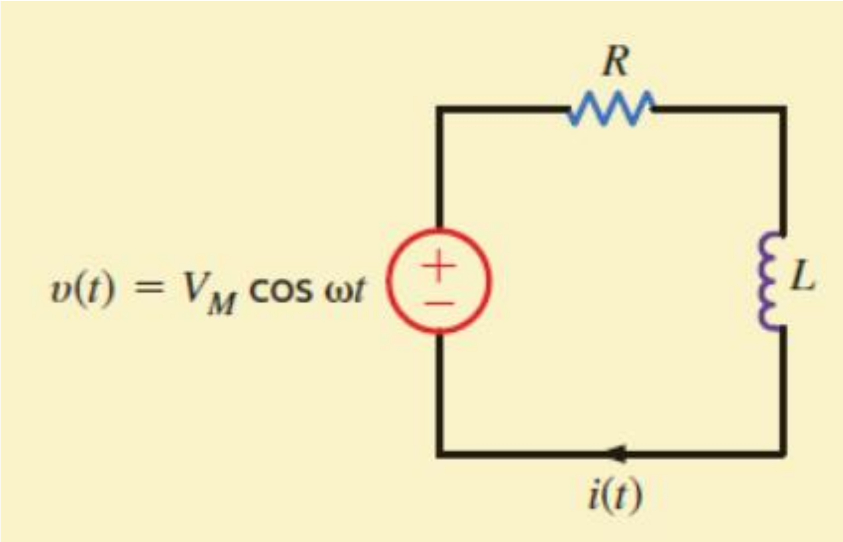
Last Lecture → Solving Circuits in Time Domain

Derive the expression for the current i(t) using the forcing function $v(t) = V_M e^{j\omega t}$ instead.



response

$$i(t) = I_M e^{j(\omega t + \varphi)}$$

KVL

$$v(t) = R \cdot i(t) + L\frac{di(t)}{dt}$$

Differential Equation

$$V_M e^{j\omega t} = R I_M e^{j(\omega t + \varphi)} + L\frac{d}{dt}\left(I_M e^{j(\omega t + \varphi)}\right)$$

Solution

$$\therefore I_M = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\therefore \varphi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i(t) = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

Last Lecture → Phasors

Representation of a complex number with just the magnitude and the phase angle.

- ***Forcing Function***

$$v(t) = V_M e^{j(\omega t+\theta)}$$

- ***Steady State Response***

$$i(t) = I_M e^{j(\omega t+\varphi)}$$

$e^{j(\omega t)}$

common to every term

$$v(t) = V_M \cos(\omega t + \theta) = Re\left[V_M e^{j(\omega t+\theta)}\right]$$

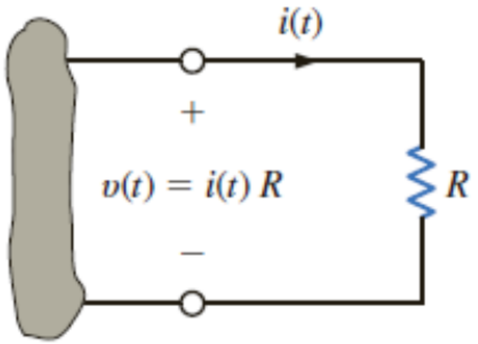
$$V = Re\left[V_M \langle\theta\ e^{j\omega t}\right] = V_M \langle\theta$$

phasor

Phasor Relationships → Resistor

- $v(t) = V_M e^{j(\omega t + \theta_v)}$
- $i(t) = I_M e^{j(\omega t + \theta_i)}$

