

Last Lecture → Nodal Analysis

- **N - nodes**
- **One node is selected as the reference node**
- ***KCL is applied to the remaining N-1 nodes***



Solution of the N-1 equations

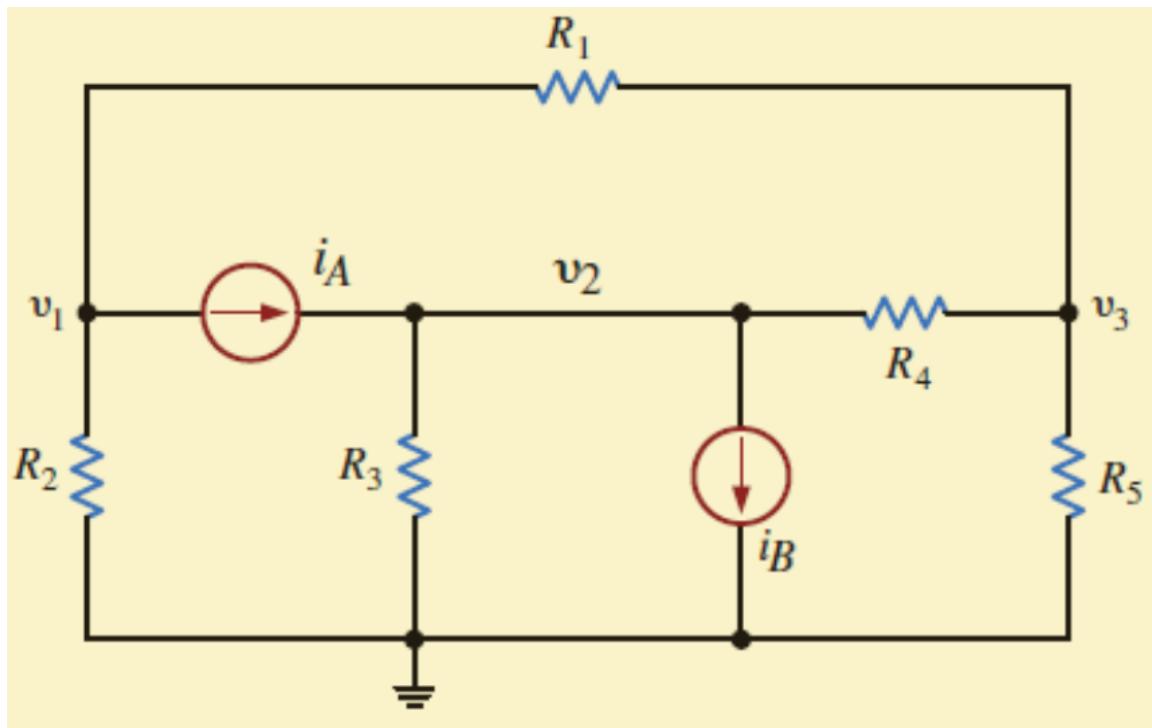
- ***N-1 unknown voltages***
- ***All currents in the circuit***

Analysis Procedure

- 1) Identify #of nodes
- 2) Select reference node
- 3) Label other node voltages
- 4) Identify branch currents
- 5) Apply KCL to nodes
- 6) Matrix Format

Last Lecture → Example 3.2

Write the equations in matrix form for the provided circuit. Assume $R_1=R_2=2\text{k}\Omega$, $R_3=R_4=4\text{k}\Omega$, $R_5=1\text{k}\Omega$, $i_A=4\text{mA}$, and $i_B=2\text{mA}$.

