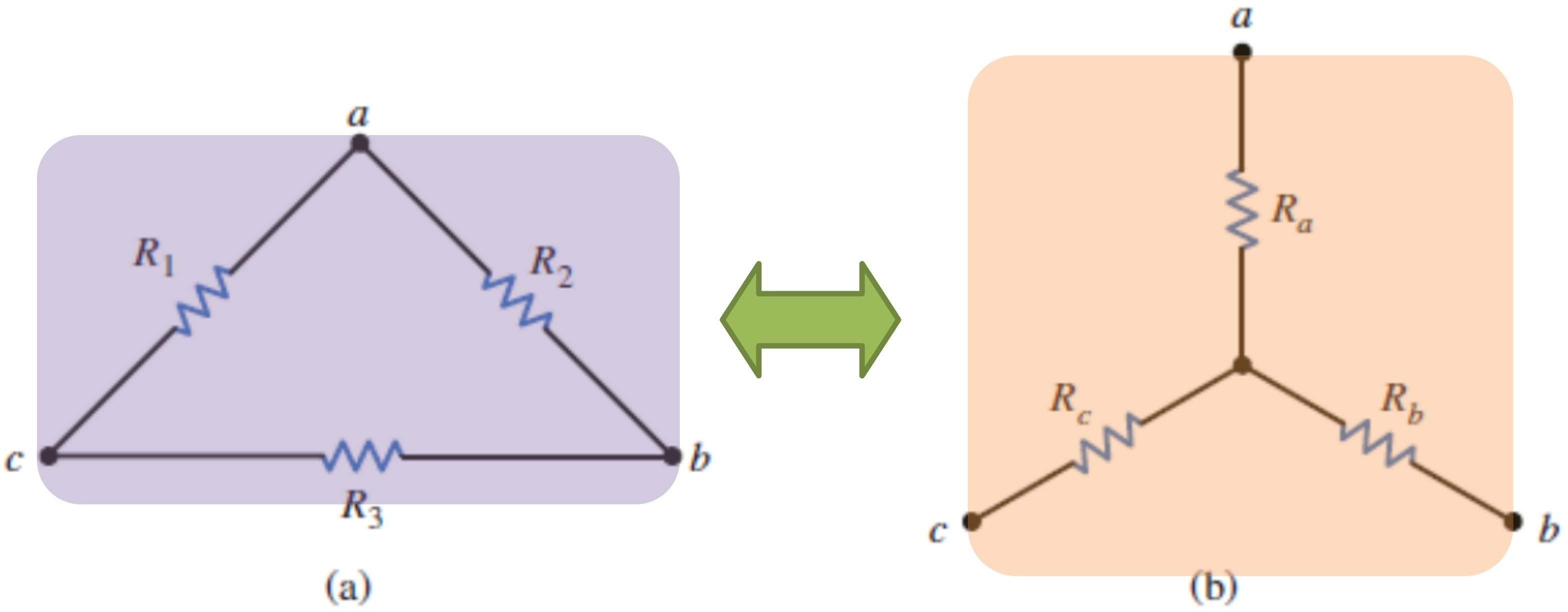
# Last Lecture → Wye ⇆ Delta Transformations

**For the two networks to be equivalent at each corresponding pair of terminals, it is necessary that the resistance at the corresponding terminals be equal (e.g., the resistance at terminals a and b with c open-circuited must be the same for both networks.**

$\Delta \leftarrow Y$

$$\begin{cases} R_1 = \dfrac{R_aR_b + R_bR_c + R_aR_c}{R_b} \\ R_2 = \dfrac{R_aR_b + R_bR_c + R_aR_c}{R_c} \\ R_3 = \dfrac{R_aR_b + R_bR_c + R_aR_c}{R_a} \end{cases}$$




$Y \leftarrow \Delta$

$$\begin{cases} R_a = \dfrac{R_1R_2}{R_1 + R_2 + R_3} \\ R_b = \dfrac{R_2R_3}{R_1 + R_2 + R_3} \\ R_c = \dfrac{R_1R_3}{R_1 + R_2 + R_3} \end{cases}$$

