quad [Var]$$

Now

- More (complicated) typing rules

- Connections between typing rules and safety of execution

Let

$$\frac{O[T_0/x] \text{ ` } e_1 : T_1}{O \text{ ` } \text{let } x : T_0 \text{ in } e_1 : T_1} \qquad \text{[Let-No-Init]}$$

$O[T_0/x]$ means O modified to return T_0 on argument x and behave as O on all other arguments:

$$O[T_0/x]\,(x) = T_0$$
$$O[T_0/x]\,(y) = O(y)$$

Let. Example.

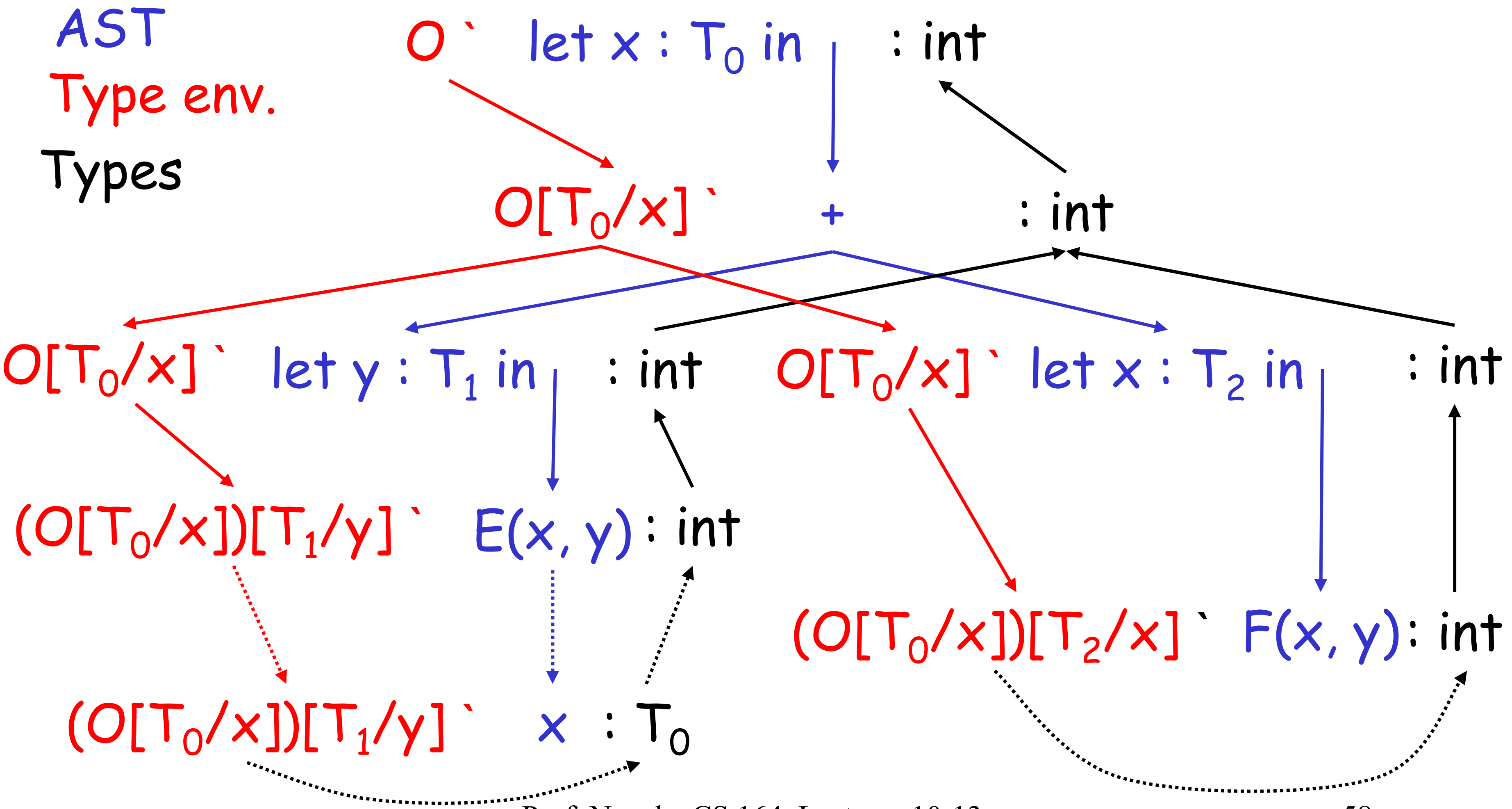
- Consider the Cool expression

  $\text{let } x : T_0 \text{ in } (\text{let } y : T_1 \text{ in } E_{x,\, y}) + (\text{let } x : T_2 \text{ in } F_{x,\, y})$

  (where $E_{x,\, y}$ and $F_{x,\, y}$ are some Cool expression that contain occurrences of “x” and “y”)

- Scope
  - of “y” is $E_{x,\, y}$
  - of outer “x” is $E_{x,\, y}$
  - of inner “x” is $F_{x,\, y}$
- This is captured precisely in the typing rule.

Let. Example.



AST

Type env.

Types

O ` let $x : T_0$ in : int

$O[T_0/x]$ ` + : int

$O[T_0/x]$ ` let $y : T_1$ in : int

$O[T_0/x]$ ` let $x : T_2$ in : int

$(O[T_0/x])[T_1/y]$ ` $E(x, y)$: int

$(O[T_0/x])[T_1/y]$ ` x : T_0

$(O[T_0/x])[T_2/x]$ ` $F(x, y)$: int

Notes

- The type environment *gives* types to the free identifiers in the current scope

- The type environment is passed down the AST from the root towards the leaves

- Types are computed up the AST from the leaves towards the root

Let with Initialization

Now consider let with initialization:

$$\frac{\begin{array}{c} O \text{ ` } e_0 : T_0 \\ O[T_0/x] \text{ ` } e_1 : T_1 \end{array}}{O \text{ ` } \mathrm{let}\ x : T_0 \leftarrow e_0\ \mathrm{in}\ e_1 : T_1} \quad \text{[Let-Init]}$$

This rule is weak. Why?

Let with Initialization

- Consider the example:

```