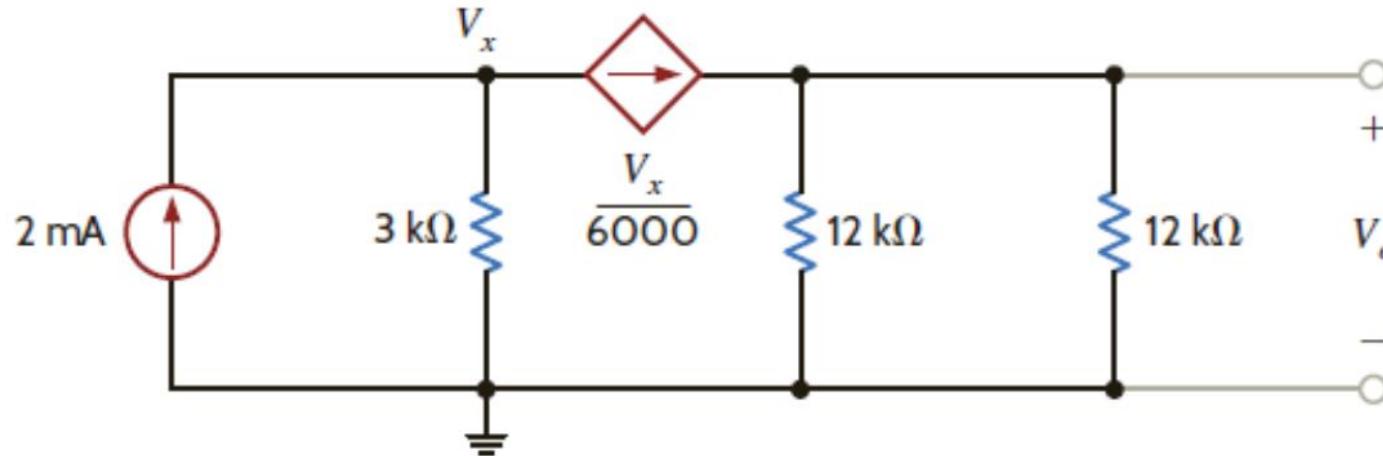


matrix eq.

$$\begin{bmatrix} I_A \\ I_A - I_B \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 \parallel R_2} & 0 & \frac{1}{R_1} \\ 0 & \frac{1}{R_3 \parallel R_4} & -\frac{1}{R_4} \\ \frac{1}{R_2} & \frac{1}{R_4} & -\frac{1}{R_1 \parallel R_4 \parallel R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Nodal Analysis → with Dependent Current Sources

E3.3: Find the voltage V_o in the network provided.

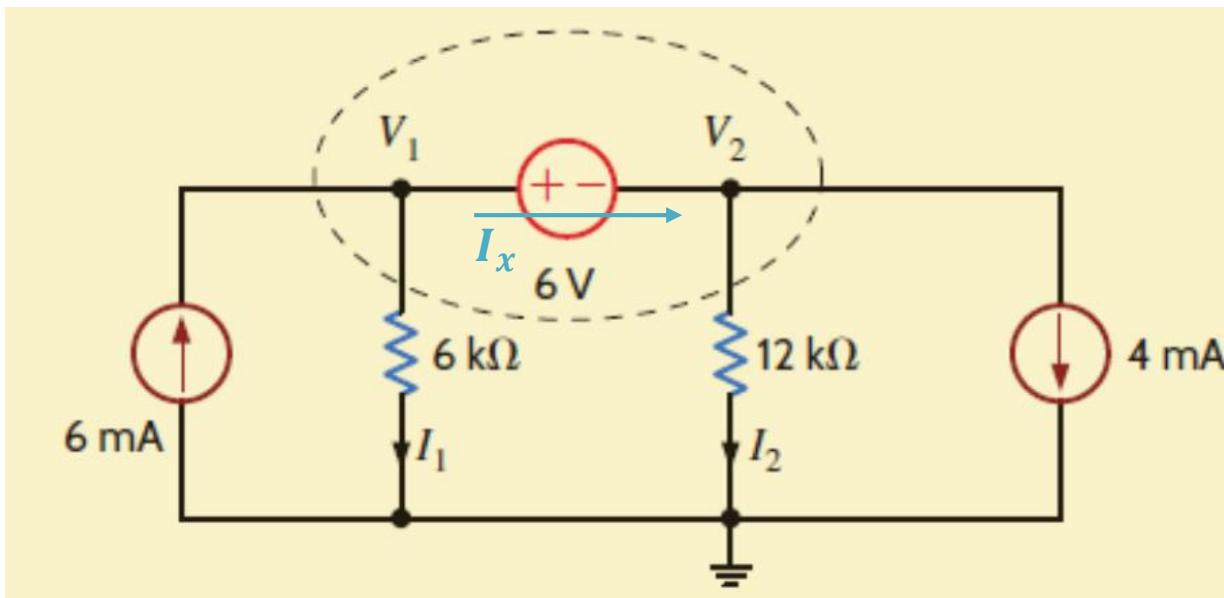


$$\left. \begin{aligned} KCL @ V_x &\rightarrow 2m - \frac{V_x}{3k} - \frac{V_x}{6k} = 0 \\ KCL @ V_o &\rightarrow \frac{V_x}{6k} - \frac{V_0}{12k} - \frac{V_0}{12k} = 0 \end{aligned} \right\} \begin{aligned} V_x &= 4 V \\ V_0 &= V_x = 4 V \end{aligned}$$

Nodal Analysis → with Independent Voltage Sources

Supernode: KCL at the surface (dashed) around the voltage source.

