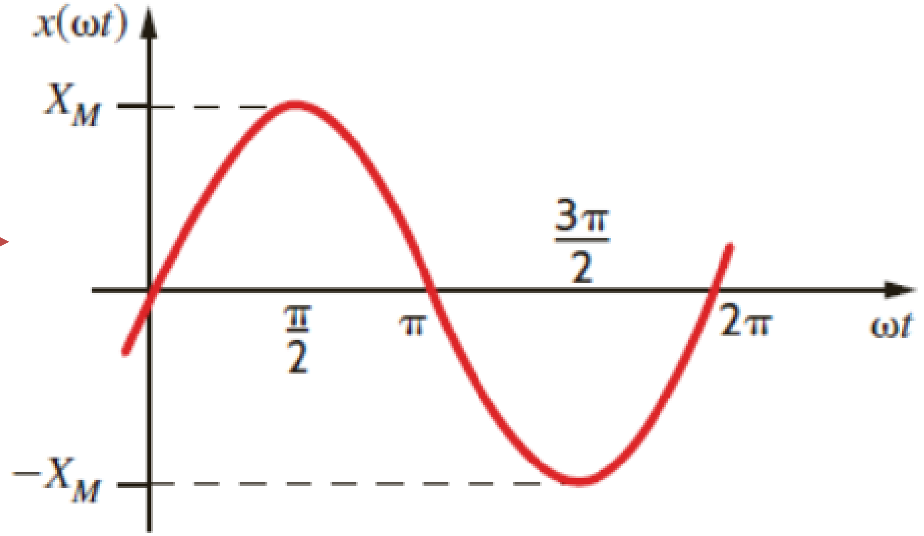
Last Lecture → Sinusoids

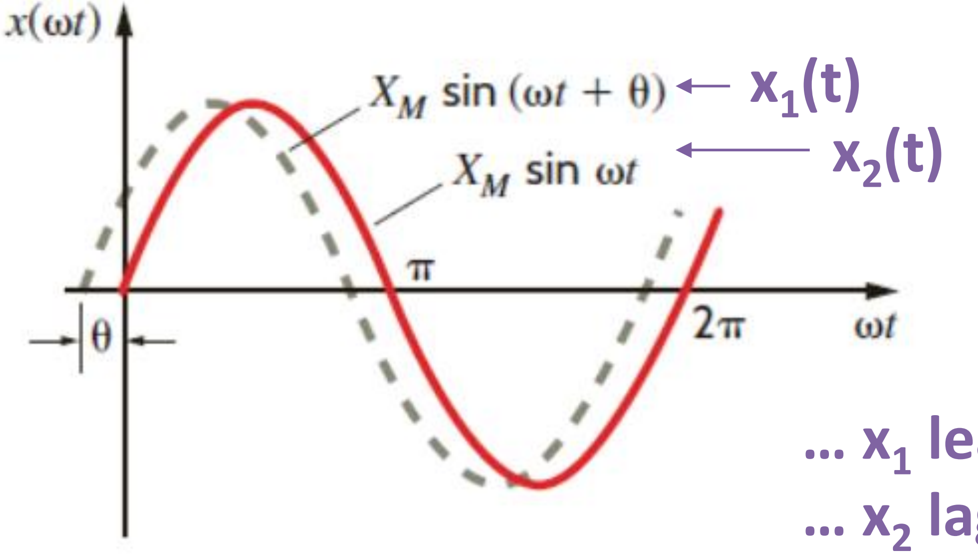
$$x(t) = X_M \cdot \sin(\omega t + \theta)$$



- X_m → amplitude / maximum value
- ω → radian / angular frequency
- θ → phase angle