class C inherits P { … }
…
let x : P ← new C in …
 …
```

- The previous let rule does not allow this code
  - We say that the rule is too weak

Subtyping

- Define a relation $X \cdot Y$ on classes to say that:
  - An object of type X could be used when one of type Y is acceptable, or equivalently
  - X conforms with Y
  - In Cool this means that X is a subtype of Y
- Define a relation $\leq$ on classes

  $X \leq X$

  $X \leq Y$ if X inherits from Y

  $X \leq Z$ if $X \leq Y$ and $Y \leq Z$

Let with Initialization (Again)

$$\frac{\begin{array}{c} O \;`\; e_0 : T \\ T \cdot T_0 \\ O[T_0/x] \;`\; e_1 : T_1 \end{array}}{O \;`\; \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \; [\text{Let-Init}]$$

- Both rules for let are correct
- But more programs type check with the latter

Let with Subtyping. Notes.

- There is a tension between
  - Flexible rules that do not constrain programming
  - Restrictive rules that ensure safety of execution

Expressiveness of Static Type Systems

- A static type system enables a compiler to detect many common programming errors
- The cost is that some correct programs are disallowed
  - Some argue for dynamic type checking instead
  - Others argue for more expressive static type checking
- But more expressive type systems are also more complex

Dynamic And Static Types

- The dynamic type of an object is the class *C* that is used in the "new *C*" expression that creates the object
  - A run-time notion
  - Even languages that are not statically typed have the notion of dynamic type
- The static type of an expression is a notation that captures all possible dynamic types the expression could take
  - A compile-time notion

Dynamic and Static Types. (Cont.)

