
Chapter 7

Frequency Response and Bode Plots

This chapter discusses frequency response in terms of both amplitude and phase. This topic will enable us to determine which frequencies are dominant and which frequencies are virtually suppressed. The design of electric filters is based on the study of the frequency response. We will also discuss the Bode method of linear system analysis using two separate plots; one for the magnitude of the transfer function, and the other for the phase, both versus frequency. These plots reveal valuable information about the frequency response behavior.

Note: Throughout this text, the common (base 10) logarithm of a number x will be denoted as $\log(x)$ while its natural (base e) logarithm will be denoted as $\ln(x)$. However, we should remember that in MATLAB the $\log(x)$ function displays the natural logarithm, and the common (base 10) logarithm is defined as $\log_{10}(x)$.

7.1 Decibels

The ratio of any two values of the same quantity (power, voltage or current) can be expressed in *decibels* (dB). For instance, we say that an amplifier has 10 dB power gain or a transmission line has a power loss of 7 dB (or gain -7 dB). If the gain (or loss) is 0 dB , the output is equal to the input. We should remember that a negative voltage or current gain A_V or A_I indicates that there is a 180° phase difference between the input and the output waveforms. For instance, if an amplifier has a gain of -100 (dimensionless number), it means that the output is 180° out-of-phase with the input. For this reason we use absolute values of power, voltage and current when these are expressed in dB terms to avoid misinterpretation of gain or loss.

By definition,

$$dB = 10 \log \left| \frac{P_{out}}{P_{in}} \right| \quad (7.1)$$

Therefore,

10 dB represents a power ratio of 10

$10n\text{ dB}$ represents a power ratio of 10^n

20 dB represents a power ratio of 100

30 dB represents a power ratio of $1,000$

60 dB represents a power ratio of $1,000,000$

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Also,

1 dB represents a power ratio of approximately 1.25

3 dB represents a power ratio of approximately 2

7 dB represents a power ratio of approximately 5

From these, we can estimate other values. For instance, $4 \text{ dB} = 3 \text{ dB} + 1 \text{ dB}$ which is equivalent to a power ratio of approximately $2 \times 1.25 = 2.5$. Likewise, $27 \text{ dB} = 20 \text{ dB} + 7 \text{ dB}$ and this is equivalent to a power ratio of approximately $100 \times 5 = 500$.

Since $y = \log x^2 = 2 \log x$ and $P = V^2/R = I^2R$, if we let $R = 1$ the dB values for the voltage and current ratios become:

$$dB_v = 10 \log \left| \frac{V_{out}}{V_{in}} \right|^2 = 20 \log \left| \frac{V_{out}}{V_{in}} \right| \quad (7.2)$$

and

$$dB_i = 10 \log \left| \frac{I_{out}}{I_{in}} \right|^2 = 20 \log \left| \frac{I_{out}}{I_{in}} \right| \quad (7.3)$$

Example 7.1

Compute the gain in dB_W for the amplifier shown in Figure 7.1.

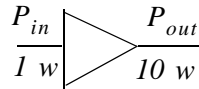


Figure 7.1. Amplifier for Example 7.1

Solution:

$$dB_W = 10 \log \frac{P_{out}}{P_{in}} = 10 \log \frac{10}{1} = 10 \log 10 = 10 \times 1 = 10 \text{ dB}_W$$

Example 7.2

Compute the gain in dB_V for the amplifier shown in Figure 7.2 given that $\log 2 = 0.3$.

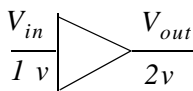


Figure 7.2. Amplifier for Example 7.2.

Solution:

$$dB_V = 20 \log \frac{V_{out}}{V_{in}} = 20 \log \frac{2}{1} = 20 \log 0.3 = 20 \times 0.3 = 6 \text{ dB}_V$$

7.2 Bandwidth and Frequency Response

Electric and electronic circuits, such as filters and amplifiers, exhibit a band of frequencies over which the output remains nearly constant. Consider, for example, the magnitude of the output voltage $|V_{out}|$ of an electric or electronic circuit as a function of radian frequency ω as shown in Figure 7.3.

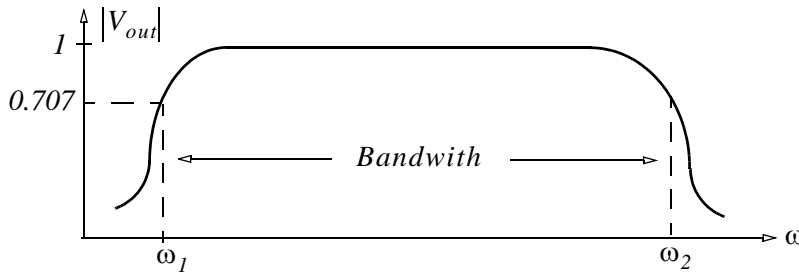


Figure 7.3. Definition of the bandwidth.

As shown in Figure 7.3, the *bandwidth* is $BW = \omega_2 - \omega_1$ where ω_1 and ω_2 are the *lower* and *upper cutoff frequencies* respectively. At these frequencies, $|V_{out}| = \sqrt{2}/2 = 0.707$ and these two points are known as the *3 dB down* or *half-power points*. They derive their name from the fact that since power $p = v^2/R = i^2R$, for $R = 1$ and for $v = 0.707|V_{out}|$ or $i = 0.707|I_{out}|$ the power is $1/2$, that is, it is “halved”. Alternately, we can define the bandwidth as the frequency band between half-power points.

Most amplifiers are used with a feedback path which returns (feeds) some or all its output to the input as shown in Figure 7.4.

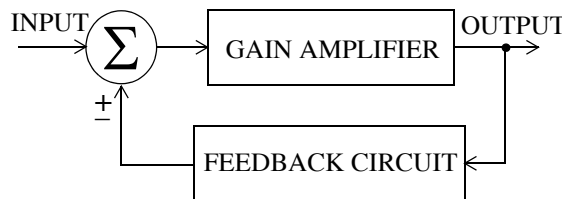


Figure 7.4. Amplifier with partial output feedback

Figure 7.5 shows an amplifier where the entire output is fed back to the input.

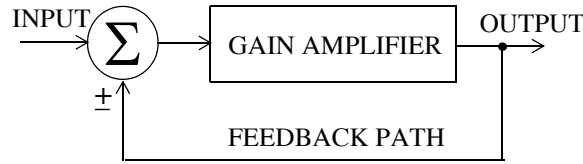


Figure 7.5. Amplifier with entire output feedback

The symbol Σ (Greek capital letter sigma) inside the circle indicates the summing point where the output signal, or portion of it, is combined with the input signal. This summing point may be also indicated with a large plus (+) symbol inside the circle. The positive (+) sign below the summing point implies *positive feedback* which means that the output, or portion of it, is added to the input. On the other hand, the negative (–) sign implies *negative feedback* which means that the output, or portion of it, is subtracted from the input. Practically, all amplifiers are used with negative feedback since positive feedback causes circuit instability.

7.3 Octave and Decade

Let us consider two frequencies ω_1 and ω_2 defining the frequency interval $\omega_2 - \omega_1$, and let

$$\omega_2 - \omega_1 = \log_{10} \omega_2 - \log_{10} \omega_1 = \log_{10} \frac{\omega_2}{\omega_1} \quad (7.4)$$

If these frequencies are such that $\omega_2 = 2\omega_1$, we say that these frequencies are separated by one *octave* and if $\omega_2 = 10\omega_1$, they are separated by one *decade*.

Let us now consider a transfer function $G(s)$ whose magnitude is evaluated at $s = |j\omega|$, that is,

$$|G(s)| = \left. \frac{C}{s^k} \right|_{s=|j\omega|} = |G(\omega)| = \frac{C}{\omega^k} \quad (7.5)$$

Taking the log of both sides of (7.5) and multiplying by 20, we get

$$20 \log_{10} |G(\omega)| = 20 \log_{10} C - 20 \log_{10} \omega^k = -20k \log_{10} \omega + 20 \log_{10} C$$

or

$$|G(\omega)|_{dB} = -20k \log_{10} \omega + 20 \log_{10} C \quad (7.6)$$

Relation (7.6) is an equation of a straight line in a semilog plot with abscissa $\log_{10} \omega$ where

$$\text{slope} = -20k \frac{dB}{\text{decade}}$$

and *intercept* = C dB shown in Figure 7.6.

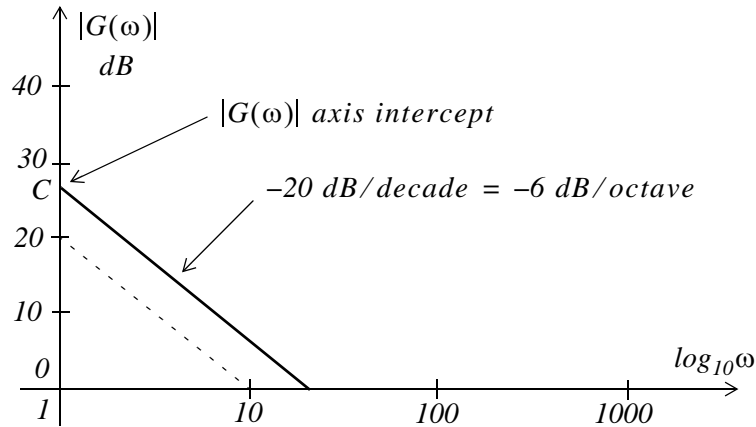


Figure 7.6. Straight line with slope $-20 \text{ dB/decade} = -6 \text{ dB/octave}$

With these concepts in mind, we can now proceed to discuss *Bode Plots* and *Asymptotic Approximations*.

7.4 Bode Plot Scales and Asymptotic Approximations

Bode plots are magnitude and phase plots where the abscissa (frequency axis) is a logarithmic (base 10) scale, and the radian frequency ω is equally spaced between powers of 10 such as 10^{-1} , 10^0 , 10^1 , 10^2 and so on.

The ordinate (*dB* axis) of the magnitude plot has a scale in *dB* units, and the ordinate of the phase plot has a scale in degrees as shown in Figure 7.7.

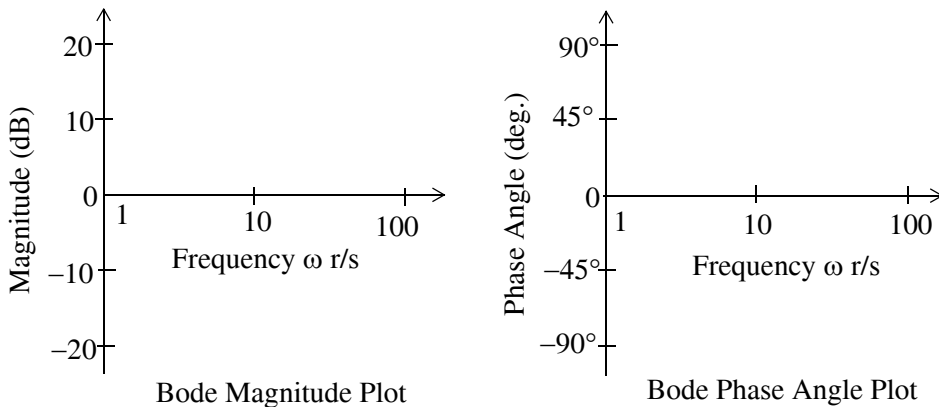


Figure 7.7. Magnitude and phase plots

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It is convenient to express the magnitude in dB so that a transfer function $G(s)$, composed of products of terms can be computed by the sum of the dB magnitudes of the individual terms. For example,

$$\frac{20 \cdot \left(1 + \frac{j\omega}{100}\right)}{1 + j\omega} = 20 \text{ dB} + \left(1 + \frac{j\omega}{100}\right) \text{ dB} + \frac{1}{1 + j\omega} \text{ dB}$$

and the Bode plots then can be approximated by straight lines called *asymptotes*.

7.5 Construction of Bode Plots when the Zeros and Poles are Real

Let us consider the transfer function

$$G(s) = \frac{A \cdot (s + z_1) \cdot (s + z_2) \cdot \dots \cdot (s + z_m)}{s \cdot (s + p_1) \cdot (s + p_2) \cdot (s + p_3) \cdot (s + p_n)} \quad (7.7)$$

where A is a real constant, and the zeros z_i and poles p_i are real numbers. We will consider complex zeros and poles in the next section. Letting $s = j\omega$ in (7.7) we get

$$G(j\omega) = \frac{A \cdot (j\omega + z_1) \cdot (j\omega + z_2) \cdot \dots \cdot (j\omega + z_m)}{j\omega \cdot (j\omega + p_1) \cdot (j\omega + p_2) \cdot (j\omega + p_3) \cdot (j\omega + p_n)} \quad (7.8)$$

Next, we multiply and divide each numerator factor $j\omega + z_i$ by z_i and each denominator factor $j\omega + p_i$ by p_i and we get:

$$G(j\omega) = \frac{A \cdot z_1 \left(\frac{j\omega}{z_1} + 1\right) \cdot z_2 \left(\frac{j\omega}{z_2} + 1\right) \cdot \dots \cdot z_m \left(\frac{j\omega}{z_m} + 1\right)}{j\omega \cdot p_1 \left(\frac{j\omega}{p_1} + 1\right) \cdot p_2 \left(\frac{j\omega}{p_2} + 1\right) \cdot \dots \cdot p_n \left(\frac{j\omega}{p_n} + 1\right)} \quad (7.9)$$

Letting

$$K = \frac{A \cdot z_1 \cdot z_2 \cdot \dots \cdot z_m}{p_1 \cdot p_2 \cdot \dots \cdot p_n} = A \frac{\prod_{i=1}^m z_i}{\prod_{i=1}^n p_i} \quad (7.10)$$

we can express (7.9) in dB magnitude and phase form,

Construction of Bode Plots when the Zeros and Poles are Real

$$|G(\omega)| = 20\log|K| + 20\log\left(\frac{j\omega}{z_1} + 1\right) + 20\log\left(\frac{j\omega}{z_2} + 1\right) + \dots + 20\log\left(\frac{j\omega}{z_m} + 1\right) \quad (7.11)$$

$$- 20\log|j\omega| - 20\log\left(\frac{j\omega}{p_1} + 1\right) - 20\log\left(\frac{j\omega}{p_2} + 1\right) - \dots - 20\log\left(\frac{j\omega}{p_n} + 1\right)$$

$$\angle G(\omega) = \angle K + \angle\left(\frac{j\omega}{z_1} + 1\right) + \angle\left(\frac{j\omega}{z_2} + 1\right) + \dots + \angle\left(\frac{j\omega}{z_m} + 1\right) \quad (7.12)$$

$$- \angle j\omega - \angle\left(\frac{j\omega}{p_1} + 1\right) - \angle\left(\frac{j\omega}{p_2} + 1\right) - \dots - \angle\left(\frac{j\omega}{p_n} + 1\right)$$

The constant K can be positive or negative. Its magnitude is $|K|$ and its phase angle is 0° if $K > 0$, and -180° if $K < 0$. The magnitude and phase plots for the constant K are shown in Figure 7.8.