- T → period
- $f = \frac{1}{T}$ → # cycles per second

$$\omega = \frac{2\pi}{T} = 2\pi f$$

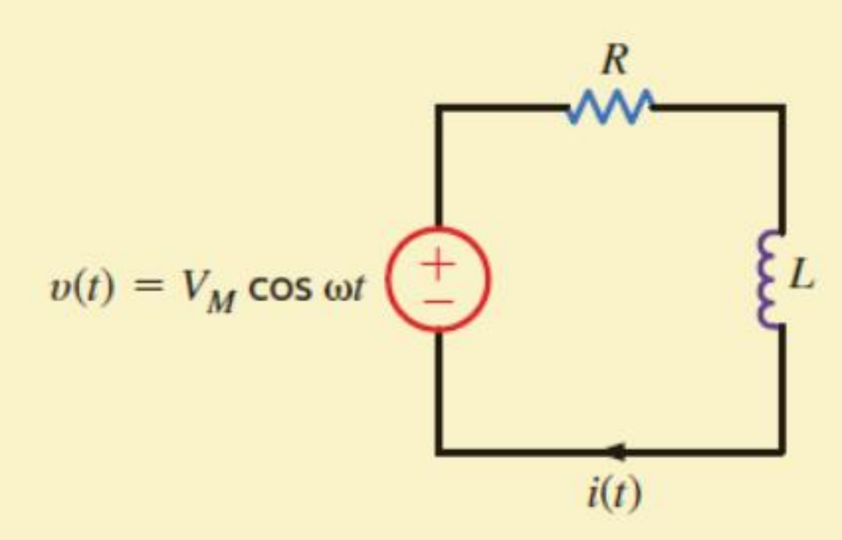


… x_1 leads x_2 by θ

… x_2 lags x_1 by θ

Last Lecture → Solving Circuits in Time Domain

Derive the expression for the current i(t) using the forcing function $v(t) = V_M e^{j\omega t}$ instead.



response

$$i(t) = I_M e^{j(\omega t + \varphi)}$$

KVL

$$v(t) = R \cdot i(t) + L\frac{di(t)}{dt}$$

Differential Equation

$$V_M e^{j\omega t} = R I_M e^{j(\omega t+\varphi)} + L\frac{d}{dt}\left(I_M e^{j(\omega t+\varphi)}\right)$$

Solution

$$\therefore I_M = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\therefore \varphi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i(t) = \frac{V_M}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

Last Lecture → Phasors

Representation of a complex number with just the magnitude and the phase angle.

- ***Forcing Function***

$$v(t) = V_M e^{j(\omega t+\theta)}$$

- ***Steady State Response***

$$i(t) = I_M e^{j(\omega t+\varphi)}$$

$e^{j(\omega t)}$

common to every term

$$v(t) = V_M \cos(\omega t + \theta) = Re\left[V_M e^{j(\omega t+\theta)}\right]$$

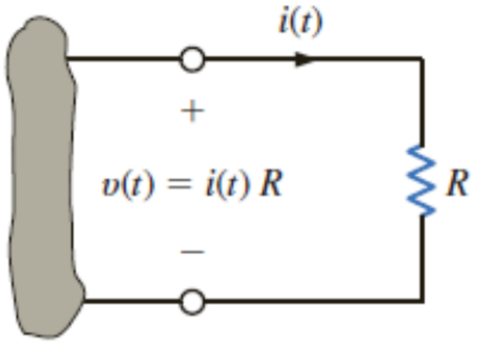
$$V = Re\left[V_M \langle\theta\ e^{j\omega t}\right] = V_M \langle\theta$$