# Last Lecture → Nodal Analysis



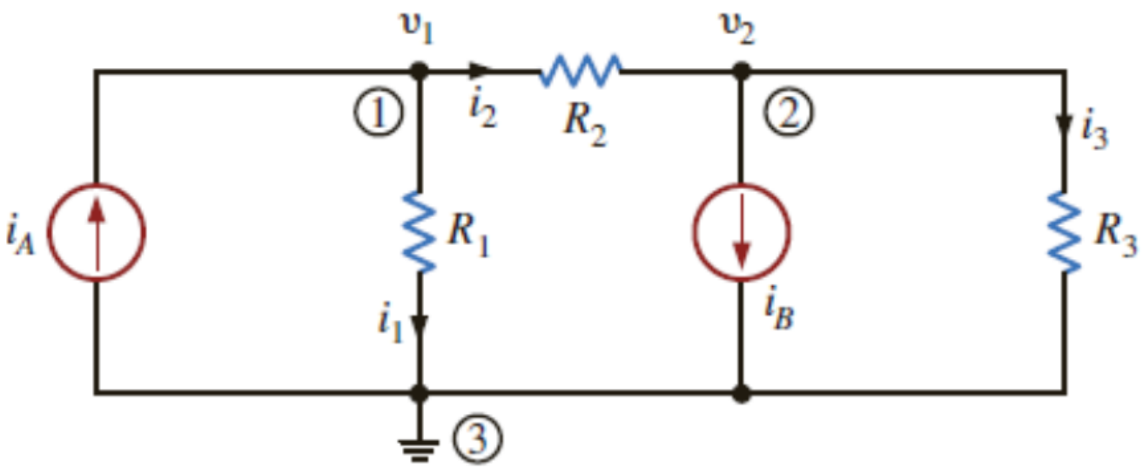
1) **Identify #of nodes:** 3 node circuit
2) **Select reference node:** bottom node, 3
3) **Label other node voltages:** $v_1$, $v_2$
4) **Identify branch currents:** $i_1$, $i_2$, $i_3$
5) **Apply KCL to nodes:** 1, 2 → 2 independent equations

**assume:** $I_A = 1mA$, $I_B = 4mA$, $R_1 = 12k\Omega$, $R_2 = 6k\Omega$, $R_3 = 6k\Omega$

**KCLs:**

$$I_A = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2}\right]$$

$$I_B = I_2 + I_3 = \frac{V_1 - V_2}{R_2} - \frac{V_2}{R_3} = V_1\left[\frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2} + \frac{1}{R_3}\right]$$