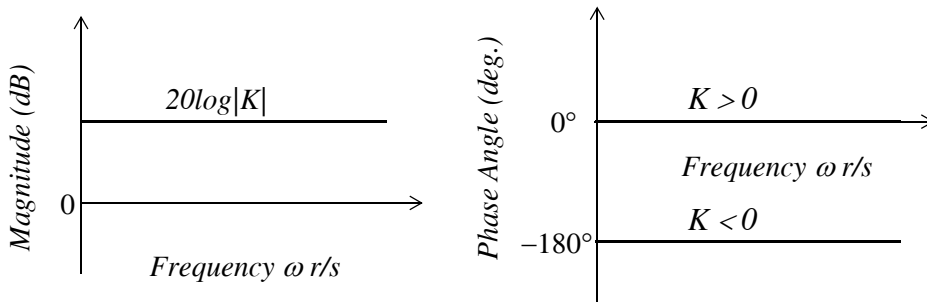


Figure 7.8. Magnitude and phase plots for the constant K

For a zero of order n , that is, $(j\omega)^n$ at the origin, the Bode plots for the magnitude and phase are as shown in Figures 7.9 and 7.10 respectively.

For a pole of order n , that is, $1/(j\omega)^n = (j\omega)^{-n}$ at the origin, the Bode plots are as shown in Figures 7.11 and 7.12 respectively.

Next, we consider the term $G(j\omega) = (a + j\omega)^n$.

The magnitude of this term is

$$|G(j\omega)| = \sqrt{(a^2 + \omega^2)^n} = (a^2 + \omega^2)^{n/2} \quad (7.13)$$

and taking the log of both sides and multiplying by 20 we get

$$20\log|G(j\omega)| = 10n\log(a^2 + \omega^2) \quad (7.14)$$

It is convenient to normalize (7.14) by letting

$$u \equiv \omega/a \quad (7.15)$$

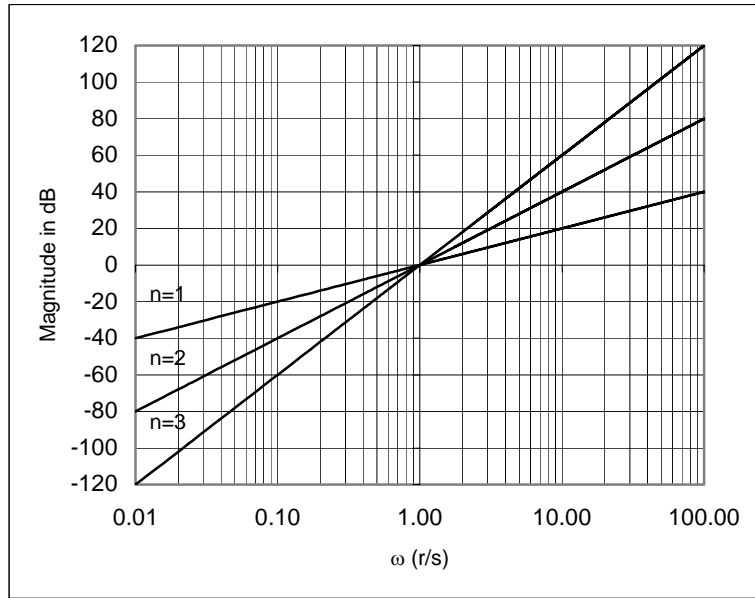


Figure 7.9. Magnitude for zeros of Order n at the origin

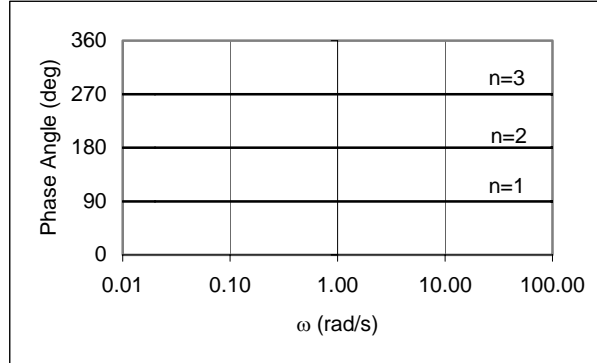


Figure 7.10. Phase for zeros of Order n at the origin

Then, (7.14) becomes

$$\begin{aligned}
 20\log|G(j\omega)| &= 10n\log\left(a^2 \cdot \frac{a^2 + \omega^2}{a^2}\right) = 10n\log a^2 + 10n\log(1 + u^2) \\
 &= 10n\log(1 + u^2) + 20n\log a
 \end{aligned}
 \tag{7.16}$$

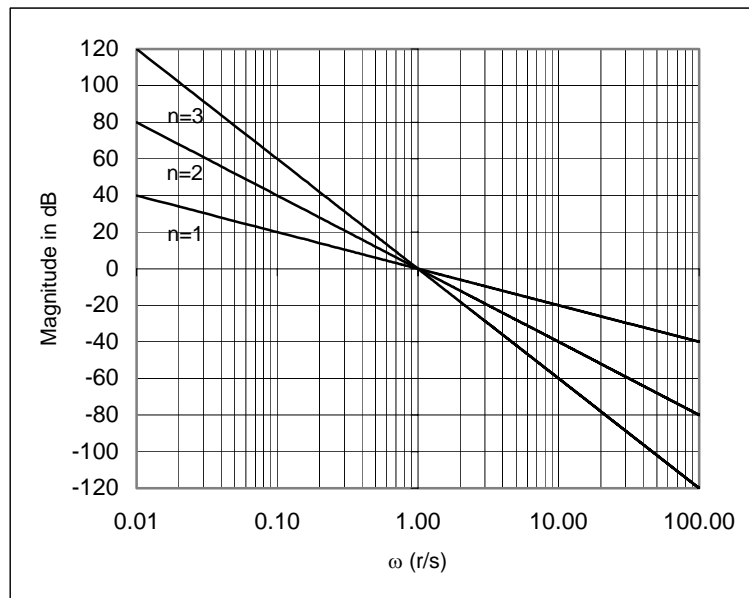


Figure 7.11. Magnitude for poles of Order n at the origin

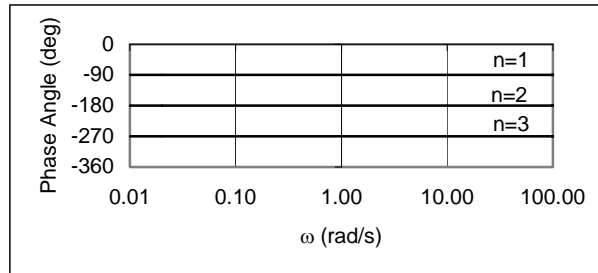


Figure 7.12. Phase for poles of Order n at the origin

For $u \ll 1$ the first term of (7.16) becomes $10n \log 1 = 0 \text{ dB}$. For $u \gg 1$, this term becomes approximately $10n \log u^2 = 20n \log u$ and this has the same form as $G(j\omega) = (j\omega)^n$ which is shown in Figure 7.9 for $n = 1, n = 2$, and $n = 3$.

The frequency at which two asymptotes intersect each other forming a corner is referred to as the *corner frequency*. Thus, the two lines defined by the first term of (7.16), one for $u \ll 1$ and the other for $u \gg 1$ intersect at the corner frequency $u = 1$.

The second term of (7.16) represents the ordinate axis intercept defined by this straight line.

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The phase response for the term $G(j\omega) = (a + j\omega)^n$ is found as follows:

We let

$$u \equiv \omega/a \quad (7.17)$$

and

$$\phi(u) = \tan^{-1} u \quad (7.18)$$

Then,

$$(a + j\omega)^n = a^n(1 + ju)^n = a^n(\sqrt{1 + u^2} \angle \tan^{-1} u)^n = a^n(1 + u^2)^{n/2} e^{jn\phi(u)} \quad (7.19)$$

Figure 7.13 shows plots of the magnitude of (7.16) for $a = 10$, $n = 1$, $n = 2$, and $n = 3$.

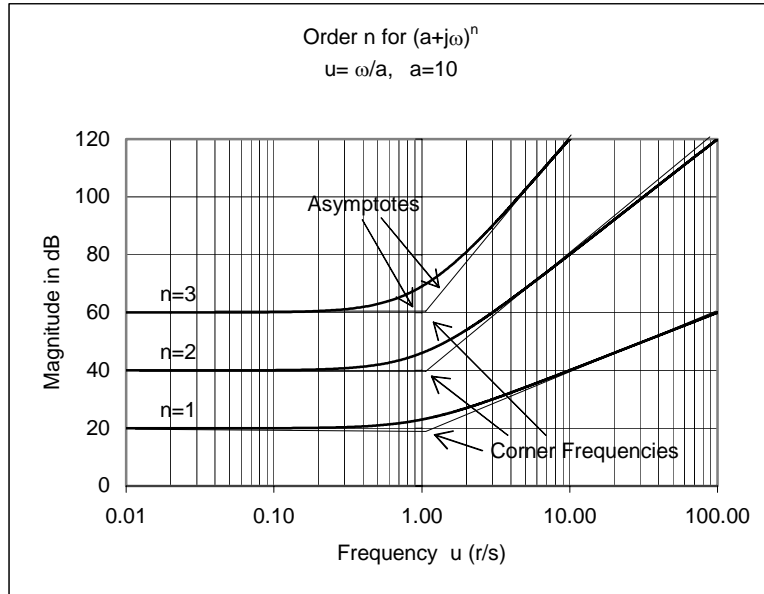


Figure 7.13. Magnitude for zeros of Order n for $(a + j\omega)^n$

As shown in Figure 7.13, a quick sketch can be obtained by drawing the straight line asymptotes given by $10\log 1 = 0$ and $10n\log u^2$ for $u \ll 1$ and $u \gg 1$ respectively.

The phase angle of (7.19) is $n\phi(u)$. Then, with (7.18) and letting

$$n\phi(u) = \theta(u) = n \tan^{-1} u \quad (7.20)$$

we get

$$\lim_{u \rightarrow 0} \theta(u) = \lim_{u \rightarrow 0} n \tan^{-1} u = 0 \quad (7.21)$$

and

$$\lim_{u \rightarrow \infty} \theta(u) = \lim_{u \rightarrow \infty} n \tan^{-1} u = \frac{n\pi}{2} \quad (7.22)$$

At the corner frequency $u = a$ we get $u = 1$ and with (7.20)

$$\theta(1) = n \tan^{-1} 1 = \frac{n\pi}{4} \quad (7.23)$$

Figure 7.14 shows the phase angle plot for (7.19).

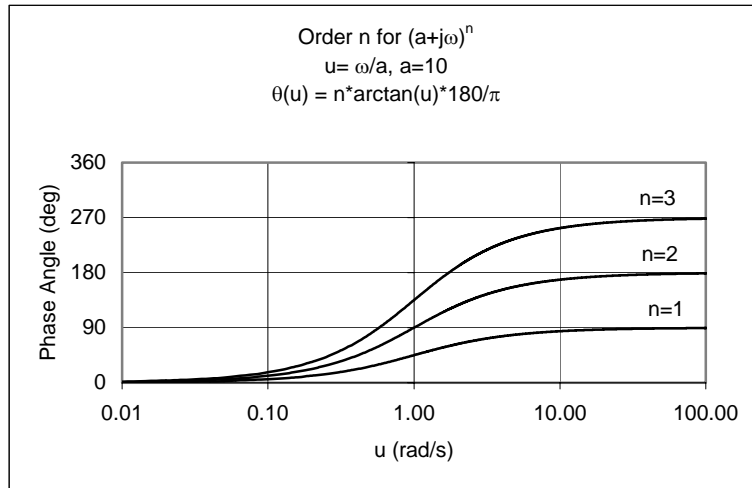


Figure 7.14. Phase for zeros of Order n for $(a + j\omega)^n$

The magnitude and phase plots for $G(j\omega) = 1/(a + j\omega)^n$ are similar to those of $G(j\omega) = (a + j\omega)^n$ except for a minus sign. In this case (7.16) becomes

$$-20 \log |G(j\omega)| = -10n \log(1 + u^2) - 20n \log a \quad (7.24)$$

and (7.20) becomes

$$\theta(u) = -n \tan^{-1} u \quad (7.25)$$

The plots for (7.24) and (7.25) are shown in Figures 7.15 and 7.16 respectively.

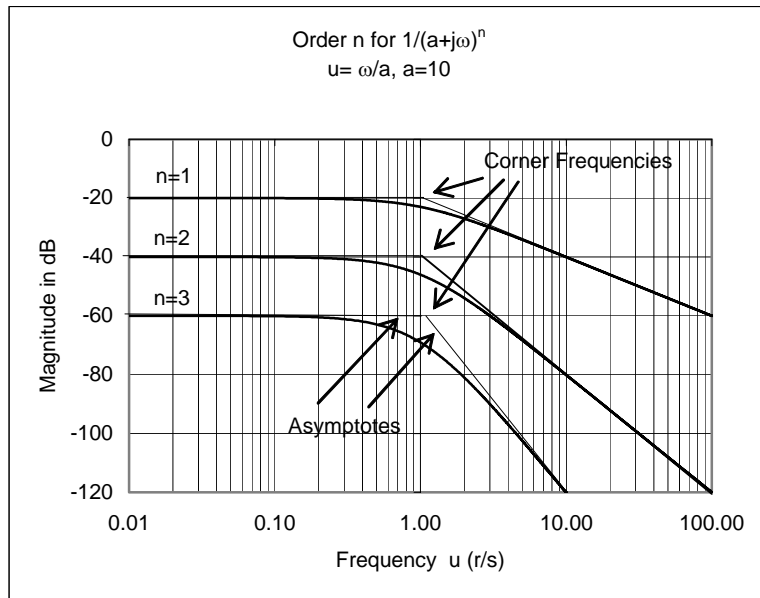


Figure 7.15. Magnitude for poles of Order n for $1/(a + j\omega)^n$

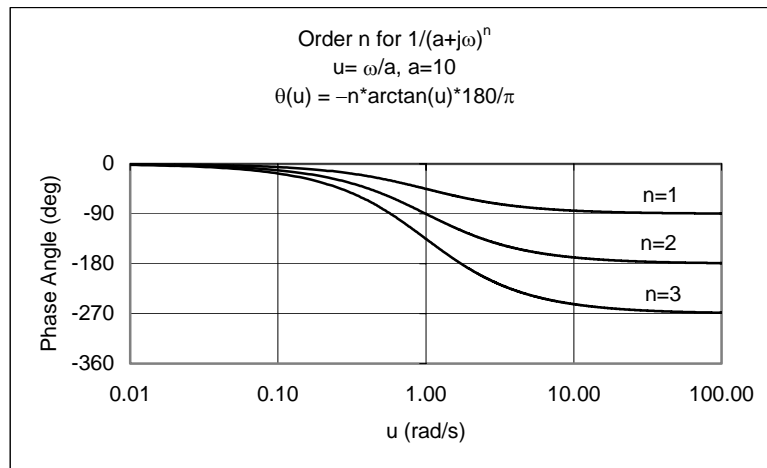


Figure 7.16. Phase for poles of Order n for $1/(a + j\omega)^n$

7.6 Construction of Bode Plots when the Zeros and Poles are Complex

The final type of terms appearing in the transfer function $G(s)$ are quadratic term of the form $as^2 + bs + c$ whose roots are complex conjugates. In this case, we express the complex conjugate roots as

$$\begin{aligned}(s + \alpha - j\beta)(s + \alpha + j\beta) &= (s + \alpha)^2 + \beta^2 \\ &= s^2 + 2\alpha s + \alpha^2 + \beta^2\end{aligned}\tag{7.26}$$

and letting

$$\alpha = \zeta\omega_n\tag{7.27}$$

and

$$\alpha^2 + \beta^2 = \omega_n^2\tag{7.28}$$

by substitution into (7.26) we get

$$s^2 + 2\alpha s + \alpha^2 + \beta^2 = s^2 + 2\zeta\omega_n s + \omega_n^2\tag{7.29}$$