Constraint equation: establishes the difference in potential between two nodes.



$$\left. \begin{aligned} KCL @ V_1 &\rightarrow 6m - \frac{V_1}{6k} - I_x = 0 \\ KCL @ V_2 &\rightarrow I_x - \frac{V_2}{12k} - 4m = 0 \end{aligned} \right\}$$

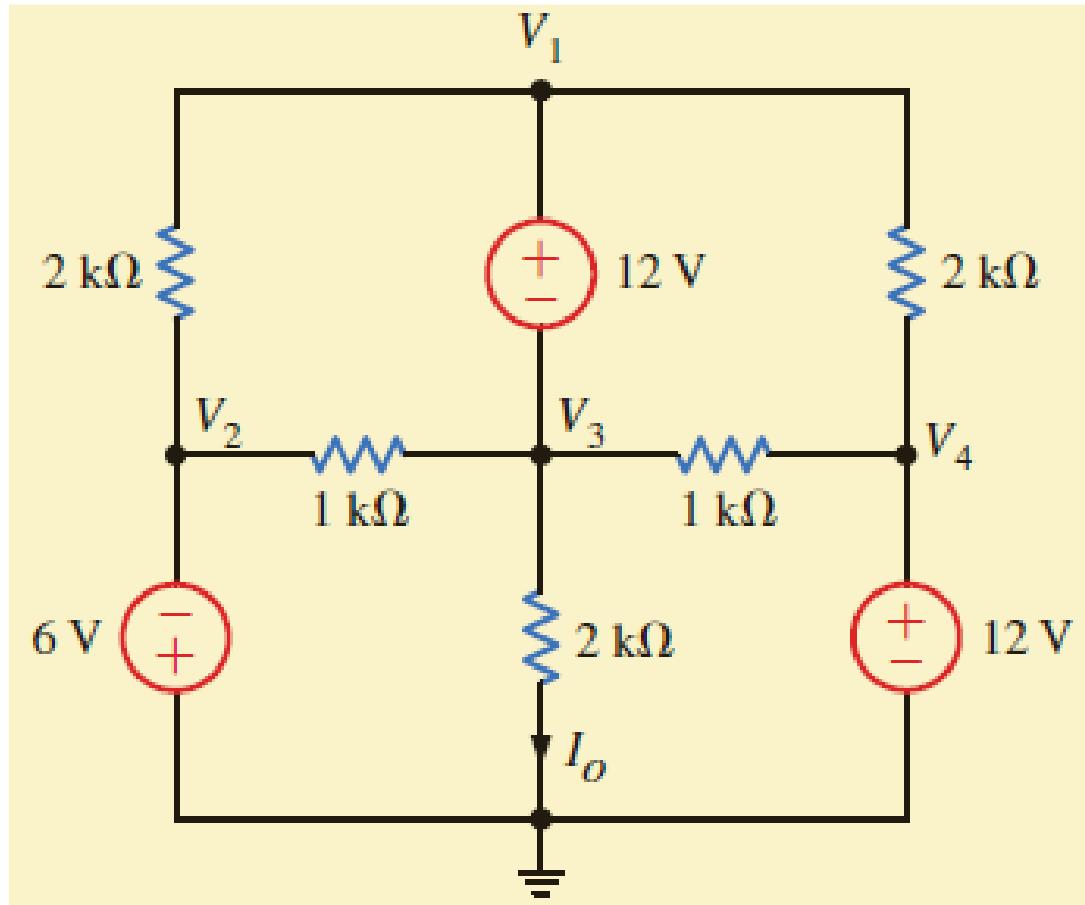
$$KCL @ Supernode \rightarrow 6m - \frac{V_1}{6k} - \frac{V_2}{12k} - 4m = 0$$

$$\begin{array}{l} V_1 = 10 \text{ V} \\ V_2 = 4 \text{ V} \\ \hline \end{array}$$

$$Constraint Eq. \rightarrow 6 = V_1 - V_2$$

Nodal Analysis → with Independent Voltage Sources

Example 3.7: Determine the current I_0 in the provided network.