- In early type systems the set of static types correspond directly with the dynamic types
- Soundness theorem: for all expressions E

  dynamic_type(E) = static_type(E)

  (in **all** executions, E evaluates to values of the type inferred by the compiler)

- This gets more complicated in advanced type systems

Dynamic and Static Types in COOL

```
class A { … }
class B inherits A {…}
class Main {
  A x ← new A;
  …
  x ← new B;
  …
}
```

x has static type A

Here, x's value has dynamic type A

Here, x's value has dynamic type B

- A variable of static type A can hold values of static type B, if B ≤ A

Dynamic and Static Types

Soundness theorem for the Cool type system:

$$\forall E.\quad \text{dynamic\_type}(E) \leq \text{static\_type}(E)$$

Why is this Ok?

- All operations that can be used on an object of type C can also be used on an object of type $C' \leq C$
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can <u>only add</u> attributes or methods
- Methods can be redefined but with same type !

Let. Examples.

- Consider the following Cool class definitions

```
Class A { a() : int { 0 }; }
Class B inherits A { b() : int { 1 }; }
```

- An instance of B has methods "a" and "b"
- An instance of A has method "a"
  - A type error occurs if we try to invoke method "b" on an instance of A

Example of Wrong Let Rule (1)

- Now consider a hypothetical let rule:

$$\frac{O \text{ ` } e_0 : T \qquad\qquad T \cdot T_0 \qquad\qquad O \text{ ` } e_1 : T_1}{O \text{ ` } \text{let } x : T_0 \ \tilde{A}\ e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following good program does not typecheck

let x : Int Ã 0 in x + 1

- Why?

Example of Wrong Let Rule (2)

- Now consider a hypothetical let rule:

$$\frac{O \text{ ` } e_0 : T \qquad\qquad T_0 \cdot T \qquad\qquad O[T_0/x] \text{ ` } e_1 : T_1}{O \text{ ` } \text{let } x : T_0 \text{ Ã } e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following bad program is well typed

  let x : B Ã new *A* in x.b()

- Why is this program bad?

Example of Wrong Let Rule (3)

- Now consider a hypothetical let rule:

$$\frac{O \text{ ` } e_0 : T \qquad T \cdot T_0 \qquad O[T/x] \text{ ` } e_1 : T_1}{O \text{ ` } \text{let } x : T_0 \text{ Ã } e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following good program is not well typed

  let x : A Ã new B in {… x Ã new A; x.a(); }

- Why is this program not well typed?

Morale.

- The typing rules use very concise notation
- They are very carefully constructed
- Virtually any change in a rule either:
  - Makes the type system unsound

    (bad programs are accepted as well typed)
  - Or, makes the type system less usable

    (perfectly good programs are rejected)

- But some good programs will be rejected anyway
  - The notion of a good program is undecidable

Assignment

More uses of subtyping:

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \mathbin{`} e_1 : T_1 \\ T_1 \cdot T_0 \end{array}}{O \mathbin{`} id \mathbin{\tilde{A}} e_1 : T_1} \quad \text{[Assign]}$$

Initialized Attributes

- Let $O_C(x) = T$ for all attributes $x{:}T$ in class C

- Attribute initialization is similar to let, except for the scope of names

$$\frac{\begin{array}{c} O_C(\mathrm{id}) = T_0 \\ O_C \text{ ` } e_1 : T_1 \\ T_1 \cdot T_0 \end{array}}{O_C \text{ ` } \mathrm{id} : T_0 \text{ Ã } e_1 \text{ ;}} \qquad \text{[Attr-Init]}$$

If-Then-Else

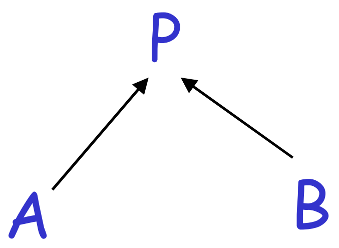
- Consider:

  if e_0 then e_1 else e_2 fi

- The result can be either e_1 or e_2
- The type is either e_1's type or e_2's type
- The best we can do is the smallest supertype larger than the type of e_1 and e_2