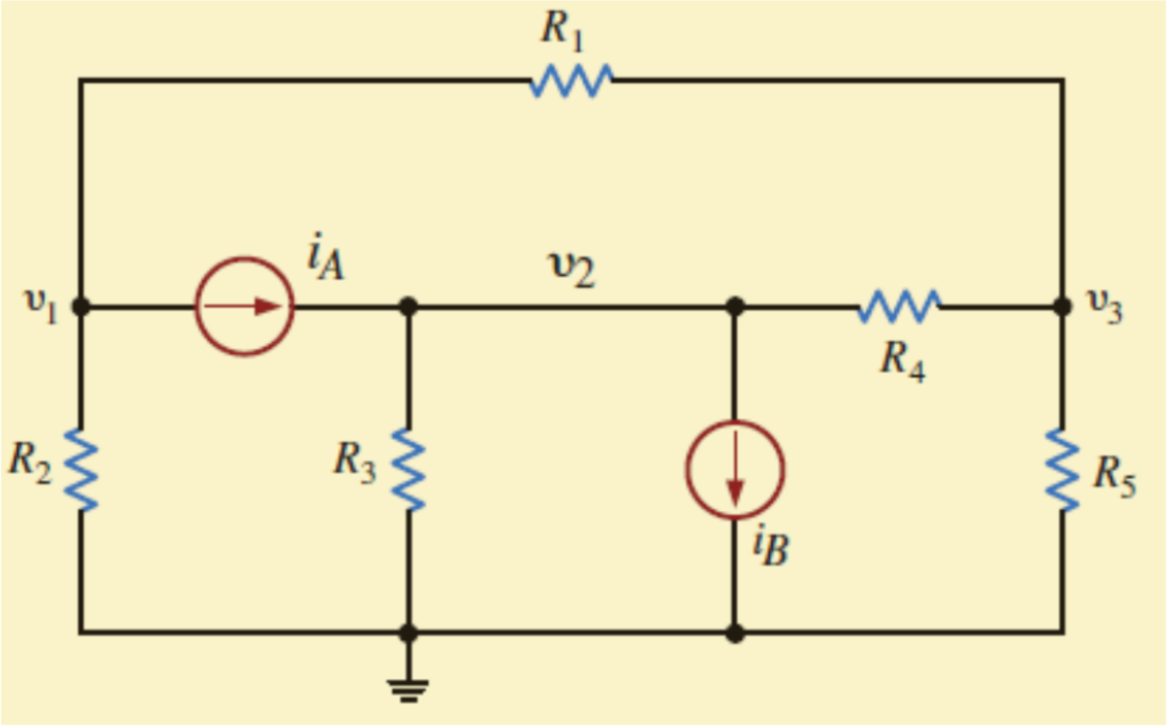

**matrix eq.**

$$\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1 \| R_2} & -\frac{1}{R_2} \\ \frac{1}{R_2} & -\frac{1}{R_2 \| R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

# Nodal Analysis → Example 3.2

**Write the equations in matrix form for the provided circuit. Assume $R_1=R_2=2k\Omega$, $R_3=R_4=4k\Omega$, $R_5=1k\Omega$, $i_A=4mA$, and $i_B=2mA$.**

1) ~~Identify #of nodes~~
2) ~~Select reference node~~
3) ~~Label other node voltages~~
4) Identify branch currents
5) Apply KCL to nodes



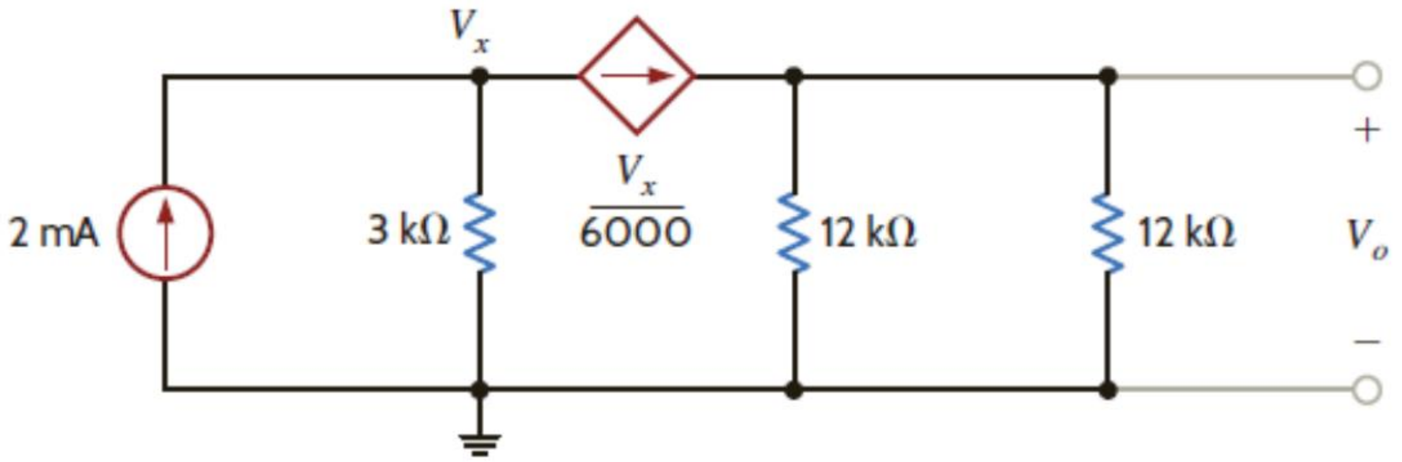

**matrix eq.**

$$\begin{bmatrix} I_A \\ I_A - I_B \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 \| R_2} & 0 & \frac{1}{R_1} \\ 0 & \frac{1}{R_3 \| R_4} & -\frac{1}{R_4} \\ \frac{1}{R_1} & \frac{1}{R_4} & -\frac{1}{R_1 \| R_4 \| R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

# Nodal Analysis → with Dependent Current Sources



**E3.3: Find the voltage $V_o$ in the network provided.**




$$KCL\ @\ Vx \rightarrow 2m - \frac{V_x}{3k} - \frac{V_x}{6k} = 0$$

$$KCL\ @\ V_0 \rightarrow \frac{V_x}{6k} - \frac{V_0}{12k} - \frac{V_0}{12k} = 0$$

