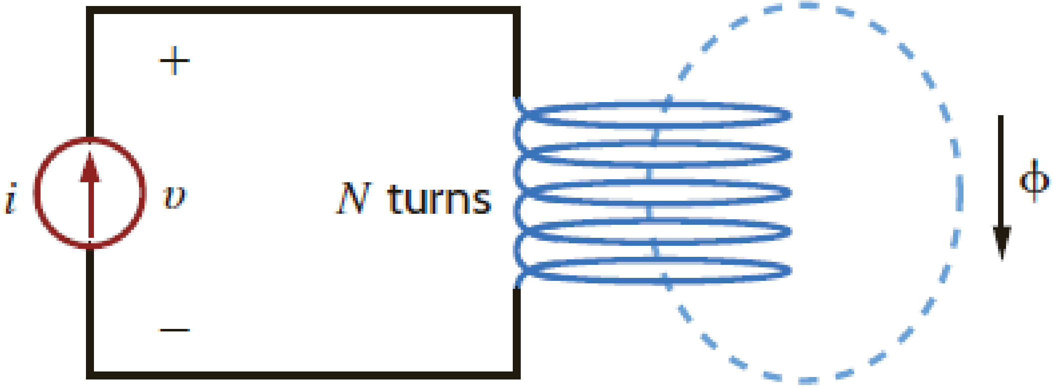
Magnetically Coupled Networks → Chapter #10

- **Mutual Inductance / Coefficient of Coupling / Turns Ration**
- **Circuit Analysis with Mutual Inductance**
- **Circuit Analysis with Ideal Transformers**

Single Coil Behavior



Flux Linkage → λ

$$\lambda = N \cdot \phi$$

coil turns (N)

magnetic flux (ϕ)