phasor

Phasor Relationships → Resistor

- $v(t) = V_M e^{j(\omega t+\theta_v)}$
- $i(t) = I_M e^{j(\omega t+\theta_i)}$

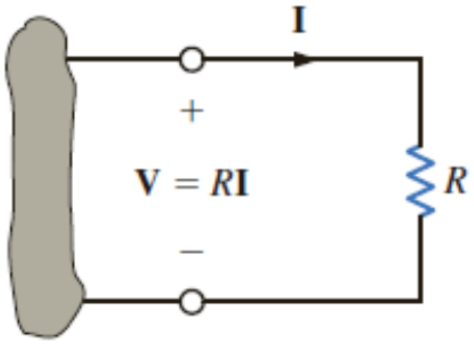
- ***Time Domain***



$v(t) = R \cdot i(t)$

$V_M e^{j(\omega t+\theta_v)} = R \cdot I_M e^{j(\omega t+\theta_i)}$

- ***Frequency Domain***



$\therefore V_M e^{j\theta_v} = R \cdot I_M e^{j\theta_i}$

↳ $V_M\angle\theta_v = R \cdot I_M\angle\theta_i$

$$\frac{V_M\angle\theta_v}{I_M\angle\theta_i} = R$$

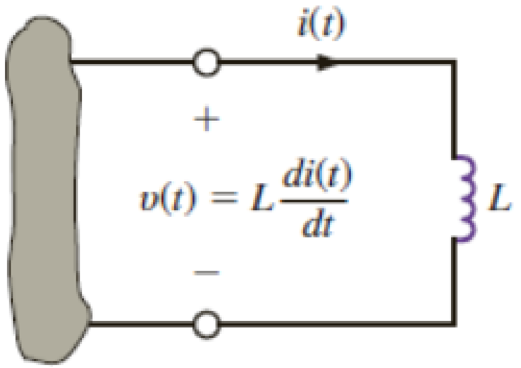
$\therefore \theta_v = \theta_i$

$\rightarrow v(t)$ *and* $i(t)$ *are in phase*

Phasor Relationships → Inductor

- $v(t) = V_M e^{j(\omega t + \theta_v)}$
- $i(t) = I_M e^{j(\omega t + \theta_i)}$

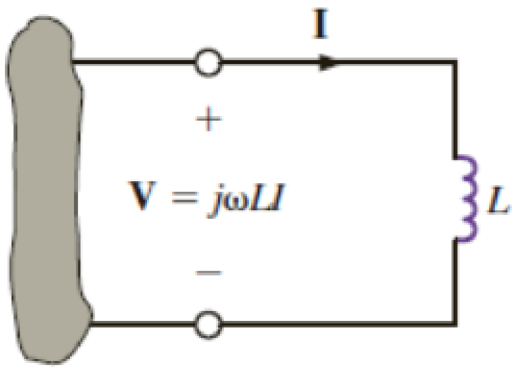
- *Time Domain*



$$v(t) = L \cdot \frac{di(t)}{dt}$$

$$V_M e^{j(\omega t + \theta_v)} = L \cdot \frac{d}{dt} I_M e^{j(\omega t + \theta_i)}$$

- *Frequency Domain*



$$\therefore V_M e^{j\theta_v} = j\omega L \cdot I_M e^{j\theta_i}$$

$$V_M \langle \theta_v = j\omega L \cdot I_M \langle \theta_i$$

$$\frac{V_M \langle \theta_v}{I_M \langle \theta_i} = j\omega L$$

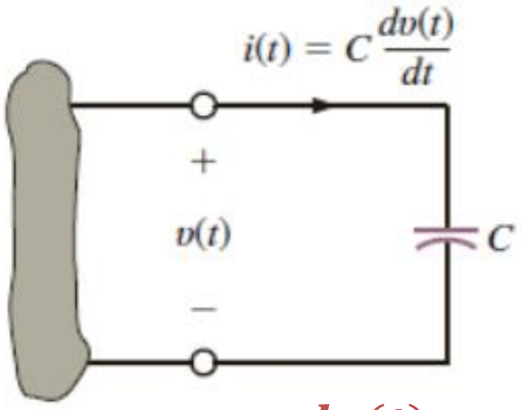
$\therefore \theta_v = \theta_i + 90^o$

$\rightarrow v(t)$ *leads* $i(t)$ *by* 90^o

Phasor Relationships → Capacitance

- $v(t) = V_M e^{j(\omega t + \theta_v)}$
- $i(t) = I_M e^{j(\omega t + \theta_i)}$

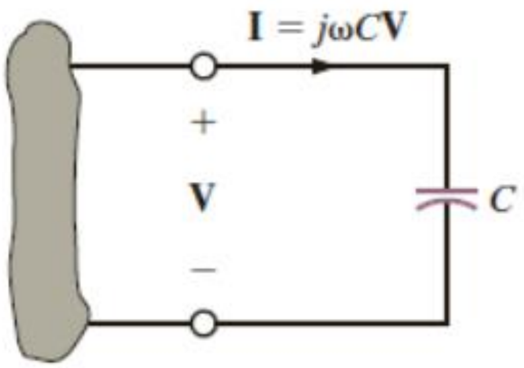
- *Time Domain*



$$i(t) = C \cdot \frac{dv(t)}{dt}$$

$$I_M e^{j(\omega t + \theta_i)} = C \cdot \frac{d}{dt} V_M e^{j(\omega t + \theta_v)}$$