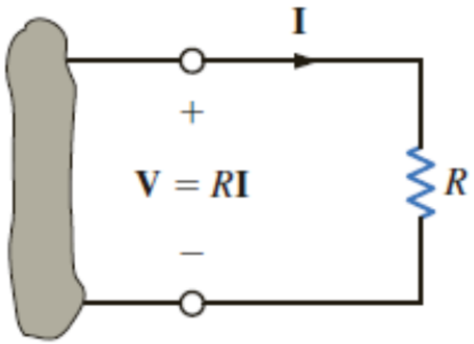
- *Time Domain*



$v(t) = R \cdot i(t)$

$V_M e^{j(\omega t + \theta_v)} = R \cdot I_M e^{j(\omega t + \theta_i)}$

- *Frequency Domain*



$\therefore V_M e^{j\theta_v} = R \cdot I_M e^{j\theta_i}$

↳ $V_M \angle \theta_v = R \cdot I_M \angle \theta_i$

$$\frac{V_M \angle \theta_v}{I_M \angle \theta_i} = R$$

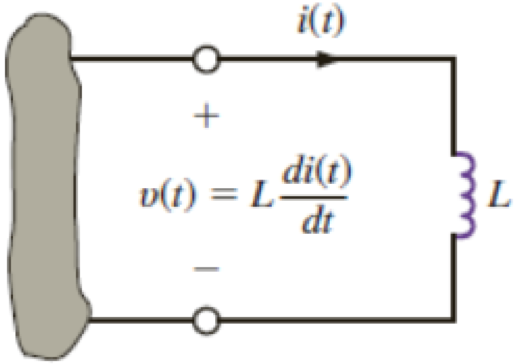
$\therefore \theta_v = \theta_i$

→ $v(t)$ *and* $i(t)$ *are in phase*

Phasor Relationships → Inductor

- $v(t) = V_M e^{j(\omega t+\theta_v)}$
- $i(t) = I_M e^{j(\omega t+\theta_i)}$

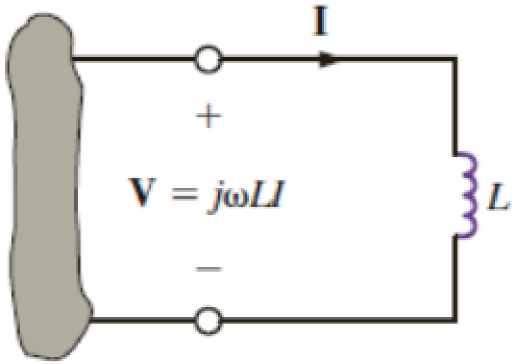
- *Time Domain*



$$v(t) = L \cdot \frac{di(t)}{dt}$$

$$V_M e^{j(\omega t+\theta_v)} = L \cdot \frac{d}{dt} I_M e^{j(\omega t+\theta_i)}$$

- *Frequency Domain*



$$\therefore V_M e^{j\theta_v} = j\omega L \cdot I_M e^{j\theta_i}$$

$$V_M \langle \theta_v = j\omega L \cdot I_M \langle \theta_i$$

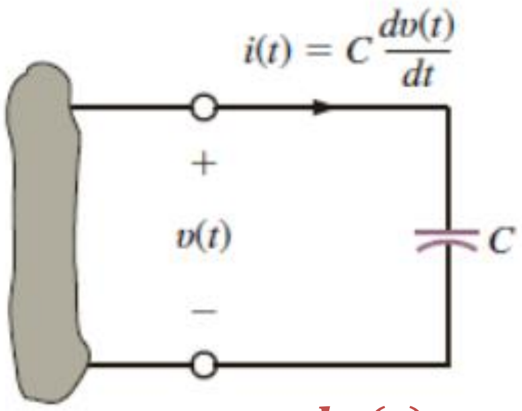
$$\frac{V_M \langle \theta_v}{I_M \langle \theta_i} = j\omega L$$

$$\therefore \theta_v = \theta_i + 90^o$$

$$\rightarrow v(t) \text{ leads } i(t) \text{ by } 90^o$$

Phasor Relationships → Capacitance

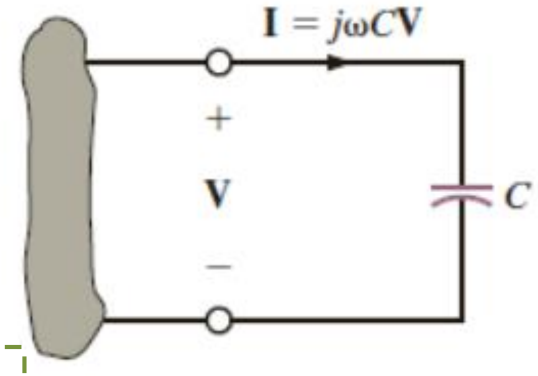
- *Time Domain*



$$i(t) = C \cdot \frac{dv(t)}{dt}$$

$$I_M e^{j(\omega t+\theta_i)} = C \cdot \frac{d}{dt} V_M e^{j(\omega t+\theta_v)}$$

- *Frequency Domain*



$$\therefore I_M e^{j\theta_i} = j\omega C \cdot V_M e^{j\theta_v}$$

$$I_M\langle\theta_i = j\omega C \cdot V_M\langle\theta_v$$

- $v(t) = V_M e^{j(\omega t+\theta_v)}$
- $i(t) = I_M e^{j(\omega t+\theta_i)}$

$$\frac{V_M\langle\theta_v}{I_M\langle\theta_i} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$\therefore \theta_v = \theta_i - 90^o$