If-Then-Else example

- Consider the class hierarchy



- … and the expression

  if … then new A else new B fi

- Its type should allow for the dynamic type to be both A or B
  - Smallest supertype is P

Least Upper Bounds

- lub(X,Y), the least upper bound of X and Y, is Z if
  - $X \le Z \wedge Y \le Z$

    Z is an upper bound
  - $X \le Z' \wedge Y \le Z' \Rightarrow Z \le Z'$

    Z is least among upper bounds
- In COOL, the least upper bound of two types is their least common ancestor in the inheritance tree

If-Then-Else Revisited

$$\frac{\begin{array}{c} O \text{ ` } e_0 : \text{Bool} \\ O \text{ ` } e_1 : T_1 \\ O \text{ ` } e_2 : T_2 \end{array}}{O \text{ ` } \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : \text{lub}(T_1, T_2)} \quad \text{[If-Then-Else]}$$

Case

- The rule for case expressions takes a lub over all branches

$$\frac{\begin{array}{c} O \text{ ` } e_0 : T_0 \\ O[T_1/x_1] \text{ ` } e_1 : T_1' \\ \ldots \\ O[T_n/x_n] \text{ ` } e_n : T_n' \end{array}}{O \text{ ` } \text{case } e_0 \text{ of } x_1 : T_1 \text{) } e_1; \ldots; x_n : T_n \text{) } e_n; \text{ esac} : \text{lub}(T_1', \ldots, T_n')} \quad \text{[Case]}$$

Outline

- Type checking method dispatch
- Type checking with SELF_TYPE in COOL

Method Dispatch

- There is a problem with type checking method calls:

$$\frac{\begin{array}{c} O \text{ ` } e_0 : T_0 \\ O \text{ ` } e_1 : T_1 \\ \ldots \\ O \text{ ` } e_n : T_n \end{array}}{O \text{ ` } e_0.f(e_1,\ldots,e_n) : ?} \qquad \text{[Dispatch]}$$

- We need information about the formal parameters and return type of f

Notes on Dispatch

- In Cool, method and object identifiers live in different name spaces
  - A method foo and an object foo can coexist in the same scope
- In the type rules, this is reflected by a separate mapping M for method signatures

$$M(C,f) = (T_1,\ldots T_n,T_{n+1})$$

means in class C there is a method f

$$f(x_1{:}T_1,\ldots,x_n{:}T_n){:}\ T_{n+1}$$

An Extended Typing Judgment

- Now we have two environments O and M

- The form of the typing judgment is

  O, M ` e : T

read as: "with the assumption that the object identifiers have types as given by O and the method identifiers have signatures as given by M, the expression e has type T"

The Method Environment

- The method environment must be added to all rules
- In most cases, M is passed down but not actually used
  - Example of a rule that does not use M:

$$\frac{O, M \text{ ` } e_1 : T_1 \qquad O, M \text{ ` } e_2 : T_2}{O, M \text{ ` } e_1 + e_2 : \mathrm{Int}} \quad [\mathrm{Add}]$$

  - Only the dispatch rules uses M

The Dispatch Rule Revisited

$$\frac{\begin{array}{c} O, M ` e_0 : T_0 \\ O, M ` e_1 : T_1 \\ \ldots \\ O, M ` e_n : T_n \\ M(T_0, f) = (T_1',\ldots,T_n', T_{n+1}') \\ T_i \cdot T_i' \quad (\text{for } 1 \cdot i \cdot n) \end{array}}{O, M ` e_0.f(e_1,\ldots,e_n) : T_{n+1}'} \qquad [\text{Dispatch}]$$

Static Dispatch

- Static dispatch is a variation on normal dispatch
- The method is found in the class explicitly named by the programmer
- The inferred type of the dispatch expression must conform to the specified type