Next, we let

$$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2\tag{7.30}$$

Then,

$$\begin{aligned}G(j\omega) &= (j\omega)^2 + j2\omega_n\omega + \omega_n^2 \\ &= (\omega_n^2 - \omega^2) + j2\omega_n\omega\end{aligned}\tag{7.31}$$

The magnitude of (7.31) is

$$|G(j\omega)| = \sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}\tag{7.32}$$

and taking the log of both sides and multiplying by 20 we get

$$20\log|G(j\omega)| = 10\log[(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2]\tag{7.33}$$

As in the previous section, it is convenient to normalize (7.33) by dividing by ω_n^4 to yield a function of the normalized frequency variable u such that

$$u \equiv \omega/\omega_n\tag{7.34}$$

Then, (7.33) is expressed as

$$20\log|G(ju)| = 10\log[(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2]$$

or

$$\begin{aligned}
 20\log|G(ju)| &= 10\log\left[\omega_n^4\left(\frac{\omega_n^2 - \omega^2}{\omega_n^2}\right)^2 + 4\zeta^2\omega_n^4\frac{\omega^2}{\omega_n^2}\right] = 10\log\left[\omega_n^4\left(\frac{\omega_n^2 - \omega^2}{\omega_n^2}\right)^2 + 4\zeta^2\omega_n^4\frac{\omega^2}{\omega_n^2}\right] \\
 &= 10\log[\omega_n^4\{(1 - u^2)^2 + 4\zeta^2 u^2\}] = 10\log\omega_n^4 + 10\log[(1 - u^2)^2 + 4\zeta^2 u^2]
 \end{aligned} \tag{7.35}$$

The first term in (7.35) is a constant which represents the ordinate axis intercept defined by this straight line. For the second term, if $u^2 \ll 1$, this term reduces to approximately $10\log 1 = 0 \text{ dB}$ and if $u^2 \gg 1$, this term reduces to approximately $10\log u^4$ and this can be plotted as a straight line increasing at 40 dB/decade . Using these two straight lines as asymptotes for the magnitude curve we see that the asymptotes intersect at the corner frequency $u = 1$. The exact shape of the curve depends on the value of ζ which is called the *damping coefficient*.

A plot of (7.35) for $\zeta = 0.2$, $\zeta = 0.4$, and $\zeta = 0.707$ is shown in Figure 7.17.

The phase shift associated with $(\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega$ is also simplified by the substitution $u \equiv \omega/\omega_n$ and thus

$$\theta(u) = \tan^{-1}\left(\frac{2\zeta u}{1 - u^2}\right) \tag{7.36}$$

The two asymptotic relations of (7.36) are

$$\lim_{u \rightarrow 0} \theta(u) = \lim_{u \rightarrow 0} \tan^{-1}\left(\frac{2\zeta u}{1 - u^2}\right) = 0 \tag{7.37}$$

and

$$\lim_{u \rightarrow \infty} \theta(u) = \lim_{u \rightarrow \infty} \tan^{-1}\left(\frac{2\zeta u}{1 - u^2}\right) = \pi \tag{7.38}$$

At the corner frequency $\omega = \omega_n$, $u = 1$ and

$$\theta(1) = \lim_{u \rightarrow 1} \tan^{-1}\left(\frac{2\zeta u}{1 - u^2}\right) = \frac{\pi}{2} \tag{7.39}$$

A plot of the phase for $\zeta = 0.2$, $\zeta = 0.4$, and $\zeta = 0.707$ is shown in Figure 7.18.

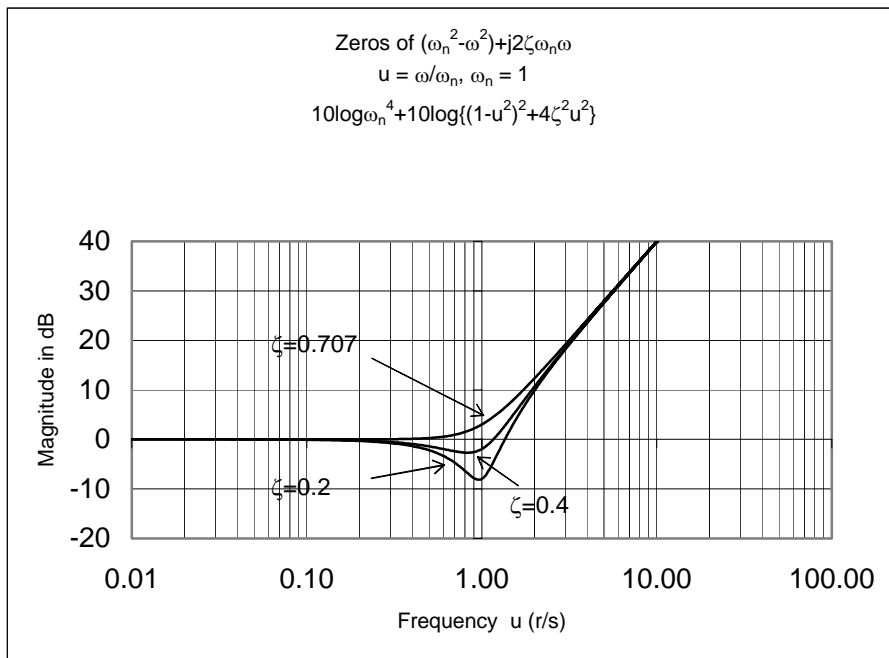


Figure 7.17. Magnitude for zeros of $10\log\omega_n^4 + 10\log[(1-u^2)^2 + 4\zeta^2u^2]$

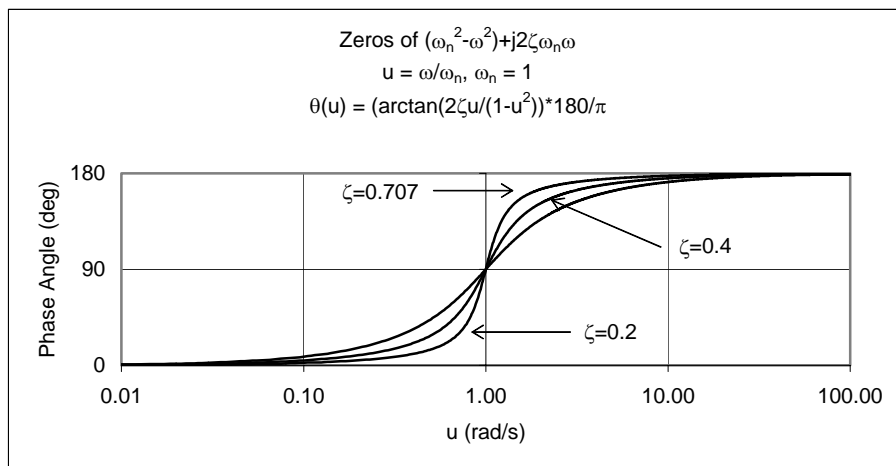


Figure 7.18. Phase for zeros of $10\log\omega_n^4 + 10\log[(1-u^2)^2 + 4\zeta^2u^2]$

The magnitude and phase plots for

$$G(j\omega) = \frac{1}{(\omega_n^2 - \omega^2) + j2\omega_n\omega}$$

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are similar to those of

$$G(j\omega) = (\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega$$

except for a minus sign. In this case, (7.35) becomes

$$-10(\log \omega_n^4) - 10\log[(1 - u^2)^2 + 4\zeta^2 u^2] \quad (7.40)$$

and (7.36) becomes

$$\theta(u) = -\tan^{-1}\left(\frac{2\zeta u}{1 - u^2}\right) \quad (7.41)$$

A plot of (7.40) for $\zeta = 0.2$, $\zeta = 0.4$, and $\zeta = 0.707$ is shown in Figure 7.19.

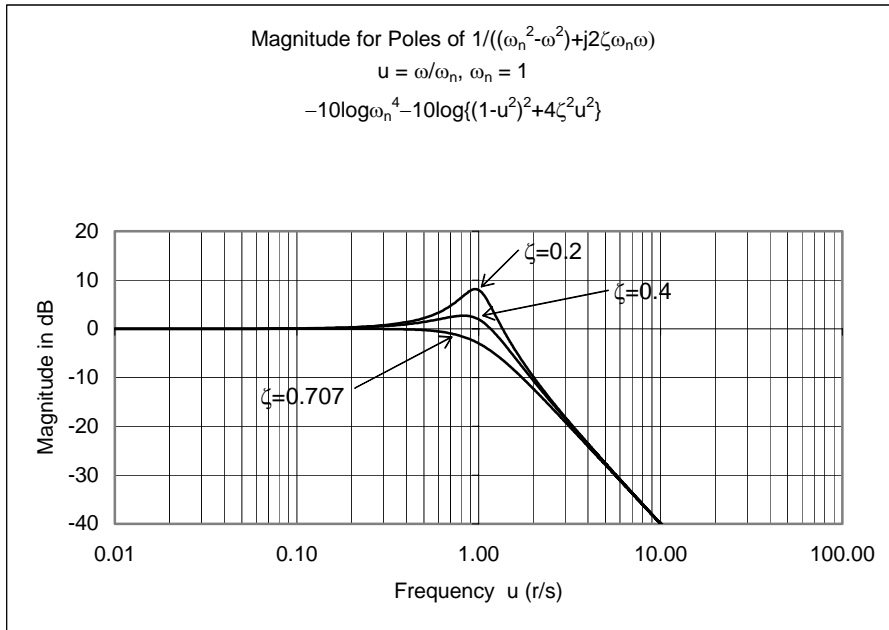


Figure 7.19. Magnitude for poles of $1/10\log \omega_n^4 + 10\log[(1 - u^2)^2 + 4\zeta^2 u^2]$

A plot of the phase for $\zeta = 0.2$, $\zeta = 0.4$, and $\zeta = 0.707$ is shown in Figure 7.20.

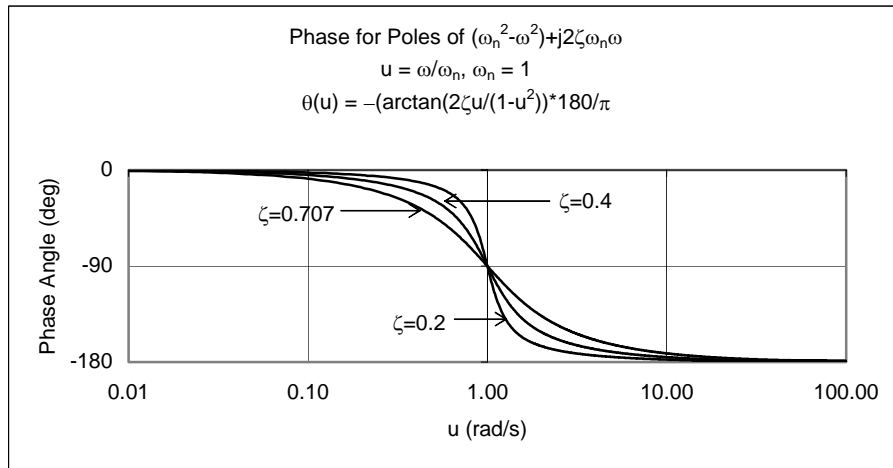


Figure 7.20. Phase for poles of $1/10 \log \omega_n^4 + 10 \log [(1 - u^2)^2 + 4\zeta^2 u^2]$

Example 7.3

For the circuit shown in Figure 7.21

- Compute the transfer function $G(s)$.
- Construct a straight line approximation for the magnitude of the Bode plot.
- From the Bode plot obtain the values of $20 \log |G(j\omega)|$ at $\omega = 30 \text{ r/s}$ and $\omega = 4000 \text{ r/s}$. Compare these values with the actual values.
- If $v_s(t) = 10 \cos(5000t + 60^\circ)$, use the Bode plot to compute the output $v_{out}(t)$.

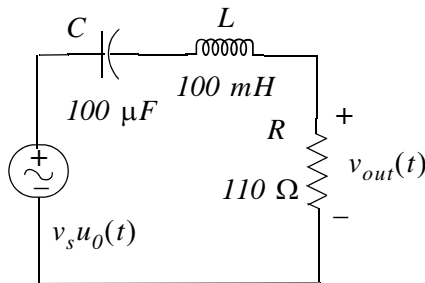


Figure 7.21. Circuit for Example 7.3.

Solution:

- We transform the given circuit to its equivalent in the s -domain shown in Figure 7.22.

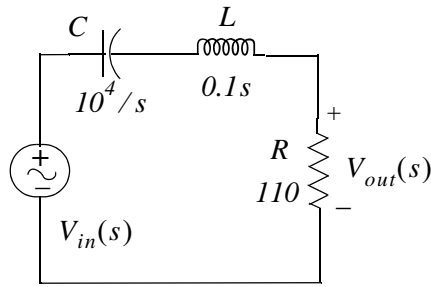


Figure 7.22. Circuit for Example 7.3 in s – domain

and by the voltage division expression,

$$V_{out}(s) = \frac{110}{10^4/s + 0.1s + 110} \cdot V_{in}(s)$$

Therefore, the transfer function is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{110s}{0.1s^2 + 110s + 10^4} = \frac{1100s}{s^2 + 1100s + 10^5} = \frac{1100s}{(s + 100)(s + 1000)} \quad (7.42)$$

b. Letting $s = j\omega$ we get

$$G(j\omega) = \frac{1100j\omega}{(j\omega + 100)(j\omega + 1000)}$$

or in standard form

$$G(j\omega) = \frac{0.011j\omega}{(1 + j\omega/100)(1 + j\omega/1000)} \quad (7.43)$$

Letting the magnitude of (7.43) be denoted as A , and expressing it in decibels we get

$$A_{dB} = 20\log|G(j\omega)| = 20\log 0.011 + 20\log|j\omega| - 20\log\left|1 + \frac{j\omega}{100}\right| - 20\log\left|1 + \frac{j\omega}{1000}\right| \quad (7.44)$$

We observe that the first term on the right side of (7.44) is a constant whose value is $20\log 0.011 = -39.17$. The second term is a straight line with slope equal to 20 dB/decade . For $\omega < 100 \text{ r/s}$ the third term is approximately zero and for $\omega > 100$ it decreases with slope equal to -20 dB/decade . Likewise, for $\omega < 1000 \text{ r/s}$ the fourth term is approximately zero and for $\omega > 1000$ it also decreases with slope equal to -20 dB/decade .

