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1 Problem Formulation

The use of Multiple Element Antennas at both ends of a link, along with appropriate signaling techniques, has showed that improve the capacity and reliability of the wireless link. The Multiple Input Multiple Output (MIMO) wireless channel behavior is very important to the performance of MIMO systems in terms of the capacity and the signaling method to be used.

2 Theoretical Framework

MIMO Radar Systems

The measurement problem of radar is to design a radar waveform to be broadcast by a radar or sonar, so as to maximize the receiver response to the signal which has interacted with an object. In radar, a known signal is sent out and the reflected signal from the object (which is a function of the distance to the object, the relative speed of the object, and the broadcast frequency of the radar), can be examined at the radar's receiver for the common elements of the out-going signal in the return signal.

When this problem is extended, integrating in the model multiple transmitters, fixed or mobile, and multiple receivers, then we have a system where multiple targets can are being "illuminated" at the same time by multiple transmitters and its respective echoes can be read by multiple receivers. This type of channels are known as **scattering channels** and are studied in MIMO systems environments. **Figure 1** shows a MIMO system environment with a scattering channel. Formally, the form of the return signal for a narrow band signal $s(t)$ is $s(t-\tau)$, where τ is the delay between the broadcast of the signal and the return time that the signal is detected at the receiver.

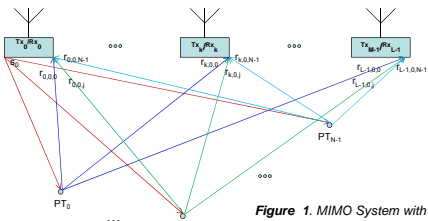


Figure 1. MIMO System with Scattering Channel

Scattering Channel Characterization:

If we consider each signal S_n transmitted by a transmitter Tx_M acting on each point target P_N and received by a receiver Rx_L as $r_{L,M,N}$ we will obtain the relationship:

$$r_{l,m,n} = h_{l,m} * S_n \quad (1)$$

Figure 2. Block Diagram of Local Channel Characterization

The set of $h_{l,m}$ impulse response signals can be represented by a H matrix given as:

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,m} \\ h_{2,1} & h_{2,2} & \dots & h_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ h_{l,1} & h_{l,2} & \dots & h_{l,m} \end{bmatrix}$$

This matrix H characterizes the scattering channel in the MIMO system.

Ambiguity Function as Characterization Function:

The pioneer in speaking about the ambiguity function was J. Ville, in the year 1948 [1]; however, the first individual working formally with the ambiguity function was P. M. Woodward [2]. Their approach was made using continuous signal analysis it was framed in the context of probability and information theory.

A discrete signal formulation of the ambiguity function is given below, in Equation 2:

$$A(F,G)[a,b] = \sum_{c=0}^{M-1} F(c)G^* \left((c+a)_M \right) e^{-j\frac{2\pi}{M}(cb)} \quad (2)$$

Where a and b belong to Z and $\langle c+a \rangle_M = \text{remainder} \{ (c+a)/M \}$. F and G are two arbitrary one-dimensional discrete complex functions that belong to $L^2(Z)$.

The result is a two-dimensional complex array that contains range and Doppler information about a point target P_N .

3 Parallel Formulation

Parallel Approach:

The computational complexity of the formulation presented equation (2) can be excessive when considering signals F and G with a great number of samples. Developing algorithms for the efficient computation of the ambiguity function is an important aspect of this work.

pMatlab can be considered a tool for parallel algorithm development based on PGAS (Partitioned Global Address Space) [4]. This category exploits the mechanisms of creation of global arrays to be distributed in more than one processor. Developing algorithms for the ambiguity function may take advantage of this PGAS paradigm.

Algorithmic Description:

A pseudo algorithmic formulation is presented below for the parallel implementation of the ambiguity function computation.