Static Dispatch (Cont.)

$$\frac{\begin{array}{c} O, M \text{`} e_0 : T_0 \\ O, M \text{`} e_1 : T_1 \\ \ldots \\ O, M \text{`} e_n : T_n \\ T_0 \cdot T \\ M(T, f) = (T_1',\ldots,T_n', T_{n+1}') \\ T_i \cdot T_i' \quad (\text{for } 1 \cdot i \cdot n) \end{array}}{O, M \text{`} e_0@T.f(e_1,\ldots,e_n) : T_{n+1}'} \qquad [\text{StaticDispatch}]$$

Handling the SELF_TYPE

Flexibility vs. Soundness

- Recall that type systems have two conflicting goals:
  - Give flexibility to the programmer
  - Prevent valid programs to "go wrong"
    - Milner, 1981: "Well-typed programs do not go wrong"
- An active line of research is in the area of inventing more flexible type systems while preserving soundness

Dynamic And Static Types. Review.

- The dynamic type of an object is the class C that is used in the "new C" expression that created it
  - A run-time notion
  - Even languages that are not statically typed have the notion of dynamic type
- The static type of an expression is a notation that captures all possible dynamic types the expression could take
  - A compile-time notion

Dynamic and Static Types. Review

Soundness theorem for the Cool type system:

$$\forall E.\quad \text{dynamic\_type}(E) \leq \text{static\_type}(E)$$

Why is this Ok?

- All operations that can be used on an object of type C can also be used on an object of type $C' \leq C$
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can only add attributes or methods
- Methods can be redefined but with same type !

An Example

```
class Count {
   i : int ← 0;
   inc () : Count {
        {
            i ← i + 1;
            self;
        }
   };
};
```

- Class Count incorporates a counter
- The inc method works for any subclass
- But there is disaster lurking in the type system

An Example (Cont.)

- Consider a subclass Stock of Count

```
class Stock inherits Count {
   name : String; -- name of item
};
```

- And the following use of Stock:

```
class Main {
  Stock a ← (new Stock).inc ();    Type checking error !
  …  a.name …
};
```

What Went Wrong?

- (new Stock).inc() has dynamic type Stock
- So it is legitimate to write

  Stock a ← (new Stock).inc ()
- But this is not well-typed

  (new Stock).inc() has static type Count
- The type checker "looses" type information
- This makes inheriting inc useless
  - So, we must redefine inc for each of the subclasses, with a specialized return type

SELF_TYPE to the Rescue

- We will extend the type system
- Insight:
  - inc returns "self"
  - Therefore the return value has same type as "self"
  - Which could be Count or any subtype of Count !
  - In the case of (new Stock).inc () the type is Stock
- We introduce the keyword SELF_TYPE to use for the return value of such functions
  - We will also need to modify the typing rules to handle SELF_TYPE

SELF_TYPE to the Rescue (Cont.)

- SELF_TYPE allows the return type of inc to change when inc is inherited
- Modify the declaration of inc to read

  inc() : SELF_TYPE { … }
- The type checker can now prove:

  O, M ` (new Count).inc() : Count

  O, M ` (new Stock).inc() : Stock
- The program from before is now well typed

Notes About SELF_TYPE

- SELF_TYPE is not a dynamic type
- It is a static type
- It helps the type checker to keep better track of types
- It enables the type checker to accept more correct programs
- In short, having SELF_TYPE increases the expressive power of the type system

SELF_TYPE and Dynamic Types (Example)

- What can be the dynamic type of the object returned by inc?
  - Answer: whatever could be the type of "self"

```
class A inherits Count { } ;
class B inherits Count { } ;
class C inherits Count { } ;
```

(inc could be invoked through any of these classes)

  - Answer: Count or any subtype of Count

SELF_TYPE and Dynamic Types (Example)