For Bode plots we use semilog paper. Instructions to construct semilog paper with Microsoft Excel are provided in Appendix D.

Construction of Bode Plots when the Zeros and Poles are Complex

In the Bode plot of Figure 7.23 the individual terms are shown with dotted lines and the sum of these with a solid line.

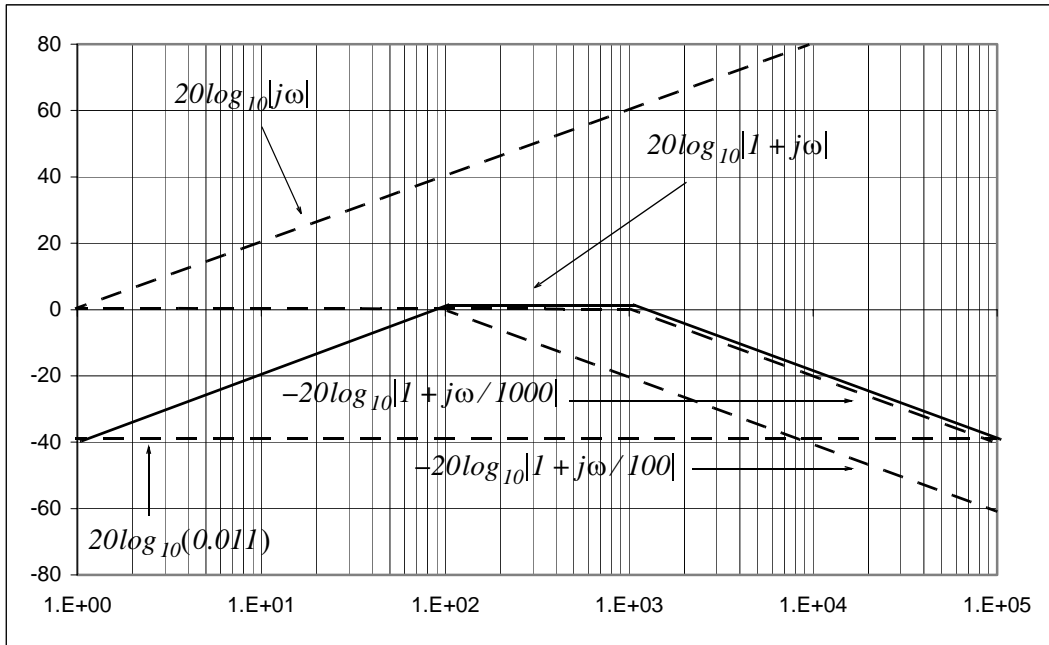


Figure 7.23. Magnitude plot of (7.44)

- c. The plot of Figure 7.23 shows that the magnitude of (7.43) at $\omega = 30 \text{ r/s}$ is approximately -9 dB and at $\omega = 4000 \text{ r/s}$ is approximately -10 dB . The actual values are found as follows:

At $\omega = 30 \text{ r/s}$, (7.43) becomes

$$G(j30) = \frac{0.011 \times j30}{(1 + j0.3)(1 + j0.03)}$$

and using MATLAB we get

```
g30=0.011*30j/((1+0.3j)*(1+0.03j));...
fprintf(' \n'); fprintf('mag = %6.2f \t',abs(g30));...
fprintf('magdB = %6.2f dB',20*log10(abs(g30))); fprintf(' \n'); fprintf(' \n')
```

mag = 0.32 magdB = -10.01 dB

Therefore,

$$|G(j30)| = 0.32$$

and

$$20 \log |G(j30)| = 20 \log 0.32 \approx -10 \text{ dB}$$

Chapter 7 Frequency Response and Bode Plots

Likewise, at $\omega = 4000 \text{ r/s}$, (7.43) becomes

$$G(j1000) = \frac{0.11(j4000)}{(1+j40)(1+j4)}$$

and using MATLAB we get

```
g4000=0.011*4000j/((1+40j)*(1+4j));...
fprintf(' \n'); fprintf('mag = %6.2f \t',abs(g4000));...
fprintf('magdB = %6.2f dB',20*log10(abs(g4000))); fprintf(' \n'); fprintf(' \n')
mag = 0.27      magdB = -11.48 dB
```

Therefore,

$$|G(j4000)| = 0.27$$

and

$$20\log|G(j4000)| = 20\log 0.27 = -11.48 \text{ dB}$$

- d. From the Bode plot of Figure 7.23, we see that the value of A_{dB} at $\omega = 5000 \text{ r/s}$ is approximately -12 dB . Then, since in general $a_{dB} = 20\log b$, and that $y = \log x$ implies $x = 10^y$, we have

$$|A| = 10^{\left(\frac{-12}{20}\right)} = 0.25$$

and therefore

$$V_{out \max} = |A||V_s| = 0.25 \times 10 = 2.5 \text{ V}$$

If we wish to obtain a more accurate value, we substitute $\omega = 5000$ into (7.43) and we get

```
g5000=0.011*5000j/((1+50j)*(1+5j));...
fprintf(' \n'); fprintf('mag = %6.2f \t',abs(g5000));...
fprintf('phase = %6.2f deg.',angle(g5000)*180/pi); fprintf(' \n'); fprintf(' \n')
mag = 0.22      phase = -77.54 deg.
```

$$G(j5000) = \frac{0.011(j5000)}{(1+j50)(1+j5)} = 0.22 \angle -77.54$$

Then,

$$V_{out \max} = |A| \times 10 = 0.22 \times 10 = 2.2 \text{ V}$$

and in the t -domain

$$v_{out}(t) = 2.2 \cos(5000t - 77.54^\circ)$$

Construction of Bode Plots when the Zeros and Poles are Complex

We can use the MATLAB function **bode(sys)** to draw the Bode plot of a Linear Time Invariant (LTI) System where **sys = tf(num,den)** creates a continuous-time transfer function **sys** with numerator **num** and denominator **den**, and **tf** creates a transfer function. With this function, the frequency range and number of points are chosen automatically. The function **bode(sys,{wmin,wmax})** draws the Bode plot for frequencies between **wmin** and **wmax** (in radians/second) and the function **bode(sys,w)** uses the user-supplied vector **w** of frequencies, in radians/second, at which the Bode response is to be evaluated. To generate logarithmically spaced frequency vectors, we use the command **logspace(first_exponent,last_exponent,number_of_values)**. For example, to generate plots for 100 logarithmically evenly spaced points for the frequency interval $10^{-1} \leq \omega \leq 10^2$ r/s, we use the statement **logspace(-1,2,100)**.

The **bode(sys,w)** function displays both magnitude and phase. If we want to display the magnitude only, we can use the **bodemag(sys,w)** function.

MATLAB requires that we express the numerator and denominator of $G(s)$ as polynomials of s in descending powers.

Let us plot the transfer function of Example 7.3 using MATLAB.

From (7.42),

$$G(s) = \frac{1100s}{s^2 + 1100s + 10^5}$$

and the MATLAB code to generate the magnitude and phase plots is

```
num=[0 1100 0]; den=[1 1100 10^5]; w=logspace(0,5,100); bode(num,den,w)
```

However, since for this example we are interested in the magnitude only, we will use the code

```
num=[0 1100 0]; den=[1 1100 10^5]; sys=tf(num,den);...  
w=logspace(0,5,100); bodemag(sys,w); grid
```

and upon execution, MATLAB displays the plot shown in Figure 7.24.

Example 7.4

For the circuit of Example 7.3

- Draw a Bode phase plot.
- Using the Bode phase plot estimate the frequency where the phase is zero degrees.
- Compute the actual frequency where the phase is zero degrees.
- Find $v_{out}(t)$ if $v_{in}(t) = 10 \cos(\omega t + 60^\circ)$ and ω is the value found in part (c).

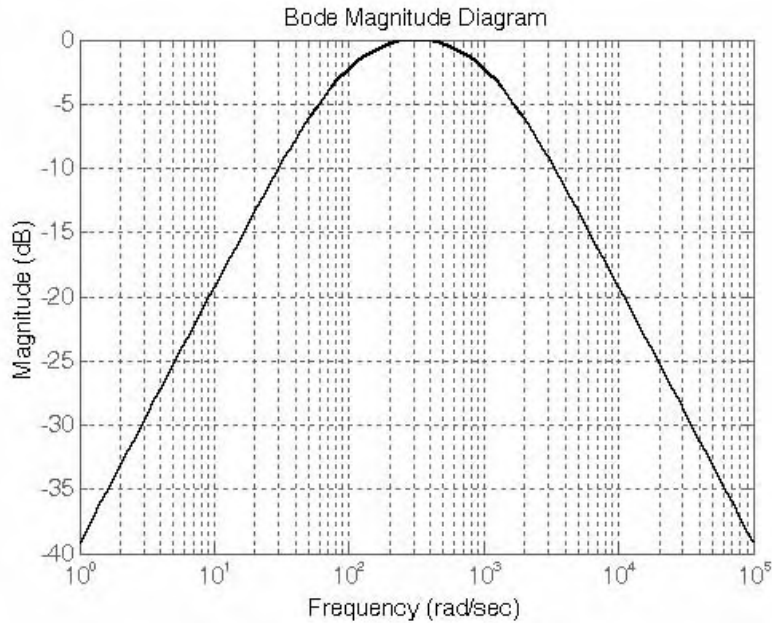


Figure 7.24. Bode plot for Example 7.3.

Solution:

a. From (7.43) of Example 7.3

$$G(j\omega) = \frac{0.011j\omega}{(1 + j\omega/100)(1 + j\omega/1000)} \quad (7.45)$$

and in magnitude-phase form

$$G(j\omega) = \frac{0.011|j\omega|}{|(1 + j\omega/100)||1 + j\omega/1000|} \angle(\alpha - \beta - \gamma)$$

where

$$\angle\alpha = 90^\circ \quad \angle-\beta = -\tan^{-1}(\omega/100) \quad \angle-\gamma = -\tan^{-1}(\omega/1000)$$

For $\omega = 100$

$$\angle-\beta = -\tan^{-1}1 = -45^\circ$$

For $\omega = 1000$

$$\angle-\gamma = -\tan^{-1}1 = -45^\circ$$

The straight-line phase angle approximations are shown in Figure 7.25.

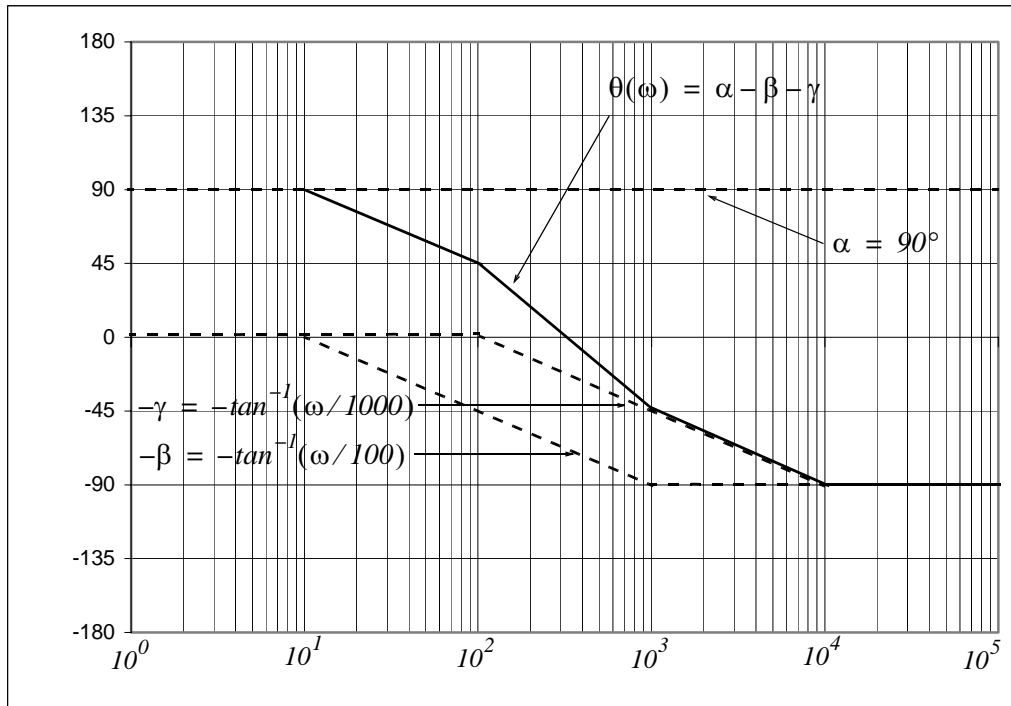


Figure 7.25. Bode plot for Example 7.4.

Figure 7.26 shows the magnitude and phase plots generated with the following MATLAB code:

```
num=[0 1100 0]; den=[1 1100 10^5]; w=logspace(0,5,100); bode(num,den,w)
```

- b. From the Bode plot of Figure 7.25 we find that the phase is zero degrees at approximately $\omega = 310 \text{ r/s}$
- c. From (7.45)

$$G(j\omega) = \frac{0.011j\omega}{(1 + j\omega/100)(1 + j\omega/1000)}$$

and in magnitude-phase form

$$G(j\omega) = \frac{0.011\omega \angle 90^\circ}{|(1 + j\omega/100)| \angle \tan^{-1}(\omega/100) |(1 + j\omega/1000)| \angle \tan^{-1}(\omega/1000)}$$

The phase will be zero when

$$\tan^{-1}(\omega/100) + \tan^{-1}(\omega/1000) = 90^\circ$$

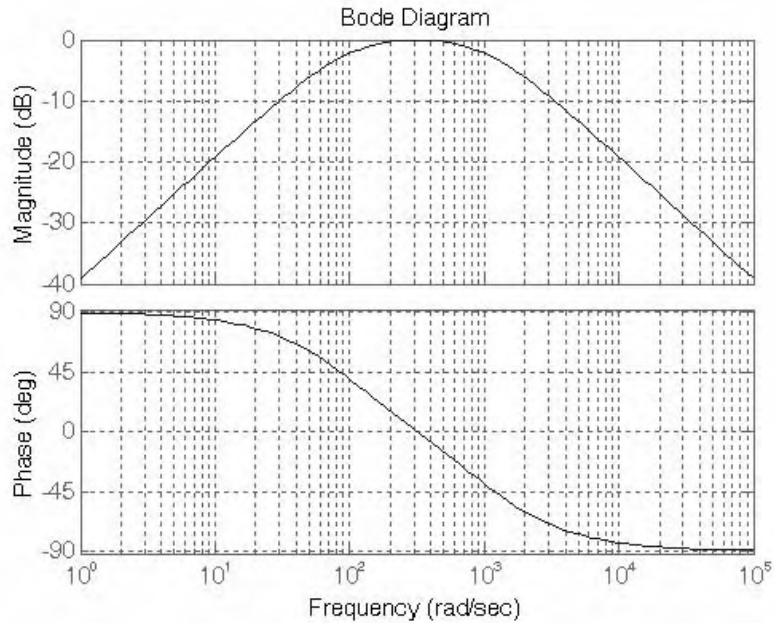


Figure 7.26. Bode plots for Example 7.4 generated with MATLAB

This is a trigonometric equation and we will solve it for ω with the **solve(equ)** MATLAB function as follows:

```
syms w; x=solve(atan(w/100)+atan(w/1000)-pi/2); combine(x)
```

```
ans =  
    316.2278
```

Therefore, $\omega = 316.23 \text{ r/s}$

d. Evaluating (7.45) at $\omega = 316.23 \text{ r/s}$ we get:

$$G(j316.23) = \frac{0.011(j316.23)}{(1 + j316.23/100)(1 + j316.23/1000)} \quad (7.46)$$

and with MATLAB

```
Gj316=0.011*316.23j/((1+316.23j/100)*(1+316.23j/1000)); fprintf(' \n');...  
fprintf('magGj316 = %5.2f \t', abs(Gj316));...  
fprintf('phaseGj316 = %5.2f deg.', angle(Gj316)*180/pi)  
magGj316 = 1.00   phaseGj316 = -0.00 deg.
```

We are given that $|V_{in}| = 10 \text{ V}$ and with $|G(j316.23)| = 1$ we get

$$|V_{out}| = |G(j316.23)||V_{in}| = 1 \times 10 = 10 \text{ V}$$

The phase angle of the input voltage is given as $\theta_{in} = 60^\circ$ and with $\theta(j316.23) = 0^\circ$ we find that the phase angle of the output voltage is

$$\theta_{out} = \theta_{in} + \theta(j316.23) = 60^\circ + 0^\circ = 60^\circ$$

and thus

$$V_{out} = 10 \angle 60^\circ$$

or

$$v_{out}(t) = 10 \cos(316.23t + 60^\circ)$$

7.7 Corrected Amplitude Plots

The amplitude plots we have considered thus far are approximate. We can make the straight line more accurate by drawing smooth curves connecting the points at one-half the corner frequency $\omega_n/2$, the corner frequency ω_n and twice the corner frequency $2\omega_n$ as shown in Figure 7.27.

