## Derivatives, Limits, Sums and Integrals

The expressions

$$\frac{du}{dt} \text{ and } \frac{d^2u}{dx^2}$$

are obtained in LaTeX by typing `\frac{du}{dt}` and `\frac{d^2 u}{dx^2}` respectively. The mathematical symbol $\partial$ is produced using `\partial`. Thus the Heat Equation

$$\frac{\partial u}{\partial t} = h^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

is obtained in LaTeX by typing

```
\[ \frac{\partial u}{\partial t}
   = h^2 \left( \frac{\partial^2 u}{\partial x^2}
      + \frac{\partial^2 u}{\partial y^2}
      + \frac{\partial^2 u}{\partial z^2} \right) \]
```

To obtain mathematical expressions such as

$$\lim_{x \to +\infty}, \inf_{x > s} \text{ and } \sup_K$$

in displayed equations we type `\lim_{x \to +\infty}`, `\inf_{x > s}` and `\sup_K` respectively. Thus to obtain

$$\lim_{x \to 0} \frac{3x^2 + 7}{x^2 + 1} = 3.$$

(in LaTeX) we type

```
\[ \lim_{x \to 0} \frac{3x^2 +7x^3}{x^2 +5x^4} = 3.\]
```

To obtain a summation sign such as

$$\sum_{i=1}^{2n}$$

we type `\sum_{i=1}^{2n}`. Thus

$$\sum_{k=1}^{n} k^2 = \frac{1}{2} n (n+1).$$

is obtained by typing

```
\[ \sum_{k=1}^n k^2 = \frac{1}{2} n (n+1).\]
```

We now discuss how to obtain *integrals* in mathematical documents. A typical integral is the following:

$$\int_a^b f(x)\,dx.$$

This is typeset using

```
\[ \int_a^b f(x)\,dx.\]
```

The integral sign $\int$ is typeset using the control sequence `\int`, and the *limits of integration* (in this case $a$ and $b$ are treated as a subscript and a superscript on the integral sign.

Most integrals occurring in mathematical documents begin with an integral sign and contain one or more instances of $d$ followed by another (Latin or Greek) letter, as in $dx$, $dy$ and $dt$. To obtain the correct appearance one should put extra space before the $d$, using `\,`. Thus

$$\int_0^{+\infty} x^n e^{-x} \,dx = n!.$$

$$\int \cos \theta \,d\theta = \sin \theta.$$

$$\int_{x^2 + y^2 \leq R^2} f(x,y)\,dx\,dy = \int_{\theta=0}^{2\pi} \int_{r=0}^R f(r\cos\theta,r\sin\theta) r\,dr\,d\theta.$$

and

$$\int_0^R \frac{2x\,dx}{1+x^2} = \log(1+R^2).$$

are obtained by typing

```
\[ \int_0^{+\infty} x^n e^{-x} \,dx = n!.\]

\[ \int \cos \theta \,d\theta = \sin \theta.\]

\[ \int_{x^2 + y^2 \leq R^2} f(x,y)\,dx\,dy
   = \int_{\theta=0}^{2\pi} \int_{r=0}^R
      f(r\cos\theta,r\sin\theta) r\,dr\,d\theta.\]
```

and

```
\[ \int_0^R \frac{2x\,dx}{1+x^2} = \log(1+R^2).\]
```

respectively.

In some multiple integrals (i.e., integrals containing more than one integral sign) one finds that LaTeX puts too much space between the integral signs. The way to improve the appearance of of the integral is to use the control sequence `\!` to remove a thin strip of unwanted space. Thus, for example, the multiple integral

$$\int_0^1 \! \int_0^1 x^2 y^2\,dx\,dy.$$

is obtained by typing

```
\[ \int_0^1 \! \int_0^1 x^2 y^2\,dx\,dy.\]
```

Had we typed

```
\[ \int_0^1 \int_0^1 x^2 y^2\,dx\,dy.\]
```

we would have obtained

$$\int_0^1 \int_0^1 x^2 y^2\,dx\,dy.$$

A particularly noteworthy example comes when we are typesetting a multiple integral such as

$$\int \!\!\! \int_D f(x,y)\,dx\,dy.$$

Here we use `\!` three times to obtain suitable spacing between the integral signs. We typeset this integral using

```
\[ \int \!\!\! \int_D f(x,y)\,dx\,dy.\]
```

Had we typed

```
\[ \int \int_D f(x,y)\,dx\,dy.\]
```

we would have obtained

$$\int \int_D f(x,y)\,dx\,dy.$$

The following (reasonably complicated) passage exhibits a number of the features which we have been discussing:

In non-relativistic wave mechanics, the wave function $\psi(\mathbf{r},t)$ of a particle satisfies the *Schrödinger Wave Equation*

$$i\hbar\frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi + V \psi.$$

It is customary to normalize the wave equation by demanding that

$$\int \!\!\! \int \!\!\! \int_{\textbf{R}^3} \left| \psi(\mathbf{r},0) \right|^2\,dx\,dy\,dz = 1.$$

A simple calculation using the Schrödinger wave equation shows that

$$\frac{d}{dt} \int \!\!\! \int \!\!\! \int_{\textbf{R}^3} \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 0,$$

and hence

$$\int \!\!\! \int \!\!\! \int_{\textbf{R}^3} \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 1$$

for all times $t$. If we normalize the wave function in this way then, for any (measurable) subset $V$ of $\textbf{R}^3$ and time $t$,

$$\int \!\!\! \int \!\!\! \int_{V} \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz$$

represents the probability that the particle is to be found within the region $V$ at time $t$.

One would typeset this in LaTeX by typing

```
In non-relativistic wave mechanics, the wave function
$\psi(\mathbf{r},t)$ of a particle satisfies the
\emph{Schr\"{o}dinger Wave Equation}
\[ i\hbar\frac{\partial \psi}{\partial t}
  = \frac{-\hbar^2}{2m} \left(
    \frac{\partial^2}{\partial x^2}
    + \frac{\partial^2}{\partial y^2}
    + \frac{\partial^2}{\partial z^2}
  \right) \psi + V \psi.\]
It is customary to normalize the wave equation by
demanding that
\[ \int \!\!\! \int \!\!\! \int_{\textbf{R}^3}
      \left| \psi(\mathbf{r},0) \right|^2\,dx\,dy\,dz = 1.\]
A simple calculation using the Schr\"{o}dinger wave
equation shows that
\[ \frac{d}{dt} \int \!\!\! \int \!\!\! \int_{\textbf{R}^3}
      \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 0,\]
and hence
\[ \int \!\!\! \int \!\!\! \int_{\textbf{R}^3}
      \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 1\]
for all times~$t$. If we normalize the wave function in this
way then, for any (measurable) subset~$V$ of $\textbf{R}^3$
```

```
and time~$t$,
\[ \int \!\!\! \int \!\!\! \int_V
      \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz\]
represents the probability that the particle is to be found
within the region~$V$ at time~$t$.
```