Steady State Power Analysis → Chapter #9

- ✓ **Instantaneous and Average Power (AC Circuits)**
- ✓ **Maximum Average Power Transfer (AC Circuits)**
- ✓ **Effective / RMS Value (periodic waveform)**
- **Real Power, Reactive Power, Complex Power, & Power Factor**
- **Power Factor Correction**

Last Lecture → Average Power

The average value of any waveform can be computed by integrating the function over a complete period and dividing this result by the period:

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\, dt$$

$$= \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_M I_M}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\ \right] dt$$

$$\therefore P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

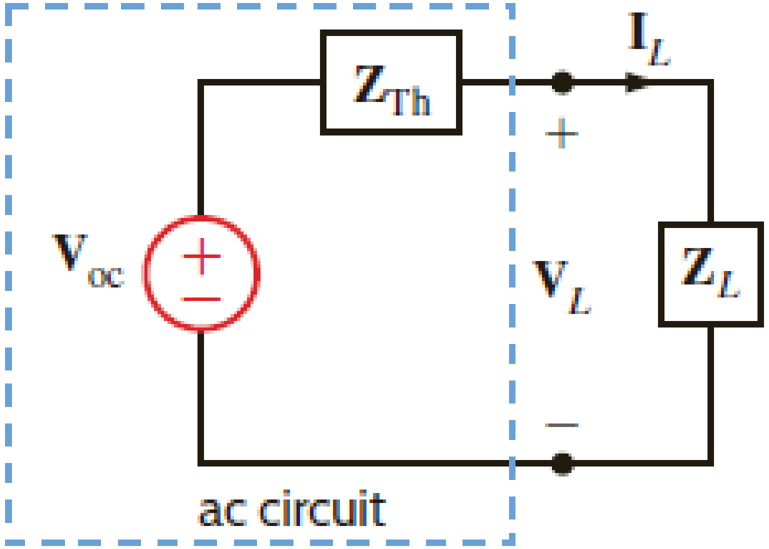
- $P_{resistive} = \frac{1}{2} V_M I_M = \frac{1}{2} R {I_M}^2 = \frac{1}{2}\frac{{V_M}^2}{R}$
- $P_{reactive} = \frac{1}{2} V_M I_M \cos(\pm 90^0) = 0$

Last Lecture → Maximum Power Transfer

Reexamining the maximum power transfer for AC sources…/

$$Z_{th} = R_{th} + jX_{th}$$

$$V_L = V_{oc}\frac{Z_L}{Z_{Th} + Z_L} \qquad I_L = \frac{V_{oc}}{Z_{Th} + Z_L}$$



$$P_L = \frac{1}{2}\frac{{V_{oc}}^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

$$Z_L = R_L + jX_L$$

$$\therefore X_L = -X_{th}$$

$$\therefore R_L = R_{th}$$

$$Z_L = R_{th} - jX_{th}$$

Last Lecture → RMS Value (Sinosoid)

- $i(t) = I_M \cos(\omega t - \theta)$
- $T = 2\pi/\omega$

$$I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T {I_M}^2 \cos^2(\omega t - \theta)\,dt}$$

$$= I_M \sqrt{\frac{1}{T}\int_0^T \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t - 2\theta)\right] dt}$$

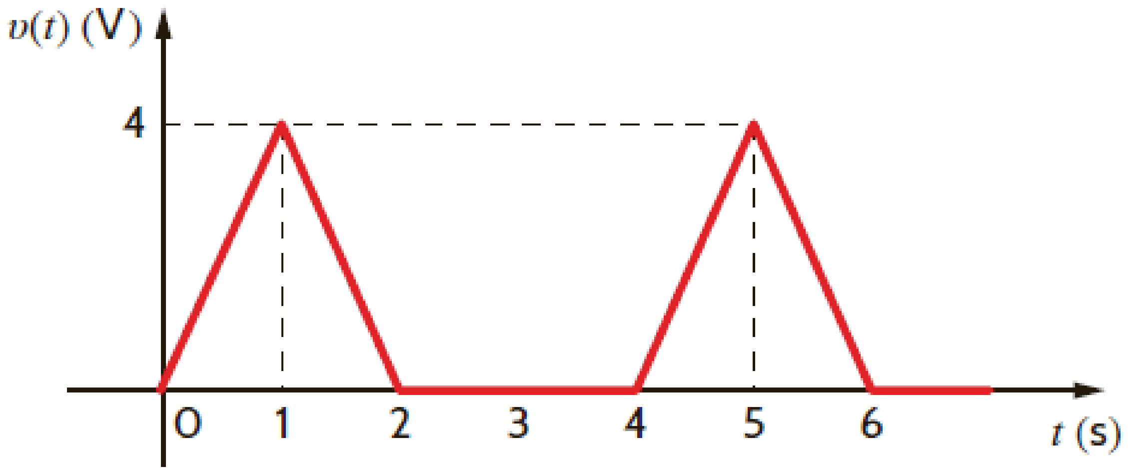
$$= I_M \sqrt{\frac{1}{T}\int_0^T \frac{1}{2}\,dt} = \frac{I_M}{\sqrt{2}}$$