At the corner frequency ω_n , the value of the amplitude A in dB is

$$A_{dB}|_{\omega = \omega_n} = \pm 20 \log |1 + j| = \pm 20 \log \sqrt{2} = \pm 3 \text{ dB} \quad (7.47)$$

where the plus (+) sign applies to a first order zero, and the minus (-) to a first order pole.

Similarly,

$$A_{dB}|_{\omega = \omega_n/2} = \pm 20 \log |1 + j/2| = \pm 20 \log \sqrt{\frac{5}{4}} = \pm 0.97 \text{ dB} \approx \pm 1 \text{ dB} \quad (7.48)$$

and

$$A_{dB}|_{\omega = 2\omega_n} = \pm 20 \log |1 + j2| = \pm 20 \log \sqrt{5} = \pm 6.99 \text{ dB} \approx \pm 7 \text{ dB} \quad (7.49)$$

As we can see from Figure 7.27, the straight line approximations, shown by dotted lines, yield 0 dB at half the corner frequency and at the corner frequency. At twice the corner frequency, the straight line approximations yield $\pm 6 \text{ dB}$ because ω_n and $2\omega_n$ are separated by one octave which is equivalent to $\pm 3 \text{ dB}$ per decade. Therefore, the corrections to be made are $\pm 1 \text{ dB}$ at half the corner frequency $\omega_n/2$, $\pm 3 \text{ dB}$ at the corner frequency ω_n , and $\pm 1 \text{ dB}$ at twice the corner frequency $2\omega_n$.

The corrected amplitude plots for a first order zero and first order pole are shown by solid lines in Figure 7.27.

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The corrections for straight-line amplitude plots when we have complex poles and zeros require different type of correction because they depend on the damping coefficient ζ . Let us refer to the plot of Figure 7.28.

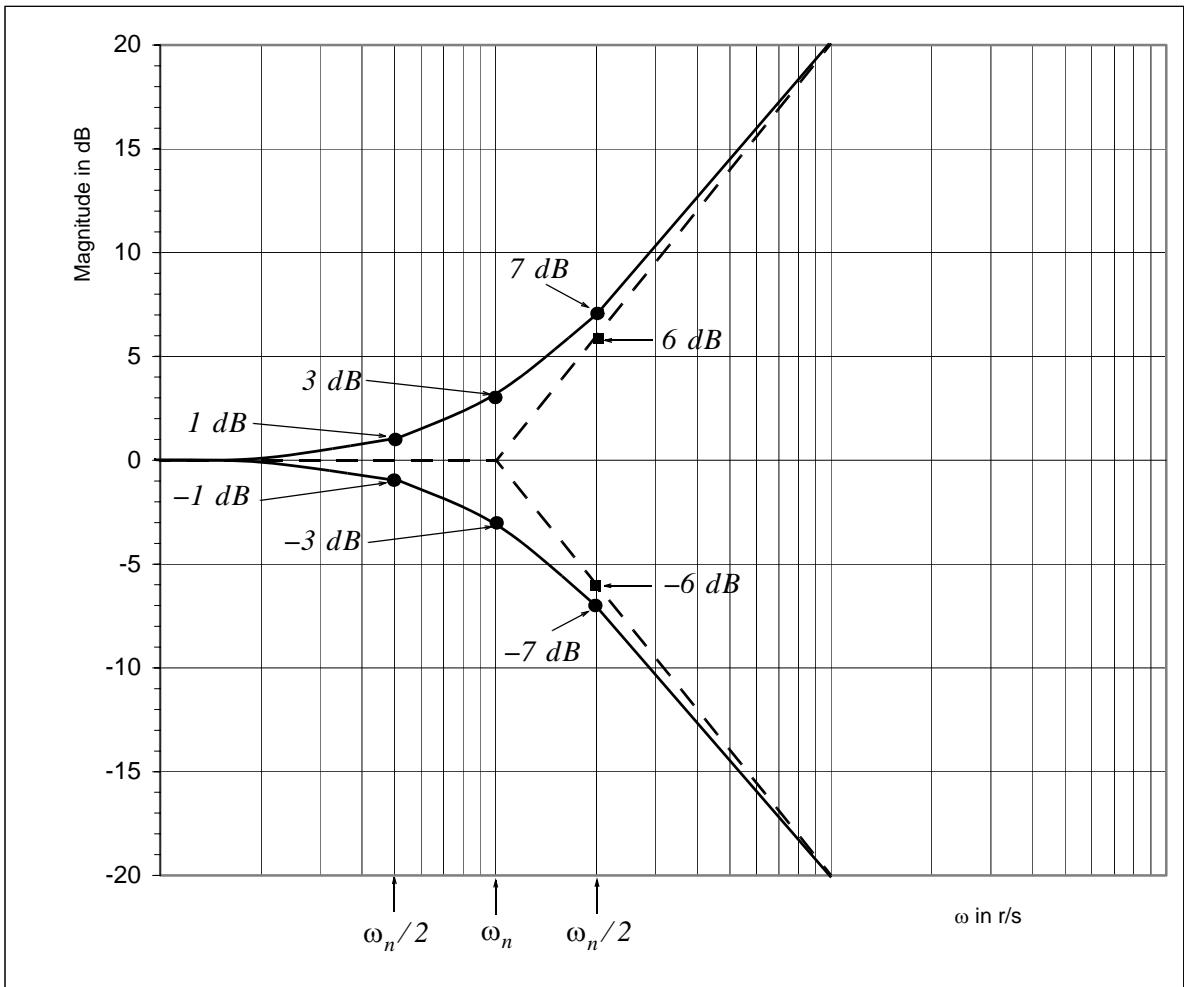


Figure 7.27. Corrections for magnitude Bode plots

We observe that as the damping coefficient ζ becomes smaller and smaller, larger and larger peaks in the amplitude occur in the vicinity of the corner frequency ω_n . We also observe that when $\zeta \geq 0.707$, the amplitude at the corner frequency ω_n lies below the straight line approximation.

We can obtain a fairly accurate amplitude plot by computing the amplitude at four points near the corner frequency ω_n as shown in Figure 7.28.

The amplitude plot of Figure 7.28 is for complex poles. In analogy with (7.30), i.e.,

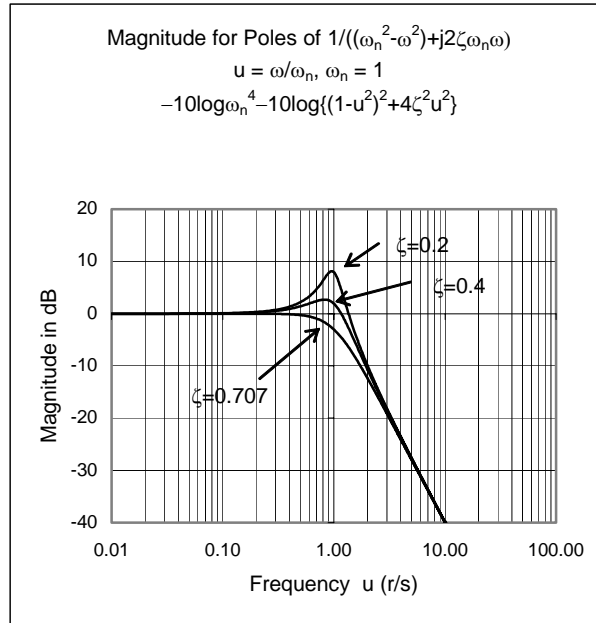


Figure 7.28. Magnitude Bode plots with complex poles

$$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

which was derived earlier for complex zeros, the transfer function for complex poles is

$$G(s) = \frac{C}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{7.50}$$

where C is a constant.

Dividing each term of the denominator of (7.50) by ω_n we get

$$G(s) = \frac{C}{\omega_n^2} \frac{1}{(s/\omega_n)^2 + 2\zeta(s/\omega_n) + 1}$$

and letting $C/\omega_n^2 = K$ and $s = j\omega$, we get

$$G(j\omega) = \frac{K}{1 - (\omega/\omega_n)^2 + j2\zeta\omega/\omega_n} \tag{7.51}$$

As before, we let $\omega/\omega_n = u$. Then (7.51) becomes

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$$G(ju) = \frac{K}{1 - u^2 + j2\zeta u} \quad (7.52)$$

and in polar form,

$$G(ju) = \frac{K}{|1 - u^2 + j2\zeta u| \angle \theta} \quad (7.53)$$

The magnitude of (7.53) in *dB* is

$$\begin{aligned} A_{dB} &= 20 \log |G(ju)| = 20 \log K - 20 \log |(1 - u^2 + j2\zeta u)| \\ &= 20 \log K - 20 \log \sqrt{(1 - u^2)^2 + 4\zeta^2 u^2} = 20 \log K - 10 \log [u^4 + 2u^2(2\zeta^2 - 1) + 1] \end{aligned} \quad (7.54)$$

and the phase is

$$\theta(u) = -\tan^{-1} \frac{2\zeta u}{1 - u^2} \quad (7.55)$$

In (7.54) the term $20 \log K$ is constant and thus the amplitude A_{dB} , as a function of frequency, is dependent only the second term on the right side. Also, from this expression, we observe that as $u \rightarrow 0$,

$$-10 \log [u^4 + 2u^2(2\zeta^2 - 1) + 1] \rightarrow 0 \quad (7.56)$$

and as $u \rightarrow \infty$,

$$-10 \log [u^4 + 2u^2(2\zeta^2 - 1) + 1] \rightarrow -40 \log u \quad (7.57)$$

We are now ready to compute the values of A_{dB} at points 1, 2, 3, and 4 of the plot of Figure 7.29.

At point 1, the corner frequency ω_n corresponds to $u = 1$. Then, from (7.54)

$$\begin{aligned} A_{dB}(\omega_n/2) &= A_{dB}\left(\frac{u}{2}\right) = -10 \log [u^4 + 2u^2(2\zeta^2 - 1) + 1] \Big|_{u=1/2} \\ &= -10 \log \left[\frac{1}{16} + 2 \cdot \frac{1}{4} (2\zeta^2 - 1) + 1 \right] = -10 \log \left[\frac{1}{16} + \zeta^2 - \frac{1}{2} + 1 \right] \\ &= -10 \log (\zeta^2 + 0.5625) \end{aligned} \quad (7.58)$$

and for $\zeta = 0.4$

$$A_{dB}(\omega_n/2) \Big|_{point 1} = -10 \log (0.4^2 + 0.5625) = 1.41 \text{ dB}$$

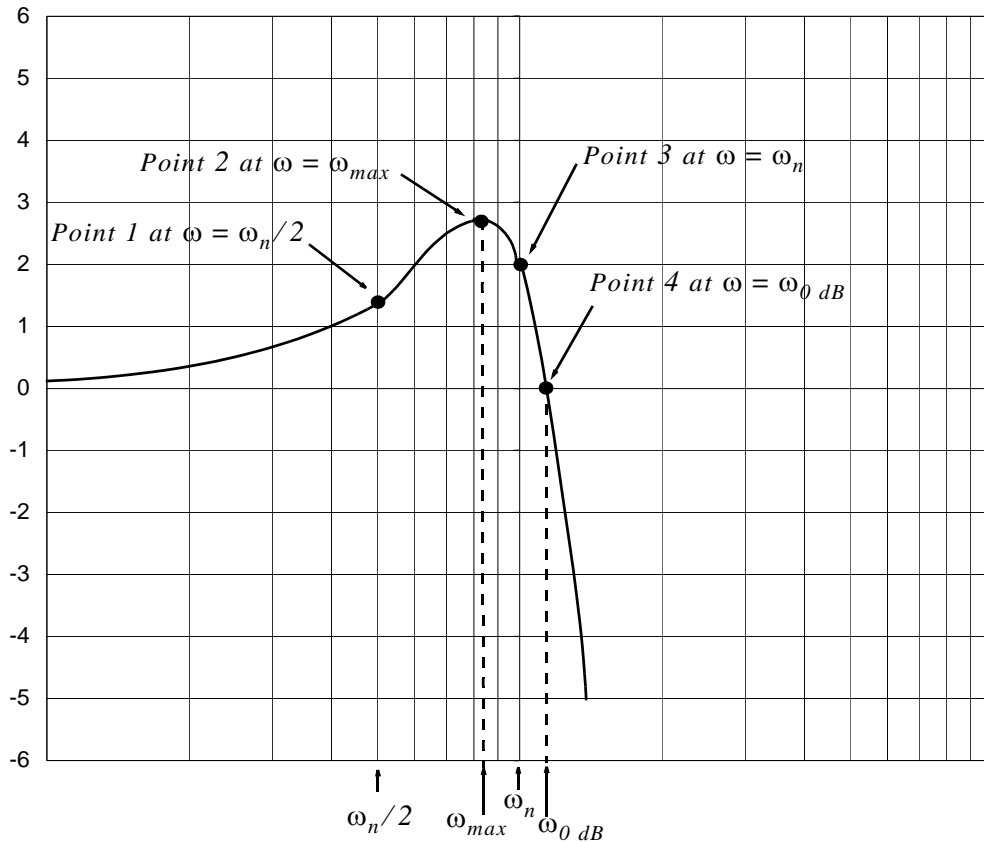


Figure 7.29. Corrections for magnitude Bode plots with complex poles when $\zeta = 0.4$

To find the amplitude at point 2, in (7.54) we let $K = 1$ and we form the magnitude in dB . Then,

$$A_{dB}|_{point\ 2} = 20 \log \frac{1}{\left| 1 - (\omega/\omega_n)^2 + j2\zeta\omega/\omega_n \right|} \quad (7.59)$$

We now recall that the logarithmic function is a monotonically increasing function and therefore (7.59) has a maximum when the absolute magnitude of this expression is maximum. Also, the square of the absolute magnitude is maximum when the absolute magnitude is maximum.

