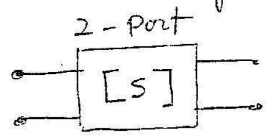
INEL 5306 - Microondas

$$\sum_{n=1}^{N} |V_n^-|^2 = \sum_{n=1}^{N} |V_n^+|^2$$

Para circuito de dos puertos:



$$|V_1^-|^2 + |V_2^-|^2 = |V_1^+|^2 + |V_2^+|^2$$

$$\left.\begin{aligned} V_1^- &= S_{11} V_1^+ + S_{12} V_2^+ \\ V_2^- &= S_{21} V_1^+ + S_{22} V_2^+ \end{aligned}\right\} \text{ Definición}$$

Entonces:

$$|S_{11} V_1^+ + S_{12} V_2^+|^2 + |S_{21} V_1^+ + S_{22} V_2^+|^2 = |V_1^+|^2 + |V_2^+|^2$$

Si hacemos $V_2^+ = 0$

$$|S_{11} V_1^+|^2 + |S_{21} V_1^+|^2 = |V_1^+|^2$$

De aquí; $|S_{11}|^2 + |S_{21}|^2 = 1$

$$S_{11} S_{11}^* + S_{21} S_{21}^* = 1$$

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1$$

La suma del producto punto de una columna por su conjugado tiene que ser $= \underline{1}$

se $|V_1^-|^2 + |V_2^-|^2 = |V_1^+|^2 + |V_2^+|^2$

Podemos decir también que,

$$(S_{11}V_1^+ + S_{12}V_2^+)(S_{11}V_1^+ + S_{12}V_2^+)^* + (S_{21}V_1^+ + S_{22}V_2^+)(S_{21}V_1^+ + S_{22}V_2^+)^*$$
$$= |V_1^+|^2 + |V_2^+|^2$$

Multiplicando,

$$\cancel{|S_{11}V_1^+|^2} + S_{11}V_1^+ S_{12}^* V_2^{+*} + S_{12}V_2^+ S_{11}^* V_1^{+*} + \cancel{|S_{12}V_2^+|^2}$$
$$+ \cancel{|S_{21}V_1^+|^2} + |S_{21}V_1^+ S_{22}^* V_2^{+*} + S_{22}V_2^+ S_{21}^* V_1^{+*} + \cancel{|S_{22}V_2^+|^2}$$
$$= |V_1^+|^2 + |V_2^+|^2 = \cancel{|S_{11}V_1^+|^2} + \cancel{|S_{21}V_1^+|^2} + \cancel{|S_{22}V_2^+|^2} + \cancel{|S_{12}V_2^+|^2}$$

Si decimos que $V_1^+ = V_2^+ = V^+$

$$|V^+|^2 (S_{11}S_{12}^* + S_{12}S_{11}^* + S_{21}S_{22}^* + S_{22}S_{21}^*) = 0$$

$$A = S_{11}S_{12}^* + S_{21}S_{22}^*$$

$$|V^+|^2 (A + A^*) = 0$$

Si $V_1^+ = V^+ \qquad V_2^+ = jV^+$

$$j|V^+|^2 (A^* - A) = 0$$

De aquí podemos inferir que $A = 0$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = 0 \qquad i \neq j$$

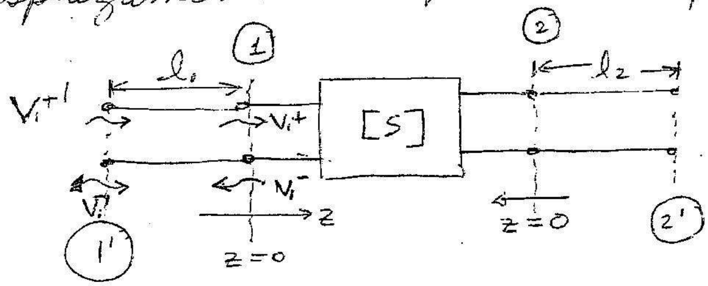
oducto Punto
de Una columna multiplicada por conjugado de otra $= 0$!!

$$\begin{bmatrix} 0.1\angle 90^\circ & 0.6\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.6\angle -45^\circ & 0 & 0 & 0.6\angle 45^\circ \\ 0.6\angle -45^\circ & 0 & 0 & 0.6\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.6\angle -45^\circ & 0 \end{bmatrix}$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$$(.1)^2 + (.6)^2 + (.6)^2 + 0 = 0.73 \neq 1$$

Desplazamiento del plano de referencia



– Matriz se midió con respecto a terminales 1 y 2. ahora se desea mover el plano de referencia a 1' y 2', ¿como se afecta [S]?

$$V_1^{+\prime} = V_1^+ e^{j\beta l_1} = V_1^+ e^{j\theta_1} \qquad V_2^{+\prime} = V_2^+ e^{j\theta_2}$$

$$V_1^{-\prime} = V_1^- e^{-j\theta_1} \qquad V_2^{-\prime} = V_2^- e^{-j\theta_2}$$

Yo se que (tomando referencia 1 y 2):

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

Puedo decir que:

$$V_1^- = V_1^{\prime -} e^{j\theta_1}; \quad V_2^- = V_2^{\prime -} e^{j\theta_2}; \quad V_1^+ = V_1^{\prime +} e^{-j\theta_1}; \quad V_2^+ = V_2^{\prime +} e^{-j\theta_2}$$

stituyendo;

$$V_1'^- e^{j\theta_1} = S_{11} V_1'^+ e^{-j\theta_1} + S_{12} V_2'^+ e^{-j\theta_1}$$