$$\therefore P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$\therefore P_R = R{I_{rms}}^2 = \frac{{V_{rms}}^2}{R}$$

Example E9.8

Calculate the rms value of the provided waveform.



Power Factor

- $P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \rightarrow average\ power\ (W)$
- $V_{rms} I_{rms} \rightarrow apparent\ power\ (VA)$

$$pf = \frac{P}{V_{rms} I_{rms}} = \cos(\theta_v - \theta_i) \rightarrow power\ factor (VA)$$

$$pf = 1 \rightarrow purely\ resistive\ load$$
$$pf = 0 \rightarrow purely\ reactive\ load$$

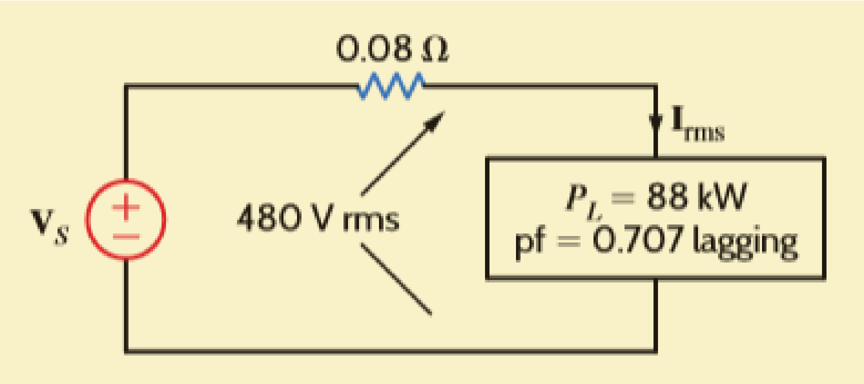
Phase of the current with respect to the voltage

- leading = $\theta_v - \theta_i < 0$
- lagging = $\theta_v - \theta_i > 0$

Example 9.10

An industrial load consumes 88kW at a pf of 0.707 lagging from a 480 V_{rms} line. The transmission line resistance from the power company's transformer to the plant is 0.08 Ω. Determine the power that must be supplied by the power company

a) **under present conditions and**
b) **if the pf is somehow change to 0.90 lagging.**



Complex Power

$$\mathbf{S} = \boldsymbol{V_{rms}}\boldsymbol{I_{rms}}^{*} \qquad (\boldsymbol{I_{rms}}^{*} \rightarrow \text{complex conjugate of } \boldsymbol{I_{rms}})$$

$$= V_{rms}I_{rms}\langle\theta_v - \theta_i$$

$$= V_{rms}I_{rms}\cos(\theta_v - \theta_i) + \mathrm{j}V_{rms}I_{rms}\sin(\theta_v - \theta_i)$$

