Steady State Power Analysis → Chapter #9

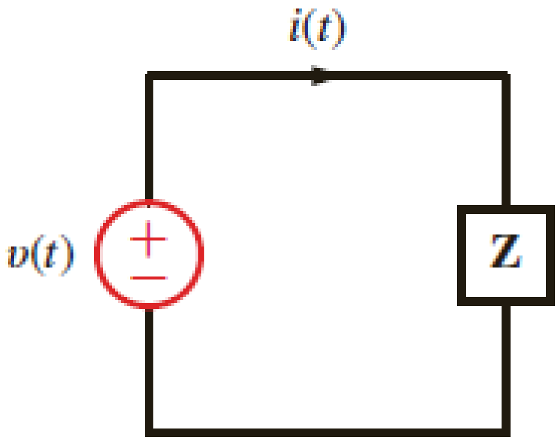
- **Instantaneous and Average Power (AC Circuits)**
- **Maximum Average Power Transfer (AC Circuits)**
- **Effective / RMS Value (periodic waveform)**
- **Real Power, Reactive Power, Complex Power, & Power Factor**
- **Power Factor Correction**

Instantaneous Power

The steady-state voltage and current in the network can be written as:

- $v(t) = V_M \cos(\omega t + \theta_v)$
- $i(t) = I_M \cos(\omega t + \theta_i)$

$$p(t) = V_M I_M \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$



$$p(t) = \frac{V_M I_M}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

constant

Freq = 2ω

Trigonometric Identity

$$\cos(\phi_1)\cos(\phi_2) = \frac{1}{2}\cos(\phi_1 - \phi_2) + \frac{1}{2}\cos(\phi_1 + \phi_2)$$

Average Power

The average value of any waveform can be computed by integrating the function over a complete period and dividing this result by the period:

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt$$

$$= \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_M I_M}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\ \right] dt$$

$$\therefore P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

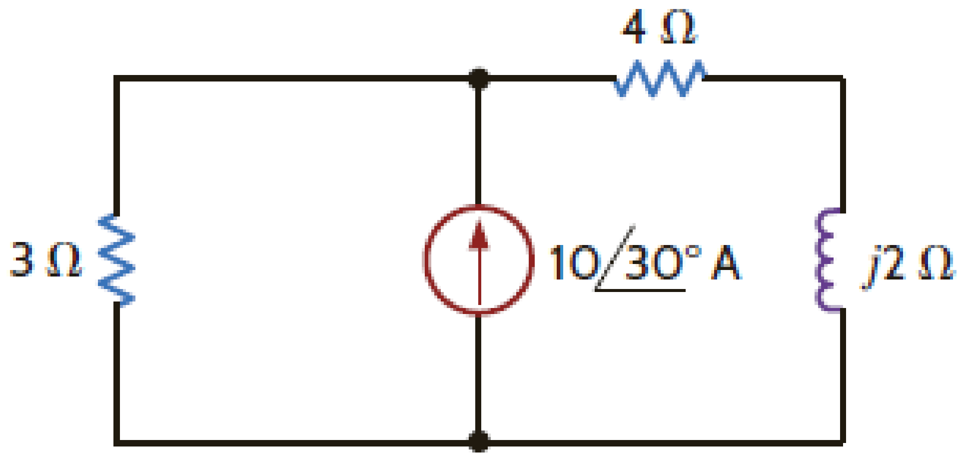
- $P_{resistive} = \frac{1}{2} V_M I_M$
- $P_{reactive} = \frac{1}{2} V_M I_M \cos(\pm 90^0) = 0$

Circuits 1

Learning Assessment E9.2

12/11/2019

Find the average power absorbed by each passive circuit element an the total average power supplied by the current source.