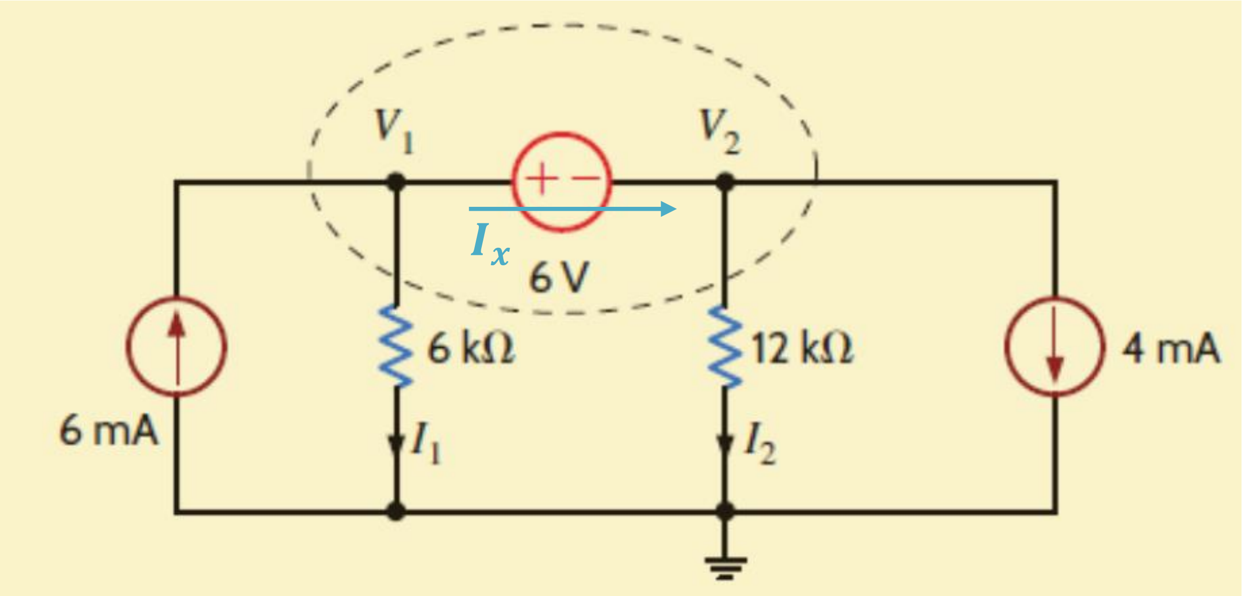
$$V_x = 4\ V$$

$$V_0 = V_x = 4\ V$$

# Nodal Analysis → with Independent Voltage Sources

**Supernode:** KCL at the surface (dashed) around the voltage source.

**Constraint equation:** establishes the difference in potential between two nodes.




$$V_1 = 10\,V$$

$$V_2 = 4\,V$$

$$Constraint\ Eq. \rightarrow 6 = V_1 - V_2$$

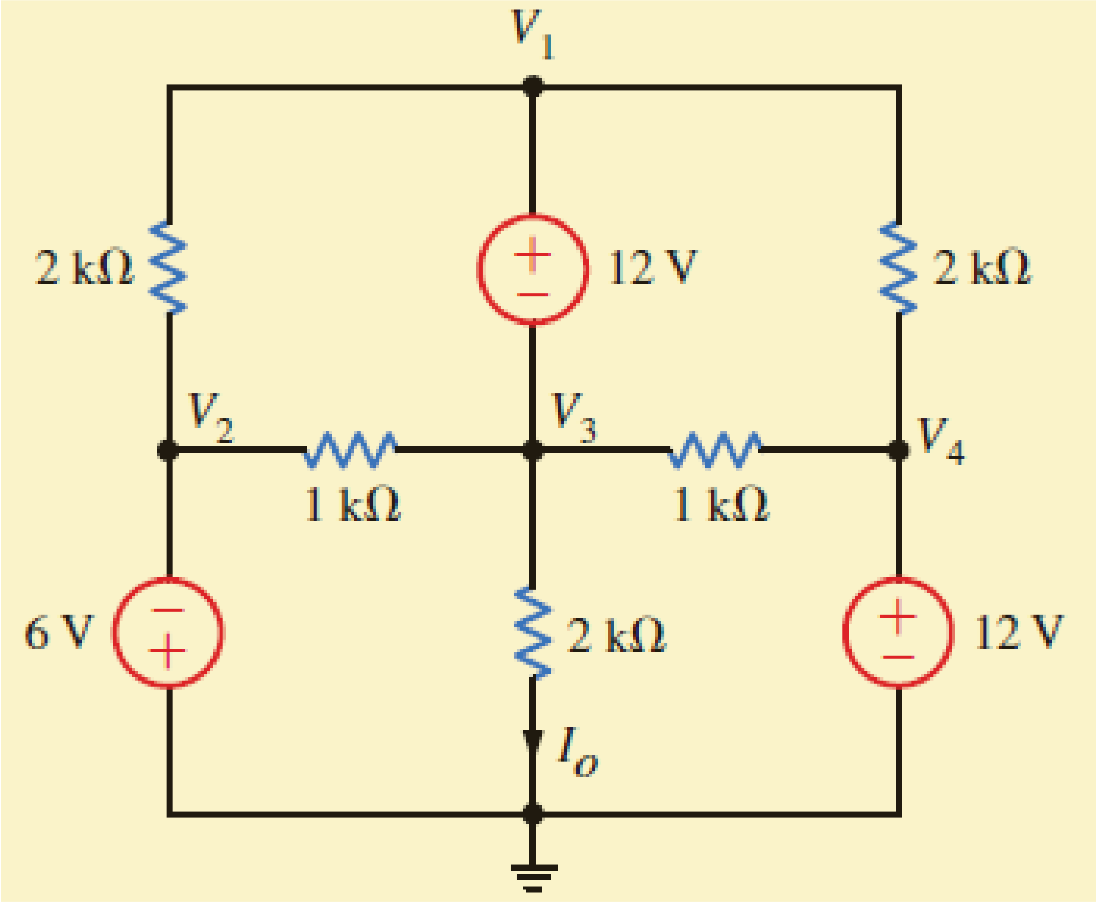
$$KCL\ @\ V_1 \rightarrow 6m - \frac{V_1}{6k} - I_x = 0$$