P → Real/Average Power

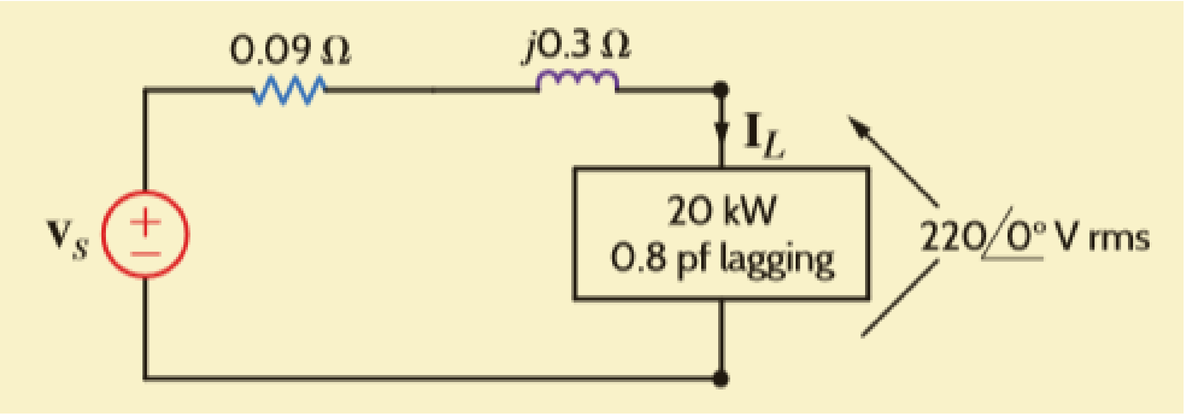
Q → Reactive Power

$$\mathbf{S} = P + jQ$$

$$\tan(\theta_v - \theta_i) = \frac{Q}{P}$$

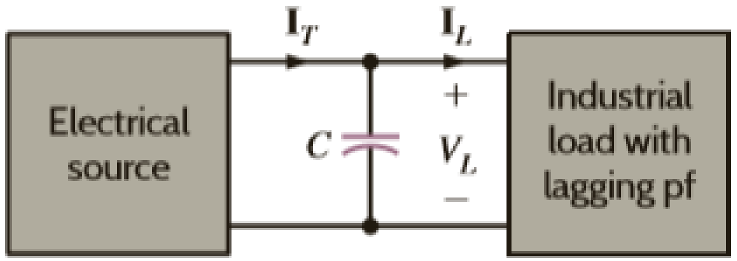
Example 9.11

A load operates at 20 kW, 0.8 pf lagging. The load voltage is 220 V_{rms} at 60 Hz. The impedance of the line is 0.09 + j0.3 Ω. Determine the voltage and the power factor at the input to the line.



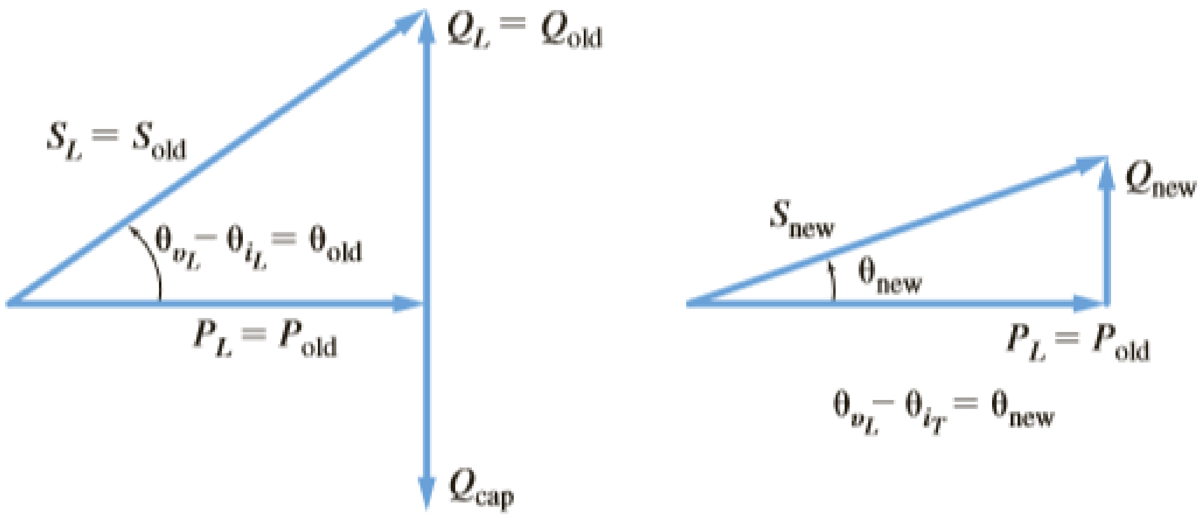
Power Factor Correction

PF can be increased by decreasing the reactive power through a capacitor bank!



$$S_{new} = S_{old} + S_{cap}$$

$$S_{cap} = -j\omega C {V_{rms}}^2$$



Example 9.14

Plastic kayaks are manufactured using a process called roto-molding. Molten plastic is injected into a mold, which is the spun on the long axis of the kayak until the plastic cools, resulting in a hollow on-piece craft. Suppose that the induction motors used to spin the molds consume 50kW at a pf of 0.8 lagging from a 220 V_{rms}, 60 Hz line. What would be the capacitor bank size to be placed in parallel to raise the pf to 0.95 lagging?