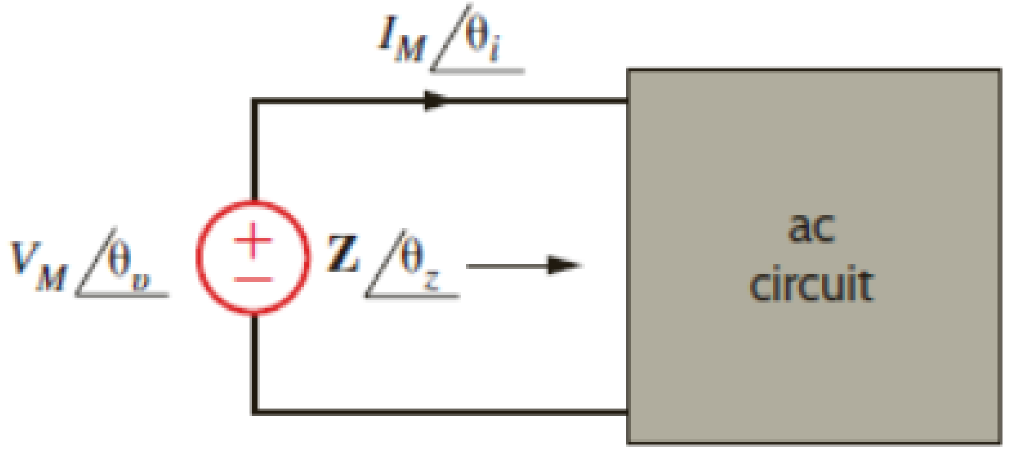
$\rightarrow v(t)$ *lags* $i(t)$ *by* 90^o

Impedance

The ratio of the phasor voltage V to the phasor current I.

$$Z = \frac{V}{I} \; [Ohms]$$

$$= \frac{V_M \angle \theta_v}{I_M \angle \theta_i} = \frac{V_M}{I_M} \angle(\theta_v - \theta_i) = Z \angle \theta_z$$



$$Z \angle \theta_z = R + jX$$

Resistance *Reactance*

Polar Coordinates

$$Z = \sqrt{R^2 + X^2}$$

$$\angle \theta_z = \tan^{-1}\left(\frac{X}{R}\right)$$

Rectangular Coordinates

$$R = Z \cos \theta_z$$

$$X = Z \sin \theta_z$$

Impedance

The ratio of the phasor voltage V to the phasor current I.

Series → Equivalent Impedance

$$Z_s = Z_1 + Z_2 + \cdots + Z_n$$

PASSIVE ELEMENT	IMPEDANCE
R	$\mathbf{Z} = R$
L	$\mathbf{Z} = j\omega L = jX_L,\ X_L = \omega L$
C	$\mathbf{Z} = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -jX_C,\ X_C = \frac{1}{\omega C}$