Step	Description	MATLAB/pMatlab Code
1	Assign a complex sequence to signal F	<code>F = rand(1,1600) + 1i*rand(1,1600);</code>
2	Assign a complex sequence to signal G	<code>G = rand(1,1800) + 1i*rand(1,1800);</code>
3	Assign the length of F to variable nF	<code>nF = size(F,2);</code>
4	Assign the length of G to variable nG	<code>nG = size(G,2);</code>
5	Calculate G Conjugate (Destructive Operation for avoid data redundancy)	<code>G = conj(G);</code>
6	Calculate M Value (for zero-padding purposes)	<code>M = 2 ^ nextpow2(max(nF,nG));</code>
7	Zero-Padding of F thru M length	<code>F = [F zeros(1,M - nF)];</code>
8	Zero-Padding of G thru M length (Now F and G have M length, where M is a power of 2)	<code>G = [G zeros(1,M - nG)];</code>
9	Enable parallel mode	<code>PARALLEL = 1;</code>
10	Default Map for serial mode	<code>Wmap = 1;</code>
11	If parallel mode is enabled , overwrite default map for enable Distributed Array mode	<code>if (PARALLEL) Wmap = map([1 Np], {}, 0:Np-1);</code> <code>End</code>
12	Create a Distributed Matrix Amb, distributed in column blocks on Np Processors.	<code>Amb = zeros(M,M,Wmap);</code>
13	Assign the local columns of Amb to Ambloc	<code>Ambloc = local(Amb);</code>
14	Recall of Column Global Indices	<code>jglobal = global_ind(Amb,2);</code>
15	Fill local correlation matrix	<code>for j=1: numel(jglobal) Ambloc(:,j) = F.*[G(jglobal(j):... M) G(1:jglobal(j)-1)];</code> <code>End</code>
16	Calculate the FFT on columns of Correlation Matrix Ambloc (Ambiguity Function Local)	<code>Ambloc = fft(Ambloc);</code>
17	Join partial results in Distributed Matrix Amb	<code>Amb = put_local(Amb,Ambloc);</code>
18	Assign the pMatlab Distributed Array Values to Matlab Regular Array	<code>A = agg(Amb);</code>

4 Times Parallel Computing

Processors	Computation Time (Average)	Launch+Comm Time
1	0.049813	0.024517
2	0.037838	15.7748
4	0.028979	17.8491
6	0.028209	21.2959
8	0.027989	23.6823

(a) 1024 Samples

Processors	Computation Time (Average)	Launch+Comm Time
1	0.31766	0.070414
2	0.17826	16.0884
4	0.10975	19.2104
6	0.085887	19.7569
8	0.072661	22.5374

(b) 2048 Samples

Processors	Computation Time (Average)	Launch+Comm Time
1	1.1349	0.24703
2	0.52469	18.3104
4	0.30457	19.8664
6	0.22913	23.3296
8	0.18436	24.9367

(c) 4096 Samples

Table 2. Times for 1024, 2048 y 4096 Samples

Running Times:

The tables on the left contain execution times for the implementation of the ambiguity function on pMatlab using 1,2,4,6, and 8 processors. This the complex signals are of lengths 1024, 2048, and 4096 samples. All times are in seconds.

The computer used for this experiment has an Intel® Core™ 2 Duo CPU E8400 running at 3.00GHz with 3.25GBytes of RAM. pMatlab version was Parallel MATLAB Toolbox v1.0.1.

5 References

- [1] J. Ville, "Theorie et application de la notion de signal analytique," *Cables et Transmission*, vol. 2, no. 1, pp. 61–74, 1948.
- [2] J. Gray, "An Interpretation of Woodward's Ambiguity Function and Its Generalization". *Electromagnetic and Sensor Systems Department Naval Surface Warfare Center Dahlgren. Radar Conference, 2010 IEEE. Code Q-23. Digital Object Identifier: 10.1109/RADAR.2010.5494499.*
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- [5] Wei-qiang Zhang, Ran Tao, Yong-feng Ma, "Fast computation of the ambiguity function," *Dept. of Electron. Eng., Beijing Inst. of Technol., China. Signal Processing, 2004. Proceedings. ICSP '04. 2004 7th International Conference on 31 Aug.-4 Sept. 2004. Digital Object Identifier : 10.1109/ICOSP.2004.1442195.*