$$\lambda = L \cdot i$$

$$\phi = \frac{L}{N} i$$

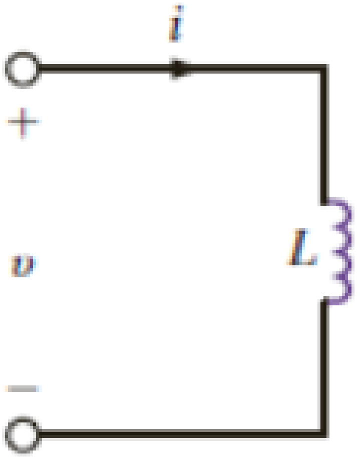
Faraday's Law → $v = f(\lambda)$

$$v = \frac{d\lambda}{dt}$$

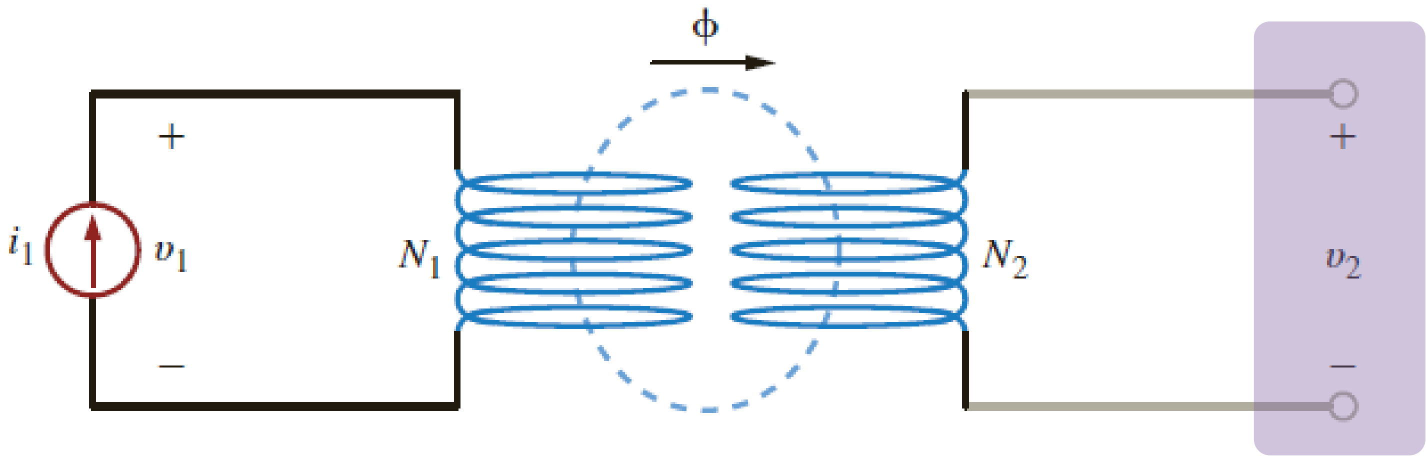
$$= L\frac{di}{dt} + i\frac{dL}{dt}$$

$L \neq f(t)$

$$v = L\frac{di}{dt}$$



Two Coils → Magnetically Coupled



No Load!
→ $i_2 = 0$

$$\lambda_1 = N_1 \cdot \phi = L_1 \cdot i_1$$

$$v_1 = \frac{d\lambda_1}{dt}$$

$$\lambda_2 = N_2 \cdot \phi$$

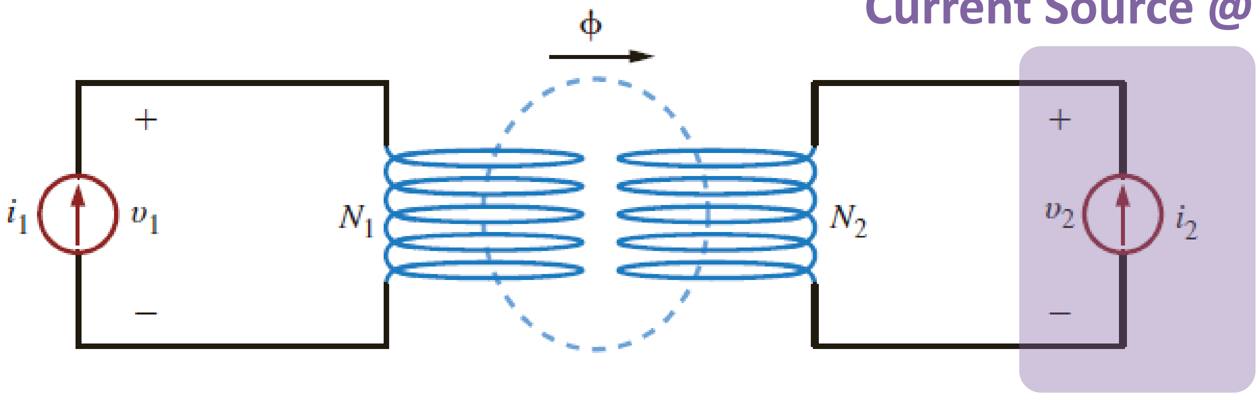
$$v_2 = \frac{d\lambda_2}{dt}$$

L_{21} → Mutual Inductance

$$v_2 = \frac{d\lambda_2}{dt} = \frac{d}{dt}(N_2 \cdot \phi) = \frac{d}{dt}\left(\frac{N_2}{N_1} \cdot \lambda_1\right) = \frac{d}{dt}\left(\frac{N_2}{N_1} \cdot L_1 \cdot i_1\right) = \frac{N_2}{N_1} L_1 \frac{di_1}{dt}$$

Induced Voltage

Magnetically Coupled Coils



$$v_1 = L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

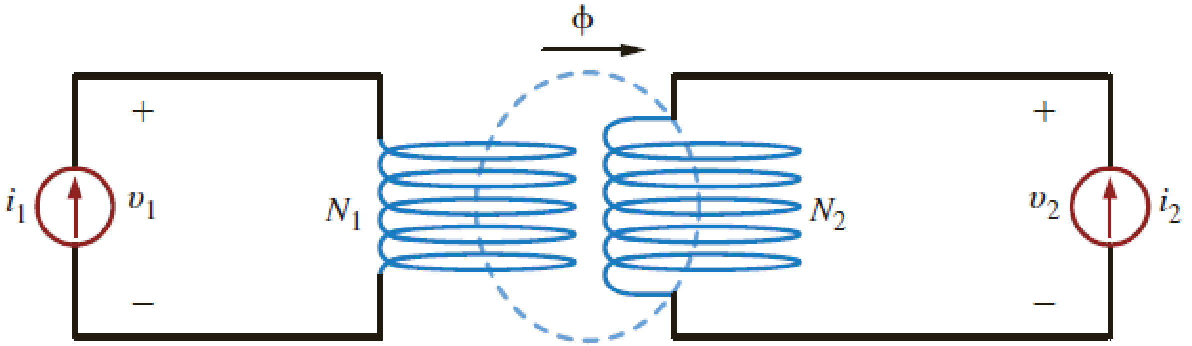
$$v_2 = L_2 \frac{di_2}{dt} + L_{21} \frac{di_1}{dt}$$