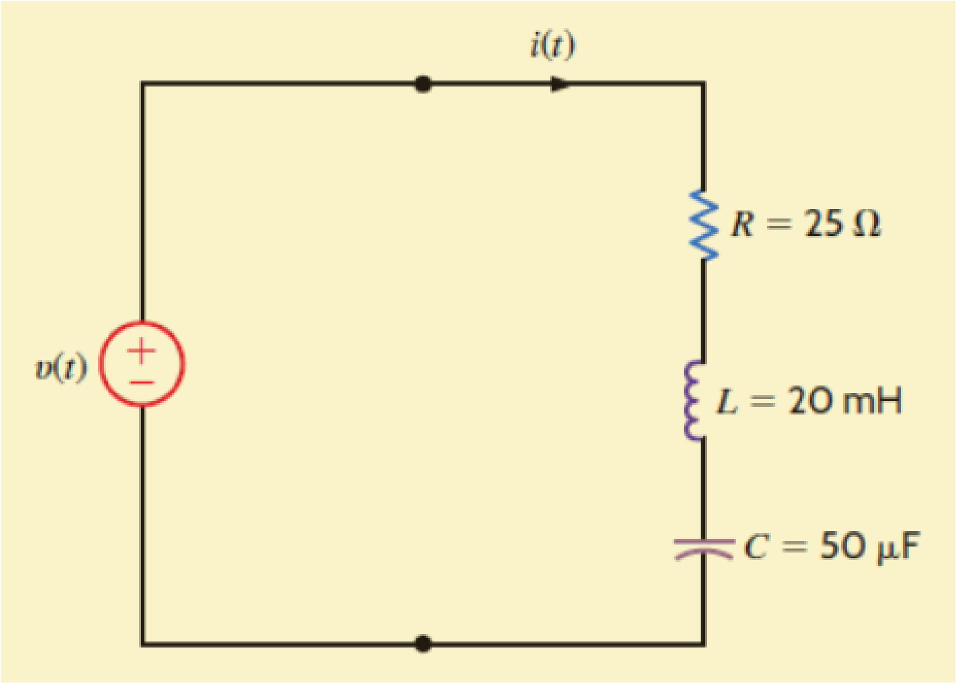
Parallel → Equivalent Impedance

$$\frac{1}{Z_p} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

KVL & KCL are valid in the frequency domain!

Example 8.9

Determine the equivalent impedance of the network provided if the frequency is f=60Hz. Then compute the current i(t) if the voltage source is $v(t) = 50\cos(\omega t + 30^o)\ V.$



Admittance

The ratio of the phasor current I to the phasor voltage V.