- In general, if SELF_TYPE appears textually in the class C as the declared type of E then it denotes the dynamic type of the "self" expression:

  $$\text{dynamic\_type}(E) = \text{dynamic\_type}(\text{self}) \leq C$$

- Note: The meaning of SELF_TYPE depends on where it appears
  - We write SELF_TYPE_C to refer to an occurrence of SELF_TYPE in the body of C

Type Checking

- This suggests a typing rule:

  $$\text{SELF\_TYPE}_C \leq C$$

- This rule has an important consequence:
  - In type checking it is always safe to replace SELF_TYPE_C by C
- This suggests one way to handle SELF_TYPE :
  - Replace all occurrences of SELF_TYPE_C by C

- This would be correct but it is like not having SELF_TYPE at all

Operations on SELF_TYPE

- Recall the operations on types
  - $T_1 \leq T_2$ T_1 is a subtype of T_2
  - $\mathrm{lub}(T_1,T_2)$ the least-upper bound of T_1 and T_2
- We must extend these operations to handle SELF_TYPE

Extending $\leq$

Let T and T' be any types but SELF_TYPE

There are four cases in the definition of $\leq$

1. $\text{SELF\_TYPE}_C \leq T$ if $C \leq T$
   - SELF_TYPE_C can be any subtype of C
   - This includes C itself
   - Thus this is the most flexible rule we can allow
2. $\text{SELF\_TYPE}_C \leq \text{SELF\_TYPE}_C$
   - SELF_TYPE_C is the type of the "self" expression
   - In Cool we never need to compare SELF_TYPEs coming from different classes

Extending $\leq$ (Cont.)

3. $T \leq SELF\_TYPE_C$ always false

   Note: $SELF\_TYPE_C$ can denote any subtype of C.

4. $T \leq T'$ (according to the rules from before)

Based on these rules we can extend lub …

Extending lub(T,T')

Let T and T' be any types but SELF_TYPE

Again there are four cases:

1. lub(SELF_TYPE_C, SELF_TYPE_C) = SELF_TYPE_C

2. lub(SELF_TYPE_C, T) = lub(C, T)

   This is the best we can do because $\text{SELF\_TYPE}_C \leq C$

3. lub(T, SELF_TYPE_C) = lub(C, T)

4. lub(T, T') defined as before

Where Can SELF_TYPE Appear in COOL?

- The parser checks that SELF_TYPE appears only where a type is expected
- But SELF_TYPE is not allowed everywhere a type can appear:

1. class T inherits T' {…}
    - T, T' cannot be SELF_TYPE
    - Because SELF_TYPE is never a dynamic type
2. x : T
    - T can be SELF_TYPE
    - An attribute whose type is SELF_TYPE_C

Where Can SELF_TYPE Appear in COOL?

3. let x : T in E
    - T can be SELF_TYPE
    - x has type SELF_TYPE_C
4. new T
    - T can be SELF_TYPE
    - Creates an object of the same type as self
5. $m@T(E_1,\dots,E_n)$
    - T cannot be SELF_TYPE

Typing Rules for SELF_TYPE

- Since occurrences of SELF_TYPE depend on the enclosing class we need to carry more context during type checking
- New form of the typing judgment:

O,M,C ` e : T

(An expression e occurring in the body of C has static type T given a variable type environment O and method signatures M)

Type Checking Rules

- The next step is to design type rules using SELF_TYPE for each language construct
- Most of the rules remain the same except that ≤ and lub are the new ones
- Example:

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \text{ ` } e_1 : T_1 \\ T_1 \cdot T_0 \end{array}}{O \text{ ` } id \text{ Ã } e_1 : T_1}$$

What's Different?

- Recall the old rule for dispatch

$$\frac{\begin{array}{c} O,M,C ` e_0 : T_0 \\ \ldots \\ O,M,C ` e_n : T_n \\ M(T_0, f) = (T_1',\ldots,T_n',T_{n+1}') \\ T_{n+1}' \neq \text{SELF\_TYPE} \\ T_i \le T_i' \qquad\qquad 1 \le i \le n \end{array}}{O,M,C ` e_0.f(e_1,\ldots,e_n) : T_{n+1}'}$$

What’s Different?