Self Term   Mutual Term

$$L_{12} = L_{21} = M$$

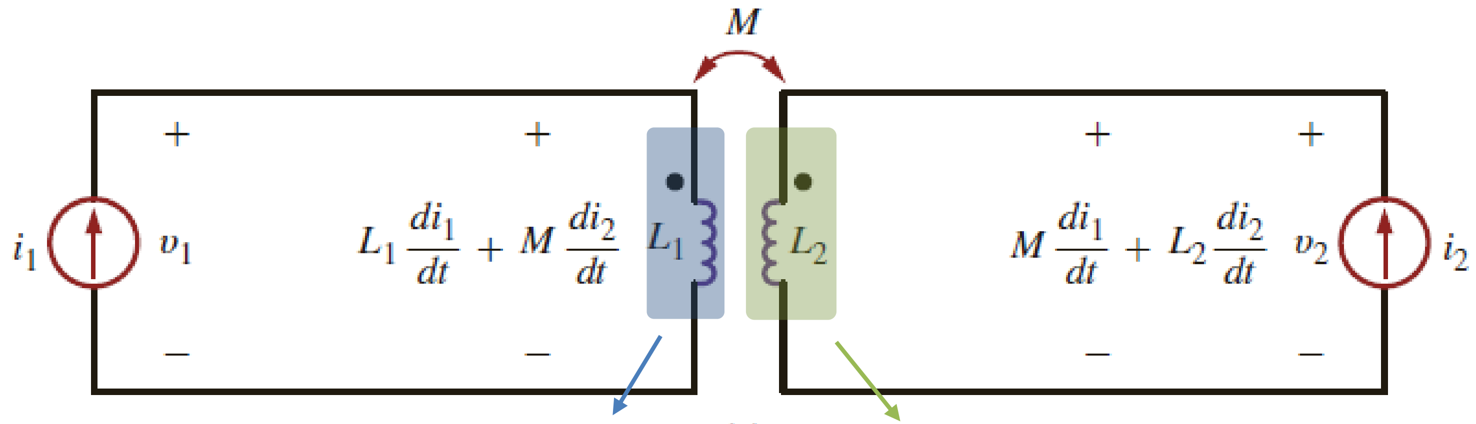
Using Superposition

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Magnetically Coupled Coils → Circuit Diagram

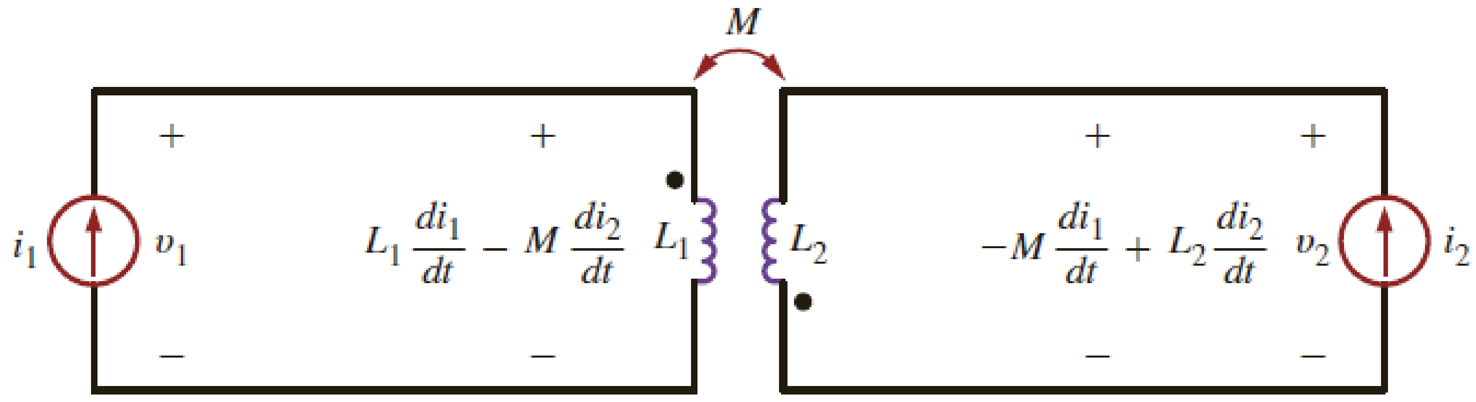


$$\boldsymbol{V_1 = jX_{L1}I_1 + jX_{LM}I_2}$$

(a)

$$\boldsymbol{V_2 = jX_{L2}I_2 + jX_{LM}I_1}$$

- **Current enters the dotted terminal → voltage at coupled coil is positive at the dotted terminal**