KCL @ supernode

$$\frac{-6 - V_1}{2k} + \frac{-6 - V_3}{1k} = \frac{V_1 - 12}{2k} + \frac{V_3 - 12}{1k} + \frac{V_3}{2k}$$

Constraint

$$12 = V_1 - V_3$$

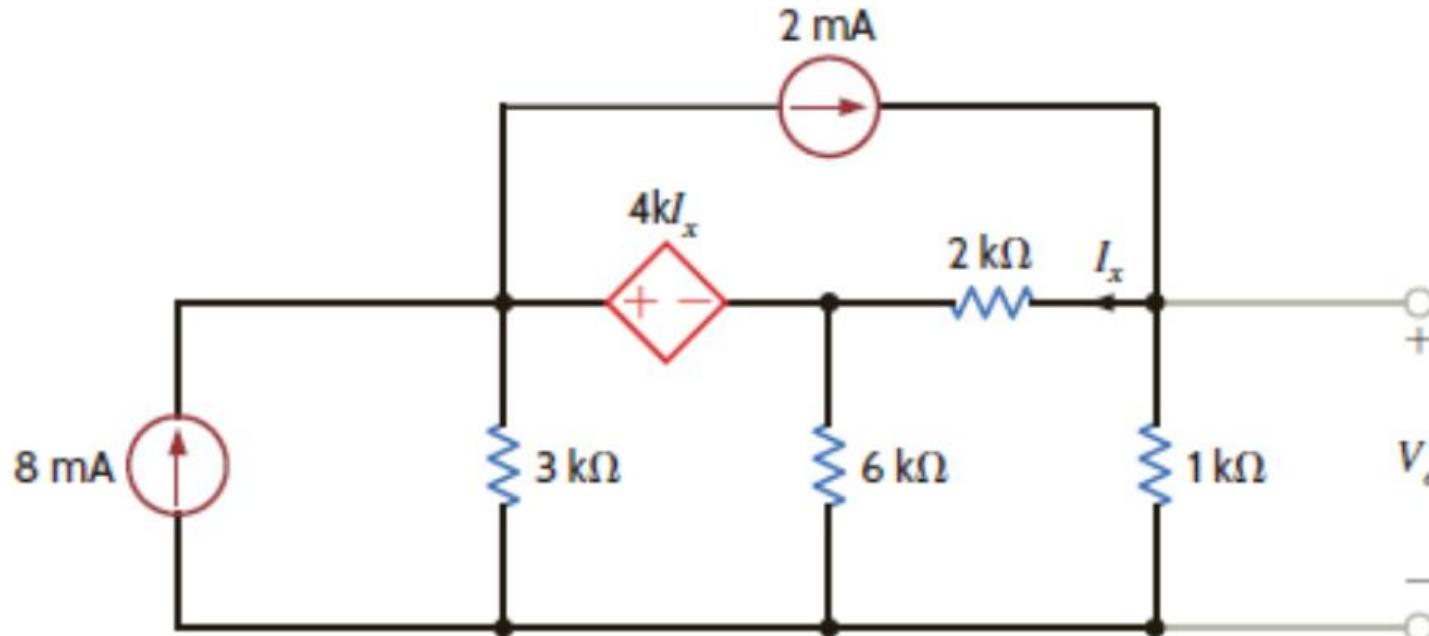
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$$V_3 = -\frac{6}{7} V$$

$$I_0 = \frac{V_3}{2k} = -\frac{3}{7} mA$$

Nodal Analysis → with Dependent Voltage Sources

E3.12: Determine the current V_0 in the provided network.



KCL @ supernode

$$8m - 2m = \frac{V_1}{3k} + \frac{V_2}{6k} + \frac{V_2 - V_0}{2k}$$

KCL @ node 2

$$2m = \frac{V_0}{1k} - \frac{V_2 - V_0}{2k}$$

Constraint

$$I_x = \frac{V_0 - V_2}{2k}$$

Dependent Source

$$V_1 - V_2 = 4k \cdot I_x$$