$$Y = \frac{I}{V} = \frac{1}{Z}\ [Siemens]$$

$$Y \angle \theta_y = G + jB$$

G ↑ *Conductance* jB ↑ *Susceptance*

Parallel → Equivalent Admittance

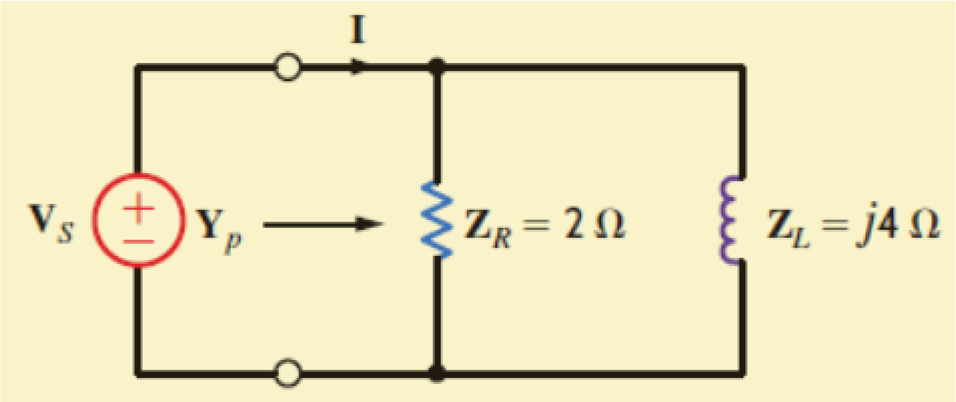
$$Y_p = Y_1 + Y_2 + \cdots + Y_n$$

Series → Equivalent Admittance

$$\frac{1}{Y_s} = \frac{1}{Y_1} + \frac{1}{Y_2} + \cdots + \frac{1}{Y_n}$$

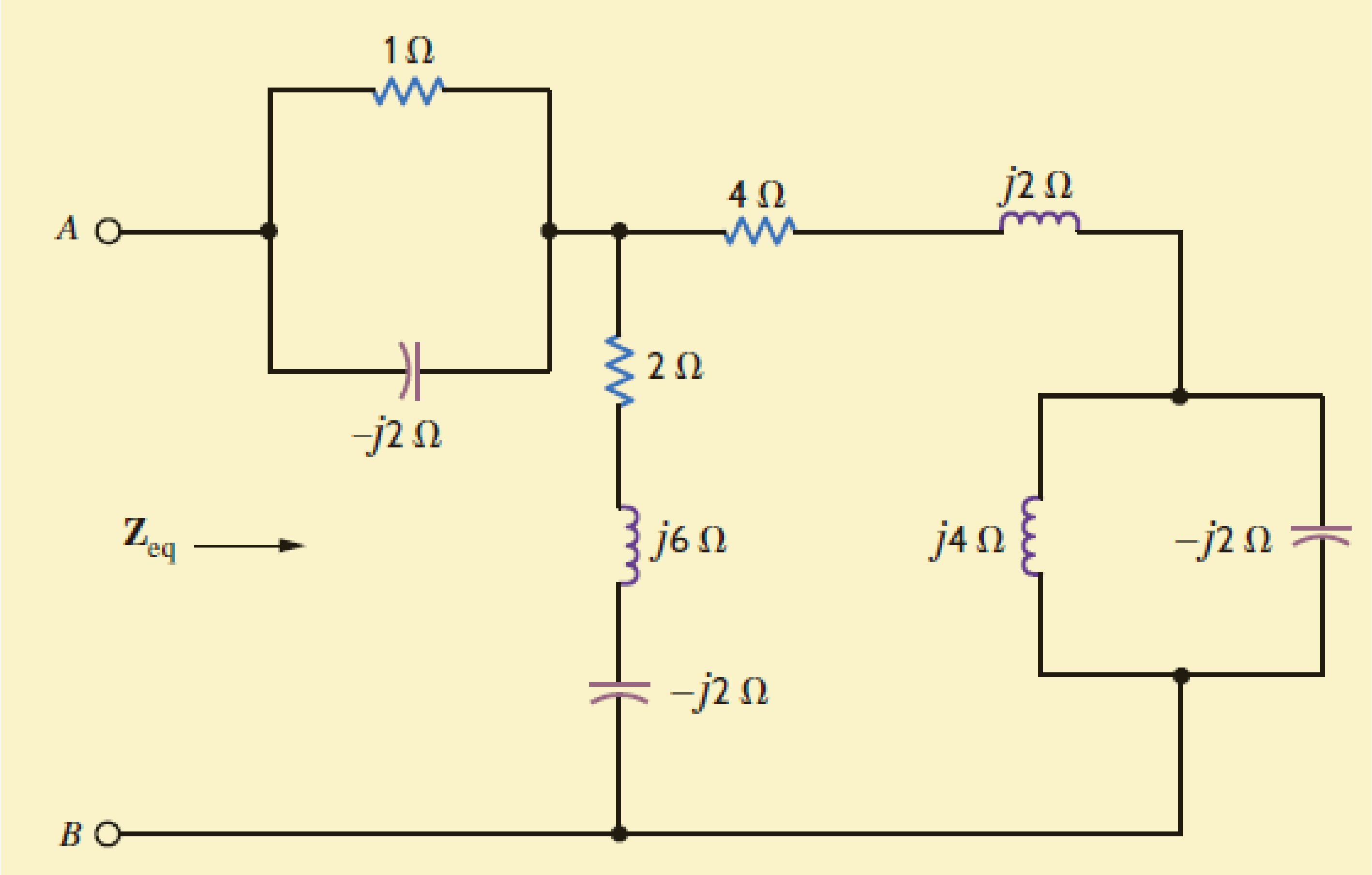
Example 8.10

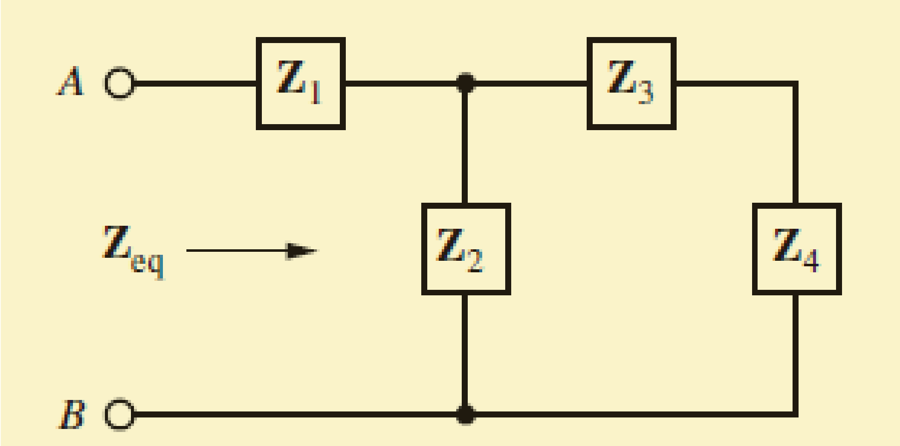
Calculate the equivalent admittance Y_p for the network provided and use it to determine the current I if $V_s = 60\langle 45^o$.



Example 8.11

For the given circuit calculate the equivalent impedance Z_{eq} at terminals A-B.