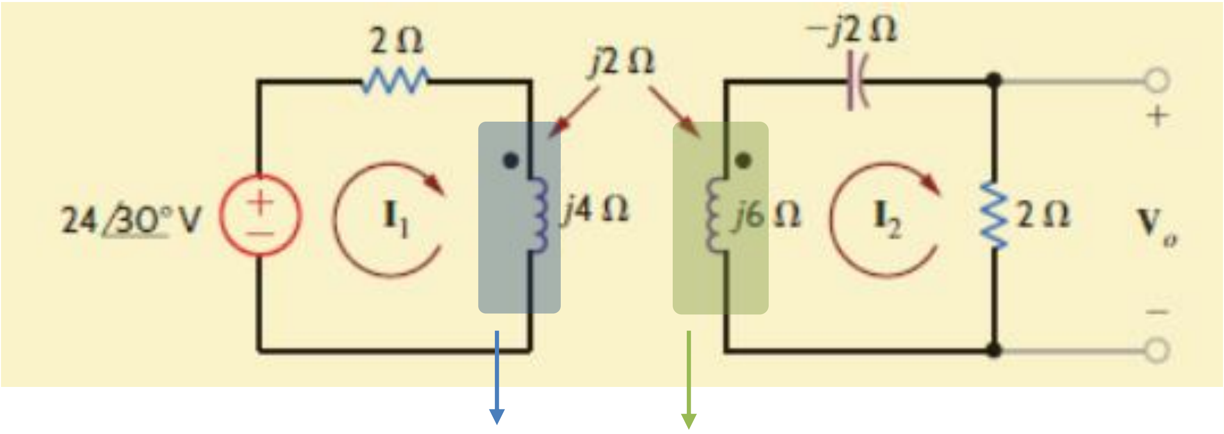


(b)

- **Current enters the undotted terminal → voltage at coupled coil is positive at the undotted terminal**

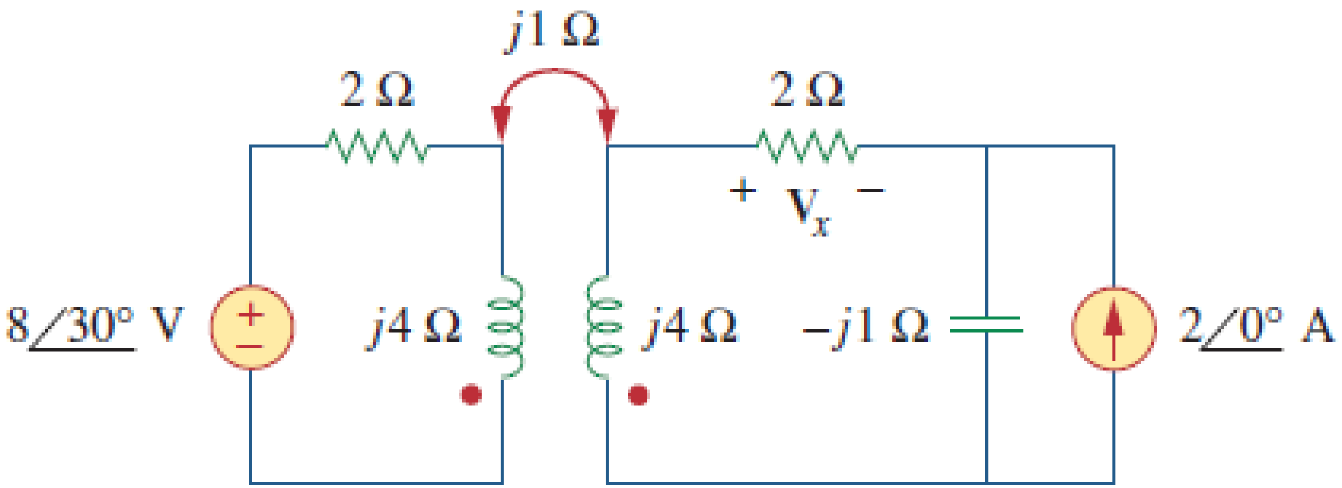
Example 10.4

Determine V_0 for the given circuit.



$V_1 = jX_{L1}I_1 - jX_{LM}I_2$ $V_2 = jX_{L2}I_2 - jX_{LM}I_1$

Find V_x in the provided circuit.



Problem

Determine the equivalent inductance L_{eq} of the circuit.