$$KCL\ @\ V_2 \rightarrow I_x - \frac{V_2}{12k} - 4m = 0$$

$$KCL\ @\ Supernode \rightarrow 6m - \frac{V_1}{6k} - \frac{V_2}{12k} - 4m = 0$$

# Nodal Analysis → with Independent Voltage Sources

**Example 3.7: Determine the current $I_0$ in the provided network.**




**KCL @ supernode**

$$\frac{-6-V_1}{2k}+\frac{-6-V_3}{1k}=\frac{V_1-12}{2k}+\frac{V_3-12}{1k}+\frac{V_3}{2k}$$

**Constraint**

$$12=V_1-V_3$$

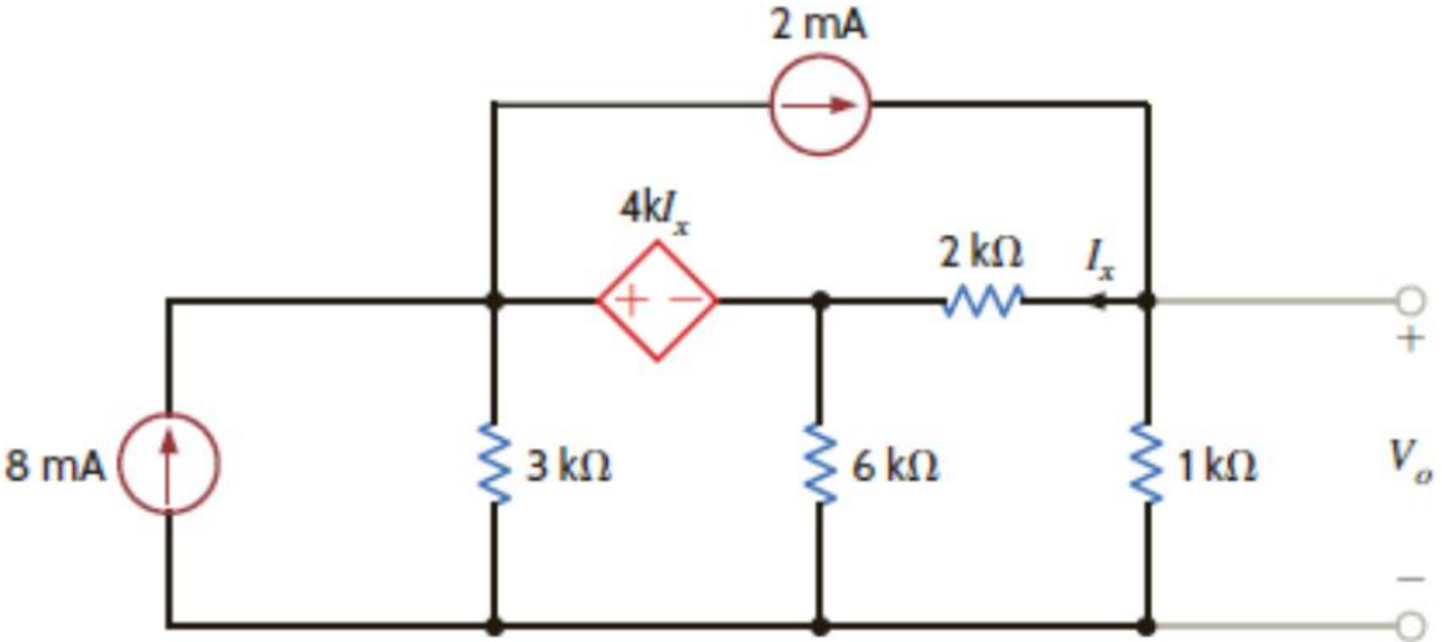
$$V_3=-\frac{6}{7}V$$

$$I_0=\frac{V_3}{2k}=-\frac{3}{7}mA$$

# Nodal Analysis → with Dependent Voltage Sources



**E3.12: Determine the current $V_0$ in the provided network.**




**KCL @ supernode**

$$8m - 2m = \frac{V_1}{3k} + \frac{V_2}{6k} + \frac{V_2 - V_0}{2k}$$

**KCL @ node 2**

$$2m = \frac{V_0}{1k} - \frac{V_2 - V_0}{2k}$$