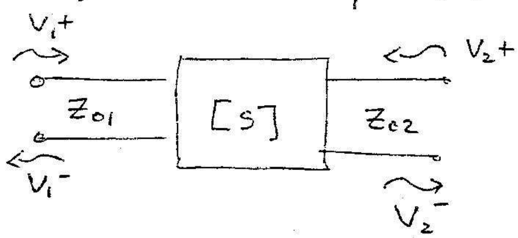
$$V_2'^- e^{j\theta_2} = S_{21} V_1'^+ e^{-j\theta_1} + S_{22} V_2'^+ e^{-j\theta_2}$$

$$V_1'^- = S_{11} V_1'^+ e^{-j2\theta_1} + S_{12} V_2'^+ e^{-j(\theta_1+\theta_2)}$$

$$V_2'^- = S_{21} V_1'^+ e^{-j(\theta_1+\theta_2)} + S_{22} V_2'^+ e^{-j2\theta_2}$$

$$\begin{bmatrix} V_1'^- \\ V_2'^- \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\theta_1} & S_{12} e^{-j(\theta_1+\theta_2)} \\ S_{21} e^{-j(\theta_1+\theta_2)} & S_{22} e^{-j2\theta_2} \end{bmatrix} \begin{bmatrix} V_1'^+ \\ V_2'^+ \end{bmatrix}$$

Matriz - S, usando componentes normalizados



$$a_1 = \frac{V_1^+}{\sqrt{Z_{01}}} \qquad b_1 = \frac{V_1^-}{\sqrt{Z_{01}}}$$

$$a_2 = \frac{V_2^+}{\sqrt{Z_{02}}} \qquad b_2 = \frac{V_2^-}{\sqrt{Z_{02}}}$$

$$V_1 = V_1^+ + V_1^- = \sqrt{Z_{01}}\,(a_1 + b_1)$$

$$I_1 = \frac{V_1^-}{Z_{01}} - \frac{V_1^-}{Z_{01}} = \frac{1}{\sqrt{Z_{01}}}\,(a_1 - b_1)$$

Potencia:

$$P_{ave} = \frac{1}{2} Re\{V I^*\}$$

$$= \frac{1}{2} Re\{|a_1|^2 - |b_1|^2 + (b_1 a_1^* - b_1^* a_1)\}$$

$$= \frac{1}{2}(|a_1|^2 - |b_1|^2)$$

En general; $S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k = 0} \quad k \neq j$

$$S_{ij} = \left.\frac{V_i^- \sqrt{Z_{0j}}}{V_j^+ \sqrt{Z_{0i}}}\right|_{V_k^+ = 0}$$

$$V_1 = \sqrt{Z_{01}}\,(a_1 + b_1)$$

$$I_1 = \frac{1}{\sqrt{Z_{01}}}(a_1 - b_1) \Longrightarrow b_1 = a_1 - \sqrt{Z_{01}}\, I_1$$

$$\therefore V_1 = \sqrt{Z_{01}}\,(a_1 + a_1 - \sqrt{Z_{01}}\, I_1)$$

$$V_1 = 2\sqrt{Z_{01}}\, a_1 - Z_{01} I_1$$

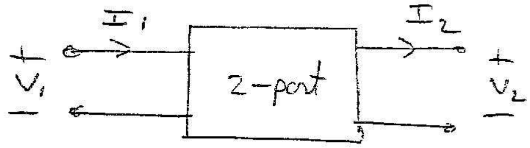
$$\Longrightarrow a_1 = \frac{V_1 + Z_{01} I_1}{2\sqrt{Z_{01}}}$$

$$b_1 = \frac{V_1 - Z_{01} I_1}{2\sqrt{Z_{01}}}$$

$$b_2 = \frac{V_2 - Z_{02} I_2}{2\sqrt{Z_{02}}}$$

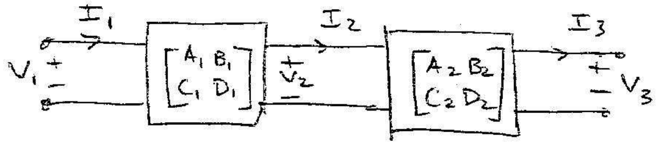
$$a_2 = \frac{V_2 + Z_{02} I_2}{2\sqrt{Z_{02}}}$$

Matriz de Transmisión (ABCD)



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = AV_2 + BI_2 \qquad I_1 = CV_2 + DI_2$$

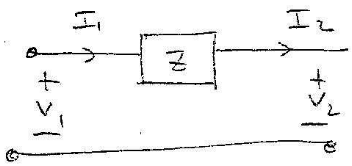


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

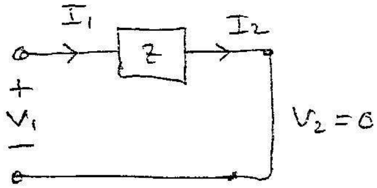
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}}_{\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_2B_2 + D_1D_2 \end{bmatrix}$$



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

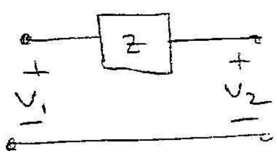
$$I_2 = I_1 = 0 \implies V_1 = V_2 \implies \underline{A = 1}$$



$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$V_1 = Z I_2$$

$$B = \frac{V_1}{I_2} = \underline{Z}$$



$$I_1 = I_2 = 0 \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

Since $I_1 = 0$ $\qquad C = 0$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} \qquad \text{como } I_1 = I_2 \implies \underline{D = 1}$$

$$[ABCD] = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$