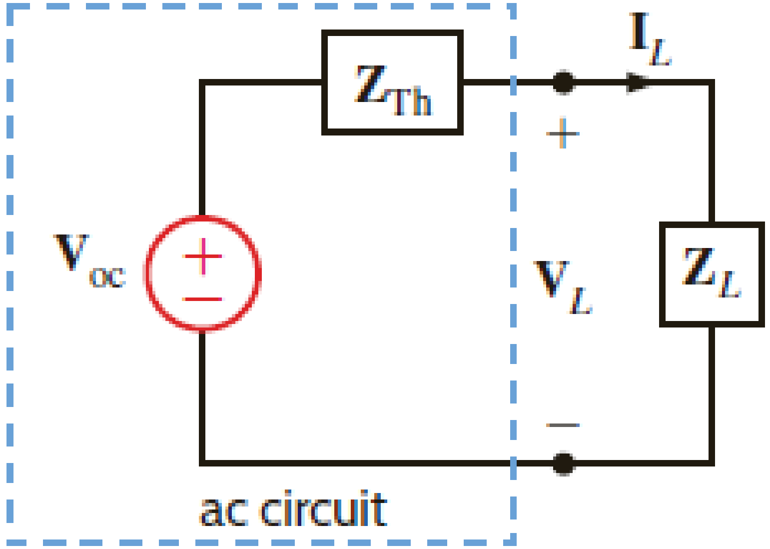


Maximum Power Transfer

Reexamining the maximum power transfer for AC sources…/

$$Z_{th} = R_{th} + jX_{th}$$

$$V_L = V_{oc}\frac{Z_L}{Z_{Th} + Z_L} \qquad I_L = \frac{V_{oc}}{Z_{Th} + Z_L}$$



$$P_L = \frac{1}{2}\frac{{V_{oc}}^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

$$Z_L = R_L + jX_L$$

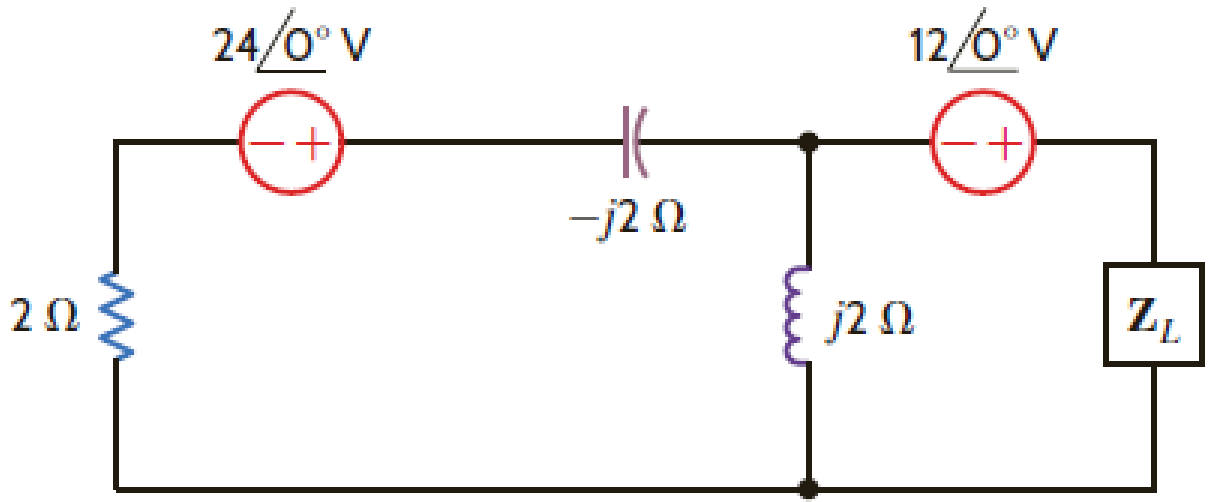
$$\left.\begin{aligned} \therefore X_L &= -X_{th} \\ \therefore R_L &= R_{th} \end{aligned}\right\} \quad Z_L = R_{th} - jX_{th}$$

Circuits 1

Learning Assessment E9.8

12/11/2019

Find Z_L for maximum average power transfer and the maximum average power transferred to the load.



Effective or RMS Values

- There are many types of periodic waveforms in circuit analysis
- An RMS value allows for effective comparison of different sources

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)R\,dt$$

$$\therefore I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt}$$

$$P = {I_{rms}}^2 R$$

rms = root mean square

RMS Value of a Sinusoid

- $i(t) = I_M \cos(\omega t - \theta)$
- $T = 2\pi/\omega$

$$I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\, dt}$$

$$= \sqrt{\frac{1}{T}\int_{0}^{T} {I_M}^2 \cos^2(\omega t - \theta)\, dt}$$

$$= I_M \sqrt{\frac{1}{T}\int_{0}^{T} \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t - 2\theta)\right] dt}$$

$$= I_M \sqrt{\frac{1}{T}\int_{0}^{T} \frac{1}{2}\, dt} = \frac{I_M}{\sqrt{2}}$$

$$P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

$$\therefore P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Trigonometric Identity

$$\cos(\phi_1)\cos(\phi_2) = \frac{1}{2}\cos(\phi_1 - \phi_2) + \frac{1}{2}\cos(\phi_1 + \phi_2)$$

Example E9.8

Determine the rms value of the current waveform provided and use this value to compute the average power delivered to a 2 Ω resistor through which this current is flossing.