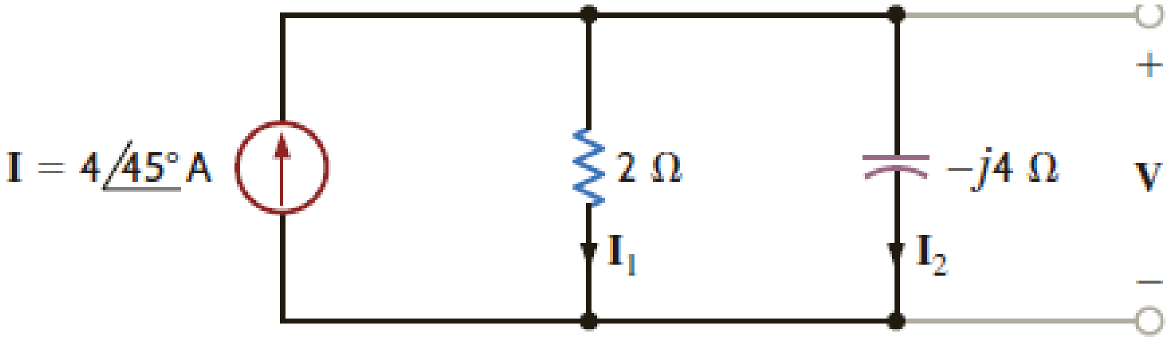


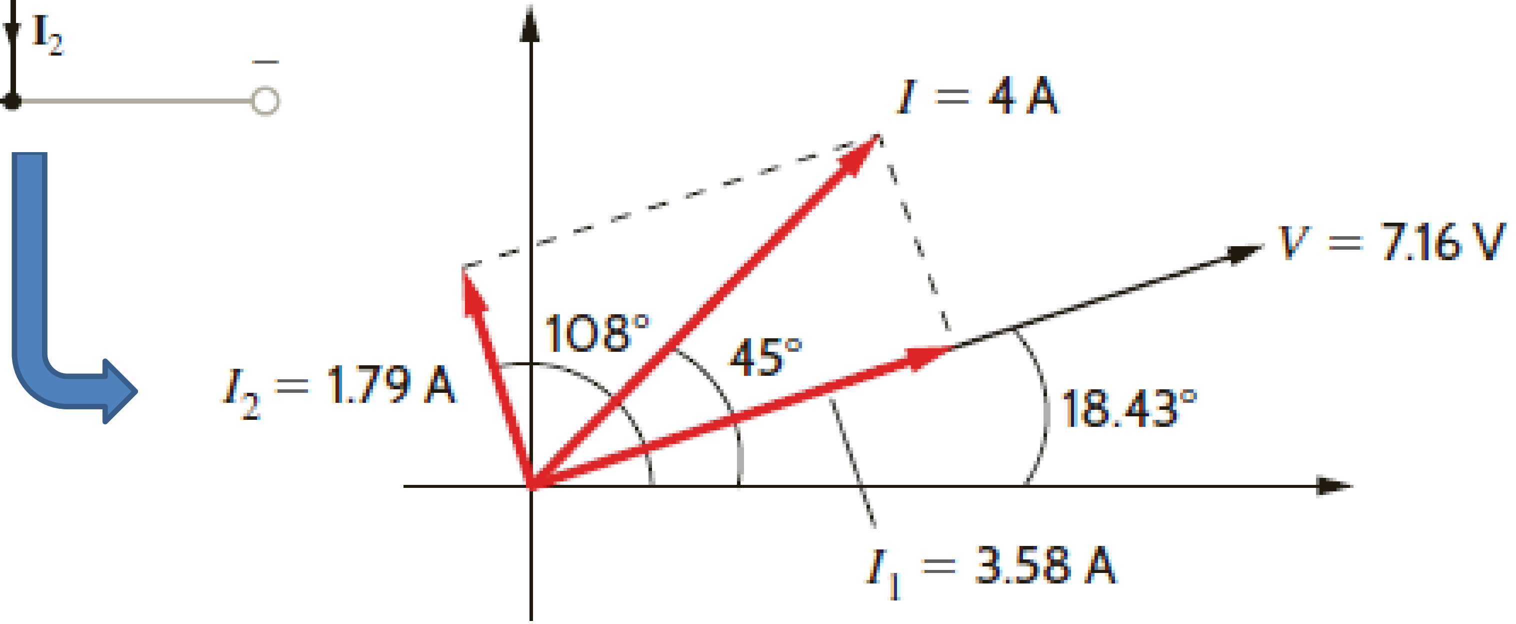
Technique for taking the reciprocal…

$$\frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$

Learning Assessment E8.12

Draw the phasor diagram to illustrate all currents and voltages for the network provided.





Problem 8.25

The admittance of the box in the figure provided is $0.1 + j0.2$ S at 500 rad/s. What is the impedance at 300 rad/s?