The square of the absolute magnitude is

$$\frac{1}{\left[1 - (\omega/\omega_n)^2 \right]^2 + 4(\zeta\omega/\omega_n)^2} \quad (7.60)$$

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or

$$\frac{1}{1 - 2\omega^2/\omega_n^2 + \omega^4/\omega_n^4 + 4\zeta^2\omega^2/\omega_n^2} \quad (7.61)$$

To find the maximum, we take the derivative with respect to ω and we set it equal to zero, that is,

$$\frac{4\omega/\omega_n^2 - 4\omega^3/\omega_n^4 - 8\zeta^2\omega/\omega_n^2}{\left\{ [1 - (\omega/\omega_n)^2]^2 + 4((\zeta\omega)/\omega_n)^2 \right\}^2} = 0 \quad (7.62)$$

The expression of (7.62) will be zero when the numerator is set to zero, that is,

$$(\omega/\omega_n^2)(4 - 4\omega^2/\omega_n^2 - 8\zeta^2) = 0 \quad (7.63)$$

Of course, we require that the value of ω must be a nonzero value. Then,

$$4 - 4\omega^2/\omega_n^2 - 8\zeta^2 = 0$$

or

$$(4\omega^2)/\omega_n^2 = 4 - 8\zeta^2$$

from which

$$\omega_{max} = \omega = \omega_n \sqrt{1 - 2\zeta^2} \quad (7.64)$$

provided that $1 - 2\zeta^2 > 0$ or $\zeta < 1/\sqrt{2}$ or $\zeta < 0.707$.

The dB value of the amplitude at point 2 is found by substitution of (7.64) into (7.54), that is,

$$\begin{aligned} A_{dB}(\omega_{max}) &= -10\log[u^4 + 2u^2(2\zeta^2 - 1) + 1] \Big|_{u = \sqrt{1 - 2\zeta^2}} \\ &= -10\log[(1 - 2\zeta^2)^2 + 2(1 - 2\zeta^2)(2\zeta^2 - 1) + 1] \\ &= -10\log[4\zeta^2(1 - \zeta^2)] \end{aligned} \quad (7.65)$$

and for $\zeta = 0.4$

$$A_{dB}(\omega_{max}) = -10\log(4 \times 0.4^2(1 - 0.4^2)) = 2.69 \text{ dB}$$

The dB value of the amplitude at point 3 is found by substitution of $\omega = \omega_n = u = 1$ into (7.54). Then,

$$\begin{aligned}
 A_{dB}(\omega_n) &= -10\log[u^4 + 2u^2(2\zeta^2 - 1) + 1] \Big|_{u=1} \\
 &= -10\log[1 + 2(2\zeta^2 - 1) + 1] \\
 &= -10\log[4\zeta^2] = -20\log(2\zeta)
 \end{aligned}
 \tag{7.66}$$

and for $\zeta = 0.4$

$$A_{dB}(\omega_n) = -20\log(2 \times 0.4) = 1.94 \text{ dB}$$

Finally, at point 4, the dB value of the amplitude crosses the 0 dB axis. Therefore, at this point we are interested not in $A_{dB}(\omega_{0 \text{ dB}})$ but in the location of $\omega_{0 \text{ dB}}$ in relation to the corner frequency ω_n . At this point we must have from (7.57)

$$0 \text{ dB} = -10\log[u^4 + 2u^2(2\zeta^2 - 1) + 1]$$

and since $\log 1 = 0$, it follows that

$$\begin{aligned}
 u^4 + 2u^2(2\zeta^2 - 1) + 1 &= 1 \\
 u^4 + 2u^2(2\zeta^2 - 1) &= 0 \\
 u^2(u^2 + 2(2\zeta^2 - 1)) &= 0
 \end{aligned}$$

or

$$u^2 + 2(2\zeta^2 - 1) = 0$$

Solving for u and making use of $u = \omega/\omega_n$ we get

$$\omega_{0 \text{ dB}} = \omega_n \sqrt{2(1 - 2\zeta^2)}$$

From (7.67),

$$\omega_{max} = \omega_n \sqrt{1 - 2\zeta^2}$$

therefore, if we already know the frequency at which the dB amplitude is maximum, we can compute the frequency at point 4 from

$$\omega_{0 \text{ dB}} = \sqrt{2}\omega_{max} \tag{7.67}$$

Example 7.5

For the circuit of Figure 7.30

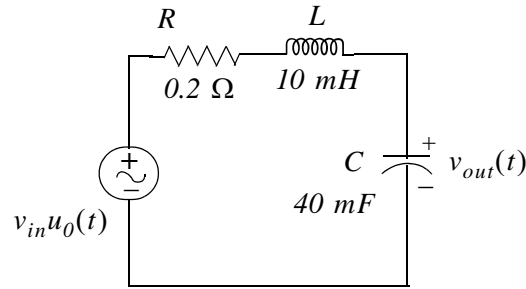


Figure 7.30. Circuit for Example 7.5.

- Compute the transfer function $G(s)$
- Find the corner frequency ω_n from $G(s)$.
- Compute the damping coefficient ζ .
- Construct a straight line approximation for the magnitude of the Bode plot.
- Compute the amplitude in dB at one-half the corner frequency $\omega_n/2$, at the frequency ω_{max} at which the amplitude reaches its maximum value, at the corner frequency ω_n , and at the frequency $\omega_{0\text{ dB}}$ where the dB amplitude is zero. Then, draw a smooth curve to connect these four points.

Solution:

- We transform the given circuit to its equivalent in the s -domain shown in Figure 7.31 where $R = 1$, $Ls = 0.05s$, and $1/Cs = 125/s$.

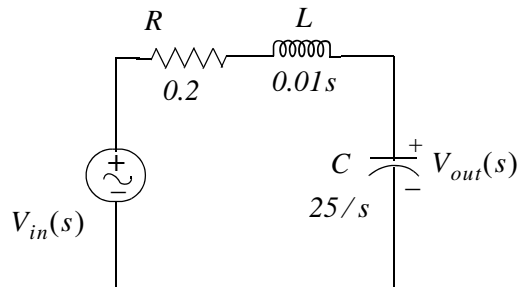


Figure 7.31. Circuit for Example 7.5 in s -domain

and by the voltage division expression,

$$V_{out}(s) = \frac{25/s}{0.2 + 0.01s + 25/s} \cdot V_{in}(s)$$

Therefore, the transfer function is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{25}{0.01s^2 + 0.2s + 25} = \frac{2500}{s^2 + 20s + 2500} \quad (7.68)$$

b. From (7.50)

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (7.69)$$

and from (7.68) and (7.69) $\omega_n^2 = 2500$ or

$$\omega_n = 50 \text{ rad/s} \quad (7.70)$$

c. From (7.68) and (7.69) $2\zeta\omega_n = 20$. Then, the damping coefficient ζ is

$$\zeta = \frac{20}{2\omega_n} = \frac{20}{2 \times 50} = 0.2 \quad (7.71)$$

d. For $\omega < \omega_n$, the straight line approximation lies along the 0 dB axis, whereas for $\omega > \omega_n$, the straight line approximation has a slope of -40 dB . The corner frequency ω_n was found in part (b) to be 50 rad/s . The dB amplitude plot is shown in Figure 7.31.

e. From (7.61),

$$A_{dB}(\omega_n/2) = -10 \log(\zeta^2 + 0.5625)$$

where from (7.74) $\zeta = 0.2$ and thus $\zeta^2 = 0.04$. Then,

$$A_{dB}(\omega_n/2) = -10 \log(0.04 + 0.5625) = -10 \log(0.6025) = 2.2 \text{ dB}$$

and this value is indicated as Point 1 on the plot of Figure 7.32.

Next, from (7.64)

$$\omega_{max} = \omega_n \sqrt{1 - 2\zeta^2}$$

Then,

$$\omega_{max} = 50 \sqrt{1 - 2 \times 0.04} = 50 \sqrt{0.92} = 47.96 \text{ rad/s}$$

Therefore, from (7.65)

$$A_{dB}(\omega_{max}) = -10 \log[4\zeta^2(1 - \zeta^2)] = -10 \log[(0.16) \times (0.96)] = 8.14 \text{ dB}$$

and this value is indicated as Point 2 on the plot of Figure 7.32.

The dB amplitude at the corner frequency is found from (7.66), that is,

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$$A_{dB}(\omega_n) = -20\log(2\zeta)$$

Then,

$$A_{dB}(\omega_n) = -20\log(2 \times 0.2) = 7.96 \text{ dB}$$

and this value is indicated as Point 3 on the plot of Figure 7.32.

Finally, the frequency at which the amplitude plot crosses the 0 dB axis is found from (7.67), that is,

$$\omega_{0 \text{ dB}} = \sqrt{2}\omega_{max}$$

or

$$\omega_{0 \text{ dB}} = \sqrt{2} \times 47.96 = 67.83 \text{ rad/s}$$

This frequency is indicated as Point 4 on the plot of Figure 7.32.

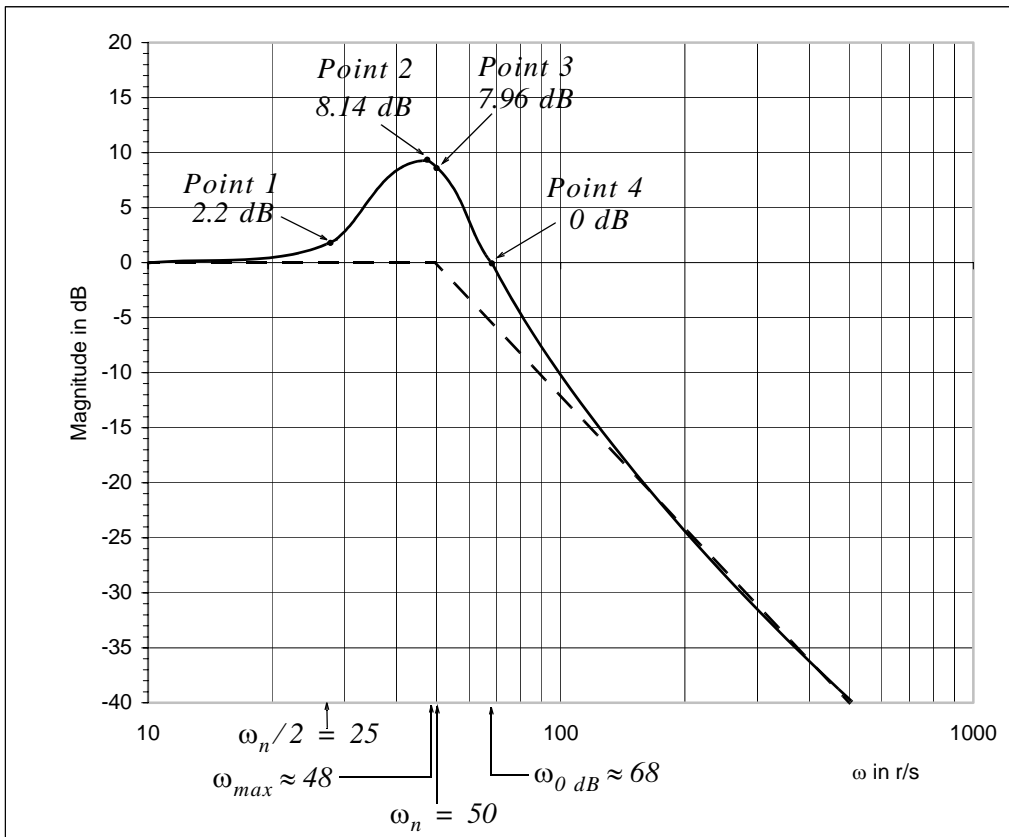


Figure 7.32. Amplitude plot for Example 7.5

The amplitude plot of Figure 7.32 reveals that the given circuit behaves as a low pass filter.

Using the transfer function of (7.68) with MATLAB, we get the Bode magnitude plot shown in Figure 7.33.

```
num=[0 0 2500]; den=[1 20 2500]; sys=tf(num,den); w=logspace(0,5,100); bodemag(sys,w)
```

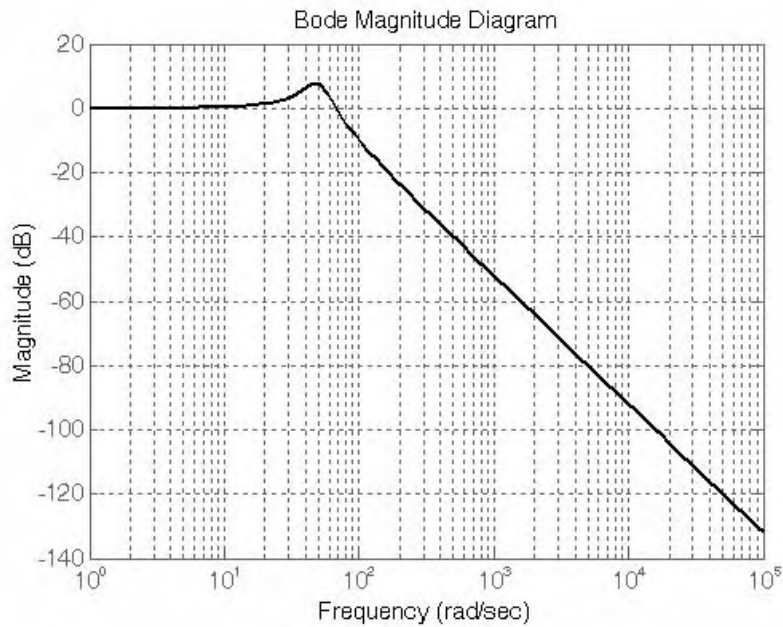


Figure 7.33. Bode plot for Example 7.5 using MATLAB

7.8 Summary

- The decibel, denoted as dB, is a unit used to express the ratio between two amounts of power, generally P_{out}/P_{in} . By definition, the number of dB is obtained from $dB_w = 10\log_{10}(P_{out}/P_{in})$. It can also be used to express voltage and current ratios provided that the voltages and currents have identical impedances. Then, for voltages we use the expression $dB_v = 20\log_{10}(V_{out}/V_{in})$, and for currents we use the expression $dB_i = 20\log_{10}(I_{out}/I_{in})$
- The bandwidth, denoted as BW , is a term generally used with electronic amplifiers and filters. For low-pass filters the bandwidth is the band of frequencies from zero frequency to the cutoff frequency where the amplitude fall to 0.707 of its maximum value. For high-pass filters the bandwidth is the band of frequencies from 0.707 of maximum amplitude to infinite frequency. For amplifiers, band-pass, and band-elimination filters the bandwidth is the range of frequencies where the maximum amplitude falls to 0.707 of its maximum value on either side of the frequency response curve.
- If two frequencies ω_1 and ω_2 are such that $\omega_2 = 2\omega_1$, we say that these frequencies are separated by one octave and if $\omega_2 = 10\omega_1$, they are separated by one decade.
- Frequency response is a term used to express the response of an amplifier or filter to input sinusoids of different frequencies. The response of an amplifier or filter to a sinusoid of frequency ω is completely described by the magnitude $|G(j\omega)|$ and phase $\angle G(j\omega)$ of the transfer function.
- Bode plots are frequency response diagrams of magnitude and phase versus frequency ω .
- In Bode plots the 3-dB frequencies, denoted as ω_n , are referred to as the corner frequencies.
- In Bode plots, the transfer function is expressed in linear factors of the form $j\omega + z_i$ for the zero (numerator) linear factors and $j\omega + p_i$ for the pole linear factors. When quadratic factors with complex roots occur in addition to the linear factors, these quadratic factors are expressed in the form $(j\omega)^2 + j2\zeta\omega_n\omega + \omega_n^2$.
- In magnitude Bode plots with quadratic factors the difference between the asymptotic plot and the actual curves depends on the value of the damping factor ζ . But regardless of the value of ζ , the actual curve approaches the asymptotes at both low and high frequencies.
- In Bode plots the corner frequencies ω_n are easily identified by expressing the linear terms as $z_i(j\omega/z_i + 1)$ and $p_i(j\omega/p_i + 1)$ for the zeros and poles respectively. For quadratic factor the corner frequency ω_n appears in the expression $(j\omega)^2 + j2\zeta\omega_n\omega + \omega_n^2$ or $(j\omega/\omega_n)^2 + j2\zeta\omega/\omega_n + 1$