- If the return type of the method is SELF_TYPE then the type of the dispatch is the type of the dispatch expression:

$$\frac{\begin{array}{c} O,M,C \text{ ` } e_0 : T_0 \\ \ldots \\ O,M,C \text{ ` } e_n : T_n \\ M(T_0, f) = (T_1',\ldots,T_n', \text{SELF\_TYPE}) \\ T_i \leq T_i' \qquad\qquad 1 \leq i \leq n \end{array}}{O,M,C \text{ ` } e_0.f(e_1,\ldots,e_n) : T_0}$$

What's Different?

- Note this rule handles the Stock example
- Formal parameters cannot be SELF_TYPE
- Actual arguments can be SELF_TYPE
  - The extended ≤ relation handles this case
- The type T_0 of the dispatch expression could be SELF_TYPE
  - Which class is used to find the declaration of f?
  - Answer: it is safe to use the class where the dispatch appears

Static Dispatch

- Recall the original rule for static dispatch

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ T_0 \leq T \\ M(T, f) = (T_1',\ldots,T_n',T_{n+1}') \\ T_{n+1}' \neq \text{SELF\_TYPE} \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0@T.f(e_1,\ldots,e_n) : T_{n+1}'}$$

Static Dispatch

- If the return type of the method is SELF_TYPE we have:

$$\frac{\begin{array}{c} O,M,C \text{ ` } e_0 : T_0 \\ \ldots \\ O,M,C \text{ ` } e_n : T_n \\ T_0 \leq T \\ M(T, f) = (T_1',\ldots,T_n',\text{SELF\_TYPE}) \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \text{ ` } e_0@T.f(e_1,\ldots,e_n) : T_0}$$

Static Dispatch

- Why is this rule correct?
- If we dispatch a method returning SELF_TYPE in class T, don't we get back a T?
- No. SELF_TYPE is the type of the self parameter, which may be a subtype of the class in which the method appears
- The static dispatch class cannot be SELF_TYPE

New Rules

- There are two new rules using SELF_TYPE

$$\frac{}{O,M,C \text{ ` } \text{self} : \text{SELF\_TYPE}_C}$$

$$\frac{}{O,M,C \text{ ` } \text{new SELF\_TYPE} : \text{SELF\_TYPE}_C}$$

- There are a number of other places where SELF_TYPE is used

Where SELF_TYPE Cannot Appear in COOL?

m(x : T) : T' { … }

- Only T' can be SELF_TYPE !

What could go wrong if T were SELF_TYPE?

```
class A {  comp(x : SELF_TYPE) : Bool  {…};  };
class B inherits A {
     b : int;
     comp(x : SELF_TYPE) : Bool { … x.b …};  };
…
  let x : A ← new B in  … x.comp(new A); …
…
```

Summary of SELF_TYPE

- The extended ≤ and lub operations can do a lot of the work. Implement them to handle SELF_TYPE
- SELF_TYPE can be used only in a few places. Be sure it isn't used anywhere else.
- A use of SELF_TYPE always refers to any subtype in the current class
  - The exception is the type checking of dispatch.
  - SELF_TYPE as the return type in an invoked method might have nothing to do with the current class

Why Cover SELF_TYPE ?

- SELF_TYPE is a research idea
  - It adds more expressiveness to the type system
- SELF_TYPE is itself not so important
  - except for the project
- Rather, SELF_TYPE is meant to illustrate that type checking can be quite subtle
- In practice, there should be a balance between the complexity of the type system and its expressiveness

Type Systems

- The rules in these lecture were COOL-specific
  - Other languages have very different rules
  - We'll survey a few more type systems later
- General themes
  - Type rules are defined on the structure of expressions
  - Types of variables are modeled by an environment
- Types are a play between flexibility and safety