**Constraint**

$$I_x = \frac{V_0 - V_2}{2k}$$

**Dependent Source**

$$V_1 - V_2 = 4k \cdot I_x$$

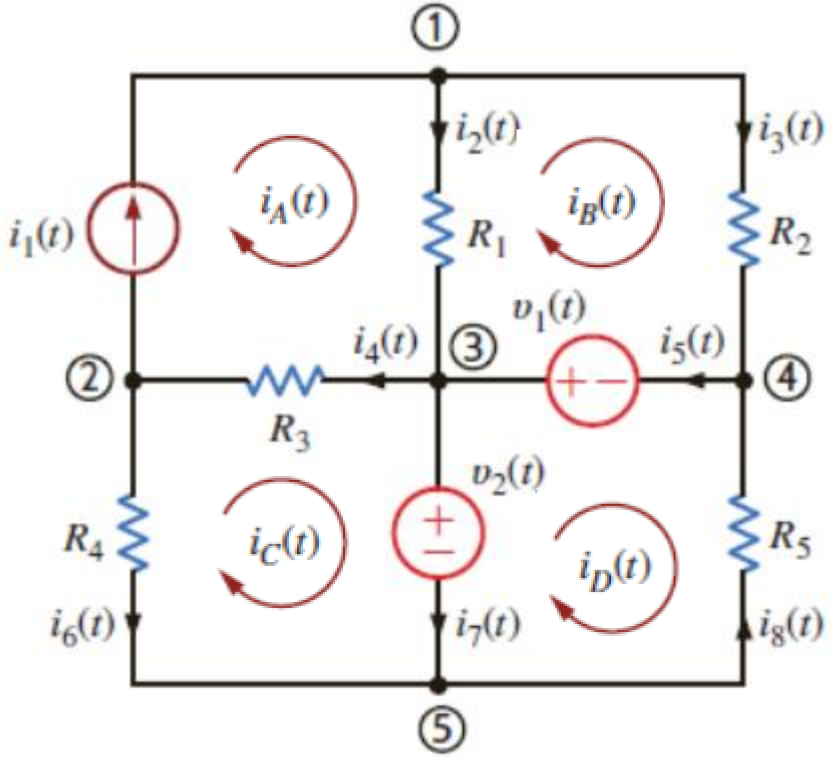
# Mesh Analysis

***Alternative # 1***

- ***B → # of branches***
- ***N → # of nodes***
- ***B-N+1 → # independent simultaneous equations***

***Alternative # 2***

- ***M → # of independent loops in a planar circuit***
- ***M → # independent simultaneous equations***




***B = 8    M = 4***
***N = 5***
***# Eq. = 4***

***Stablish the currents around the loops***

$$i_A(t) = i_1(t)$$
$$v_1 = V_{R_1} + V_{R_2}$$
$$-v_2 = V_{R_3} + V_{R_4}$$
$$v_2 - v_1 = V_5$$

***Express voltages in terms of currents: $i_A(t)$, $i_B(t)$, $i_C(t)$, and $i_D(t)$***

*KVLs according to the current around the loop*