- In both the magnitude and phase Bode plots the frequency (abscissa) scale is logarithmic. The ordinate in the magnitude plot is expressed in dB and in the phase plot is expressed in degrees.
- In magnitude Bode plots, the asymptotes corresponding to the linear terms of the form $(j\omega/z_i + 1)$ and $(j\omega/p_i + 1)$ have a slope $\pm 20 dB/decade$ where the positive slope applies to zero (numerator) linear factors, and the negative slope applies to pole (denominator) linear factors.
- In magnitude Bode plots, the asymptotes corresponding to the quadratic terms of the form $(j\omega/\omega_n)^2 + j2\zeta\omega/\omega_n + 1$ have a slope $\pm 40 dB/decade$ where the positive slope applies to zero (numerator) quadratic factors, and the negative slope applies to pole (denominator) quadratic factors.
- In phase Bode plots with linear factors, for frequencies less than one tenth the corner frequency we assume that the phase angle is zero. At the corner frequency the phase angle is $\pm 45^\circ$. For frequencies ten times or greater than the corner frequency, the phase angle is approximately $\pm 90^\circ$ where the positive angle applies to zero (numerator) linear factors, and the negative angle applies to pole (denominator) linear factors.
- In phase Bode plots with quadratic factors, the phase angle is zero for frequencies less than one tenth the corner frequency. At the corner frequency the phase angle is $\pm 90^\circ$. For frequencies ten times or greater than the corner frequency, the phase angle is approximately $\pm 180^\circ$ where the positive angle applies to zero (numerator) quadratic factors, and the negative angle applies to pole (denominator) quadratic factors.
- Bode plots can be easily constructed and verified with the MATLAB function **bode(sys)** function. With this function, the frequency range and number of points are chosen automatically. The function **bode(sys),{wmin,wmax}** draws the Bode plot for frequencies between **wmin** and **wmax** (in radian/second) and the function **bode(sys,w)** uses the user-supplied vector **w** of frequencies, in radian/second, at which the Bode response is to be evaluated. To generate logarithmically spaced frequency vectors, we use the command **logspace(first_exponent,last_exponent,number_of_values)**.

7.9 Exercises

1. For the transfer function

$$G(s) = \frac{10^5(s+5)}{(s+100)(s+5000)}$$

- Draw the magnitude Bode plot and find the approximate maximum value of $|G(j\omega)|$ in dB .
 - Find the value of ω where $|G(j\omega)| = 1$ for $\omega > 5$ r/s
 - Check your plot with the plot generated with MATLAB.
2. For the transfer function of Exercise 1
- Draw a Bode plot for the phase angle and find the approximate phase angle at $\omega = 30$ r/s , $\omega = 50$ r/s , $\omega = 100$ r/s , and $\omega = 5000$ r/s
 - Compute the actual values of the phase angle at the frequencies specified in (a).
 - Check your magnitude plot of Exercise 1 and the phase plot of this exercise with the plots generated with MATLAB.
3. For the circuit of Figure 7.34
- Compute the transfer function.
 - Draw the Bode amplitude plot for $20\log|G(j\omega)|$
 - From the plot of part (b) determine the type of filter represented by this circuit and estimate the cutoff frequency.
 - Compute the actual cutoff frequency of this filter.
 - Draw a straight line phase angle plot of $G(j\omega)$.
 - Determine the value of $\theta(\omega)$ at the cutoff frequency from the plot of part (c).
 - Compute the actual value of $\theta(\omega)$ at the cutoff frequency.

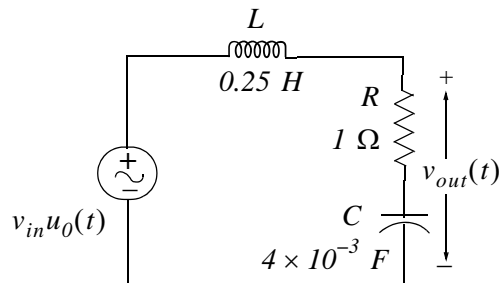


Figure 7.34. Circuit for Exercise 3

7.10 Answers to Exercises

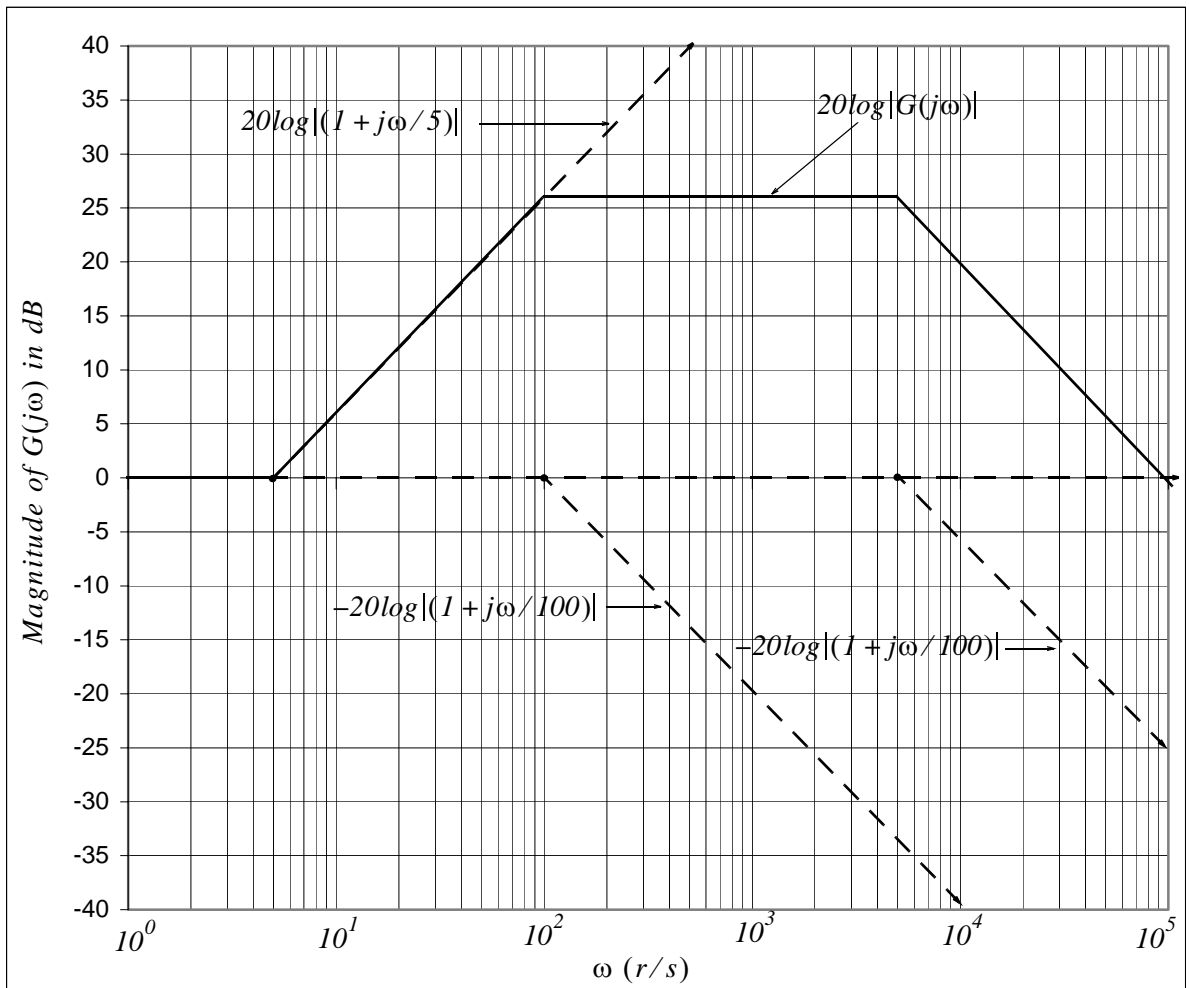
1. a.

$$G(j\omega) = \frac{10^5(j\omega + 5)}{(j\omega + 100)(j\omega + 5000)} = \frac{10^5 \times 5 \times (1 + j\omega/5)}{100 \times (1 + j\omega/100) \times 5000 \times (1 + j\omega/5000)}$$

$$= \frac{(1 + j\omega/5)}{(1 + j\omega/100) \cdot (1 + j\omega/5000)}$$

$$20\log|G(j\omega)| = 20\log|1 + j\omega/5| - 20\log|1 + j\omega/100| - 20\log|1 + j\omega/5000|$$

The corner frequencies are at $\omega = 5 \text{ r/s}$, $\omega = 100 \text{ r/s}$, and $\omega = 5000 \text{ r/s}$. The asymptotes are shown as solid lines.



From this plot we observe that $20\log|G(j\omega)|_{max} \approx 26 \text{ dB}$ for the interval $10^2 \leq \omega \leq 5 \times 10^3$

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b. By inspection, $20\log|G(j\omega)| = 0 \text{ dB}$ at $\omega = 9.85 \times 10^4 \text{ r/s}$

2. From the solution of Exercise 1

$$G(j\omega) = \frac{(1 + j\omega/5)}{(1 + j\omega/100) \cdot (1 + j\omega/5000)}$$

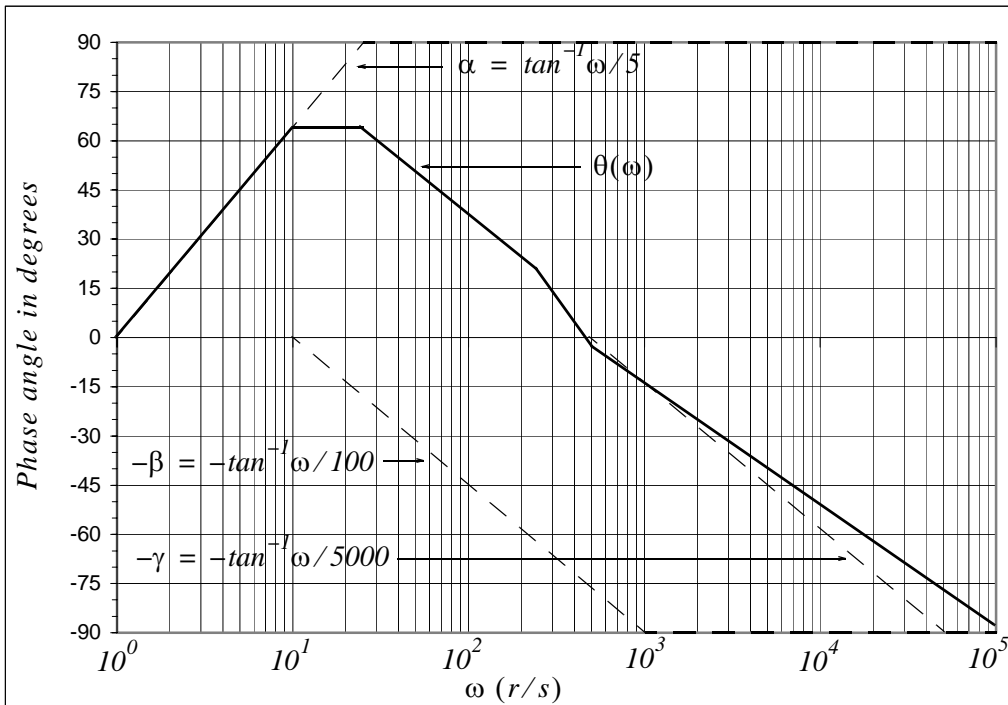
and in magnitude-phase form

$$G(j\omega) = \frac{|(1 + j\omega/5)|}{|(1 + j\omega/100)| \cdot |(1 + j\omega/5000)|} \angle(\alpha - \beta - \gamma)$$

that is, $\theta(\omega) = \alpha - \beta - \gamma$ where $\alpha = \tan^{-1}\omega/5$, $-\beta = -\tan^{-1}\omega/100$, and $-\gamma = -\tan^{-1}\omega/5000$

The corner frequencies are at $\omega = 5 \text{ r/s}$, $\omega = 100 \text{ r/s}$, and $\omega = 5000 \text{ r/s}$ where at those frequencies $\alpha = 45^\circ$, $-\beta = -45^\circ$, and $-\gamma = -45^\circ$ respectively. The asymptotes are shown as solid lines.

From the phase plot we observe that $\theta(30 \text{ r/s}) \approx 60^\circ$, $\theta(50 \text{ r/s}) \approx 53^\circ$, $\theta(100 \text{ r/s}) \approx 38^\circ$, and $\theta(5000 \text{ r/s}) \approx -39^\circ$



b. We use MATLAB for the computations.

```
theta_g30=(1+30j/5)/((1+30j/100)*(1+30j/5000));...
theta_g50=(1+50j/5)/((1+50j/100)*(1+50j/5000));...
theta_g100=(1+100j/5)/((1+100j/100)*(1+100j/5000));...
theta_g5000=(1+5000j/5)/((1+5000j/100)*(1+5000j/5000));...
fprintf(' \n');...
fprintf('theta30r = %5.2f deg. \t', angle(theta_g30)*180/pi);...
fprintf('theta50r = %5.2f deg. ', angle(theta_g50)*180/pi);...
fprintf(' \n');...
fprintf('theta100r = %5.2f deg. \t', angle(theta_g100)*180/pi);...
fprintf('theta5000r = %5.2f deg. ', angle(theta_g5000)*180/pi);...
fprintf(' \n')
```

```
theta30r = 63.49 deg. theta50r = 57.15 deg.
theta100r = 40.99 deg. theta5000r = -43.91 deg.
```

Thus, the actual values are

$$\angle G(j30) = \angle \frac{(1 + j30/5)}{(1 + j30/100) \cdot (1 + j30/5000)} = 63.49^\circ$$

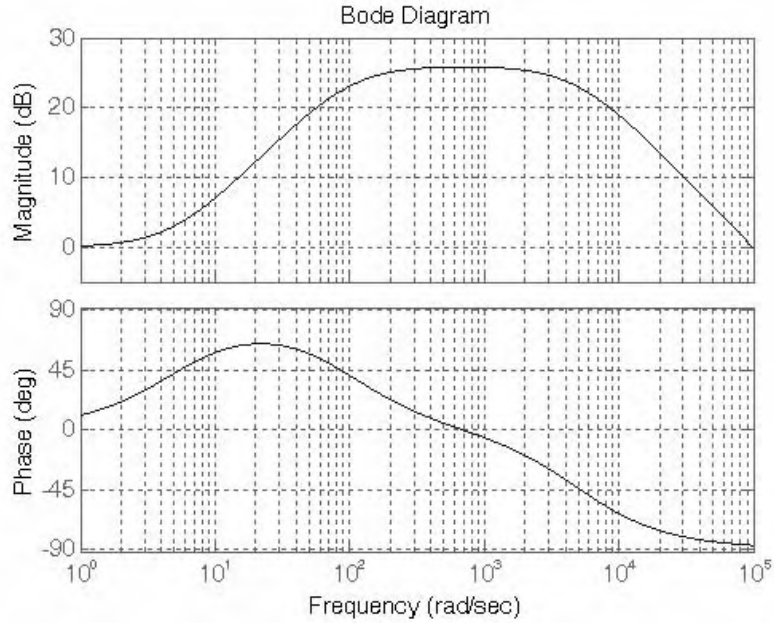
$$\angle G(j50) = \angle \frac{(1 + j50/5)}{(1 + j50/100) \cdot (1 + j50/5000)} = 57.15^\circ$$

$$\angle G(j100) = \angle \frac{(1 + j100/5)}{(1 + j100/100) \cdot (1 + j100/5000)} = 40.99^\circ$$

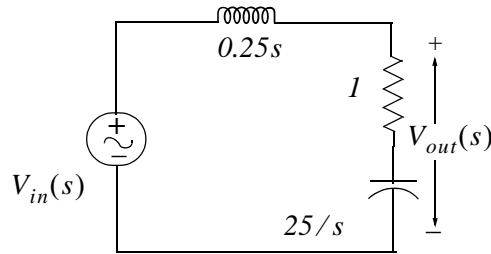
$$\angle G(j5000) = \angle \frac{(1 + j5000/5)}{(1 + j5000/100) \cdot (1 + j5000/5000)} = -43.91^\circ$$

c. The Bode plot generated with MATLAB is shown below.

```
syms s; expand((s+100)*(s+5000))
ans =
s^2+5100*s+500000
num=[0 10^5 5*10^5]; den=[1 5.1*10^3 5*10^5]; w=logspace(0,5,10^4);...
bode(num,den,w)
```



3. a. The equivalent s -domain circuit is shown below.



By the voltage division expression

$$V_{out}(s) = \frac{1 + 25/s}{0.25s + 1 + 25/s} \cdot V_{in}(s)$$

and

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{s + 25}{0.25s^2 + s + 25} = \frac{4(s + 25)}{s^2 + 4s + 100} \quad (1)$$

b. From (1) with $s = j\omega$

$$G(j\omega) = \frac{4(j\omega + 25)}{-\omega^2 + 4j\omega + 100} \quad (2)$$

From (7.53)

$$G(s) = \frac{C}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (3)$$

and from (1) and (3) $\omega_n^2 = 100$, $\omega_n = 10$, and $2\zeta\omega_n = 4$, $\zeta = 0.2$

Following the procedure of page 7-26 we let $u = \omega/\omega_n = \omega/10$. The numerator of (2) is a linear factor and thus we express it as $100(1 + j\omega/25)$. Then (2) is written as

$$G(j\omega) = \frac{100(1 + j\omega/25)}{100(-\omega^2/100 + 4j\omega/100 + 100/100)} = \frac{(1 + j\omega/25)}{1 - (\omega/10)^2 + j0.4\omega/10}$$

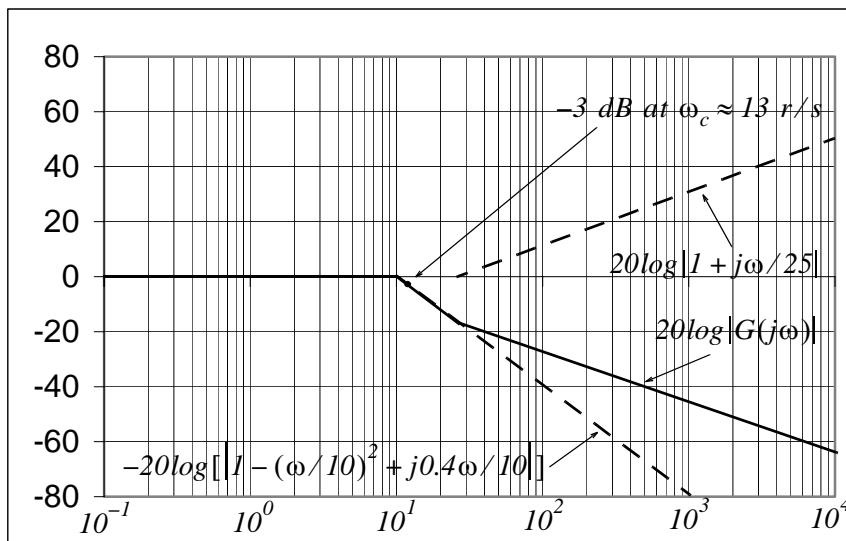
or

$$G(j\omega) = \frac{|1 + j\omega/25|\angle\theta}{|1 - (\omega/10)^2 + j0.4\omega/10|\angle\phi} \quad (4)$$

The amplitude of $G(j\omega)$ in dB is

$$20\log|G(j\omega)| = 20\log|1 + j\omega/25| - 20\log[|1 - (\omega/10)^2 + j0.4\omega/10|] \quad (5)$$

The asymptote of the first term on the right side of (5) has a corner frequency of 25 r/s and rises with slope of 20 dB/decade . The second term has a corner frequency of 10 r/s and rises with slope of -40 dB/decade . The amplitude plot is shown below.



- c. The plot above indicates that the circuit is a low-pass filter and the 3 dB cutoff frequency ω_c occurs at approximately 13 r/s .

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d. The actual cutoff frequency occurs where

$$|G(j\omega_c)| = |G(j\omega)|_{max} / \sqrt{2} = 1/(\sqrt{2}) = 0.70$$

At this frequency (2) is written as

$$G(j\omega_c) = \frac{100 + 4j\omega_c}{(100 - \omega_c^2) + 4j\omega_c}$$

and considering its magnitude we get

$$\frac{\sqrt{100^2 + (4\omega_c)^2}}{\sqrt{(100 - \omega_c^2)^2 + (4\omega_c)^2}} = \frac{1}{\sqrt{2}}$$

$$2[100^2 + (4\omega_c)^2] = (100 - \omega_c^2)^2 + (4\omega_c)^2$$

$$20000 + 32\omega_c^2 = 10000 - 200\omega_c^2 + \omega_c^4 + 16\omega_c^2$$

$$\omega_c^4 - 216\omega_c^2 - 10000 = 0$$

We will use MATLAB to find the four roots of this equation.

```
syms w; solve(w^4-216*w^2-10000)
```

```
ans =
```

```
[ 2*(27+1354^(1/2))^(1/2) ] [ -2*(27+1354^(1/2))^(1/2) ]  
[ 2*(27-1354^(1/2))^(1/2) ] [ -2*(27-1354^(1/2))^(1/2) ]
```

```
w1=2*(27+1354^(1/2))^(1/2)
```

```
w1 =  
15.9746
```

```
w2=-2*(27+1354^(1/2))^(1/2)
```

```
w2 =  
-15.9746
```

```
w3=2*(27-1354^(1/2))^(1/2)
```

```
w3 =  
0.0000 + 6.2599i
```

```
w4=-2*(27-1354^(1/2))^(1/2)
```

```
w4 =  
-0.0000 - 6.2599i
```

From these four roots we accept only the first, that is, $\omega_c \approx 16 \text{ r/s}$

e. From (4)

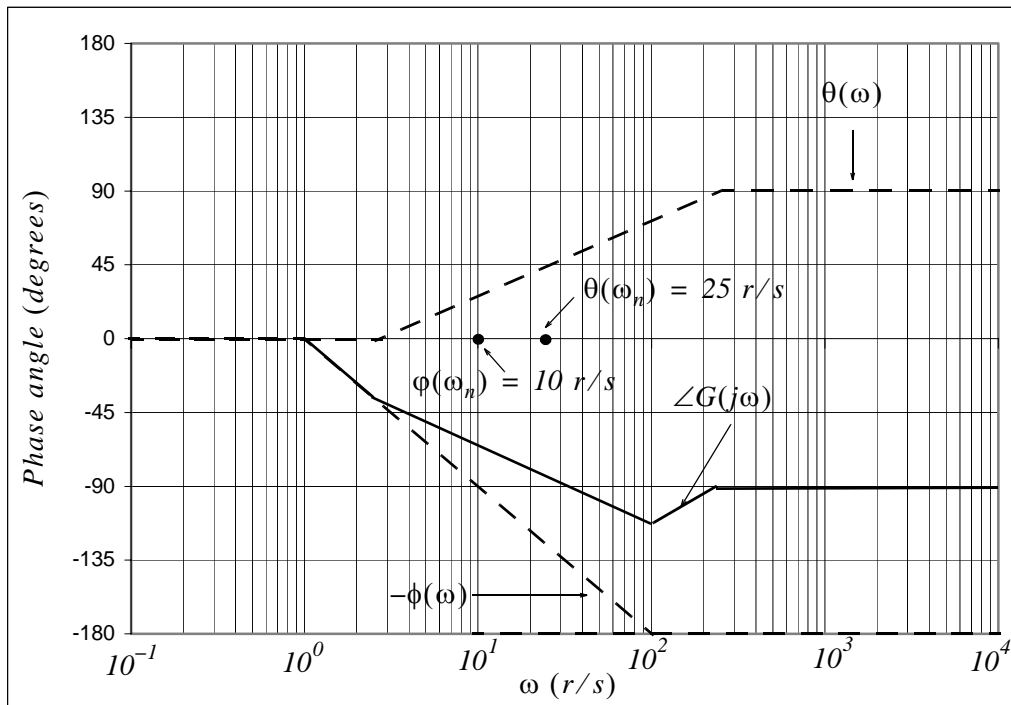
$$\theta = \tan^{-1}(\omega/25)$$

and

$$\phi = \frac{0.4\omega/10}{1 - (\omega/10)^2}$$

For a first order zero or pole not at the origin, the straight line phase angle plot approximations are as follows:

- I. For frequencies less than one tenth the corner frequency we assume that the phase angle is zero. For this exercise the corner frequency of $\theta(\omega)$ is $\omega_n = 25 \text{ r/s}$ and thus for $1 \leq \omega \leq 2.5 \text{ r/s}$ the phase angle is zero as shown on the Bode plot below.



- II For frequencies ten times or greater than the corner frequency, the phase angle is approximately $\pm 90^\circ$. The numerator phase angle $\theta(\omega)$ is zero at one tenth the corner frequency, it is 45° at the corner frequency, and 90° for frequencies ten times or greater the corner frequency. For this exercise, in the interval $2.5 \leq \omega \leq 250 \text{ r/s}$ the phase angle is zero at 2.5 r/s and rises to 90° at 250 r/s .

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III As shown in Figure 7.20, for complex poles the phase angle is zero at zero frequency, -90° at the corner frequency and approaches -180° as the frequency becomes large. The phase angle asymptotes are shown on the plot of the previous page.

f. From the plot of the previous page we observe that the phase angle at the cutoff frequency is approximately -63°

g. The exact phase angle at the cutoff frequency $\omega_c = 16 \text{ r/s}$ is found from (1) with $s = j16$.

$$G(j16) = \frac{4(j16 + 25)}{(j16)^2 + 4(j16) + 100}$$

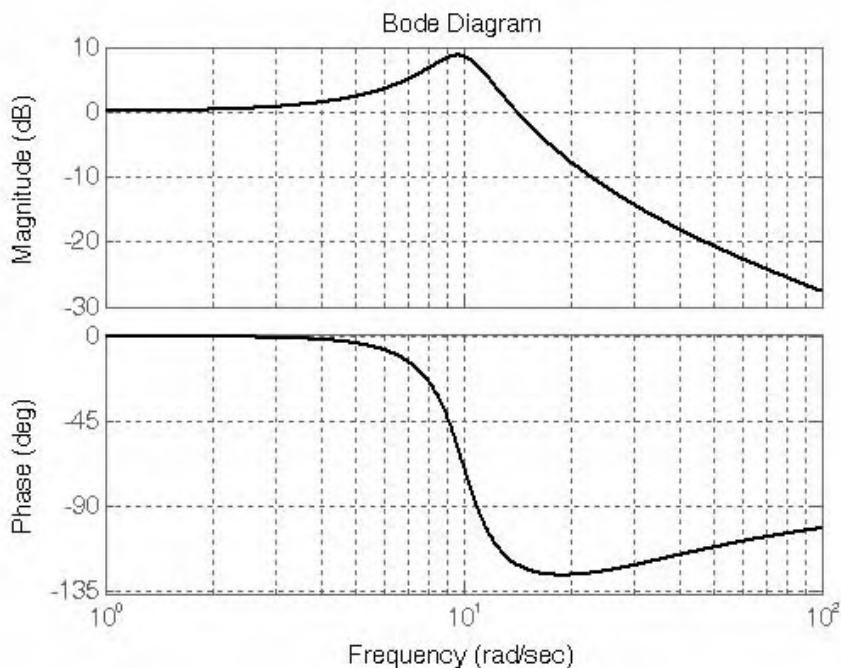
We need not simplify this expression since we can use MATLAB.

```
g16=(64j+100)/((16j)^2+64j+100); angle(g16)*180/pi
```

```
ans =  
-125.0746
```

This value is about twice as that we observed from the asymptotic plot of the previous page. Errors such as this occur because of the high non-linearity between frequency intervals. Therefore, we should use the straight line asymptotes only to observe the shape of the phase angle. It is best to use MATLAB as shown below.

```
num=[0 4 100]; den=[1 4 100]; w=logspace(0,2,1000);bode(num,den,w)
```



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