- *Frequency Domain*



$$\therefore I_M e^{j\theta_i} = j\omega C \cdot V_M e^{j\theta_v}$$

$$I_M\langle\theta_i = j\omega C \cdot V_M\langle\theta_v$$

$$\frac{V_M\langle\theta_v}{I_M\langle\theta_i} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$\therefore \theta_v = \theta_i - 90^o$

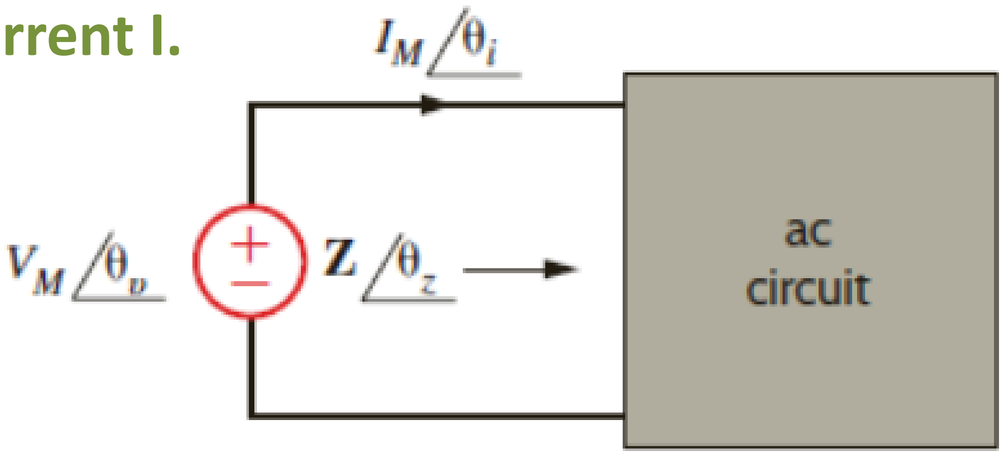
→ $v(t)$ *lags* $i(t)$ *by* 90^o

Impedance

The ratio of the phasor voltage V to the phasor current I.

$$Z = \frac{V}{I} \quad [Ohms]$$

$$= \frac{V_M \angle \theta_v}{I_M \angle \theta_i} = \frac{V_M}{I_M} \angle (\theta_v - \theta_i) = Z \angle \theta_z$$



$$Z \angle \theta_z = R + jX$$

Resistance (R) *Reactance* (X)

Polar Coordinates

$$Z = \sqrt{R^2 + X^2}$$

$$\angle \theta_z = \tan^{-1}\left(\frac{X}{R}\right)$$

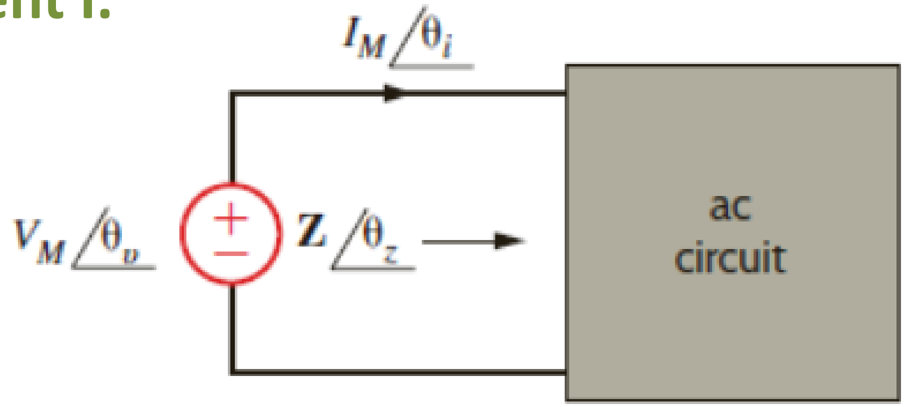
Rectangular Coordinates

$$R = Z \cos \theta_z$$

$$X = Z \sin \theta_z$$

Impedance

The ratio of the phasor voltage V to the phasor current I.



Series → Equivalent Impedance

$$Z_s = Z_1 + Z_2 + \cdots + Z_n$$

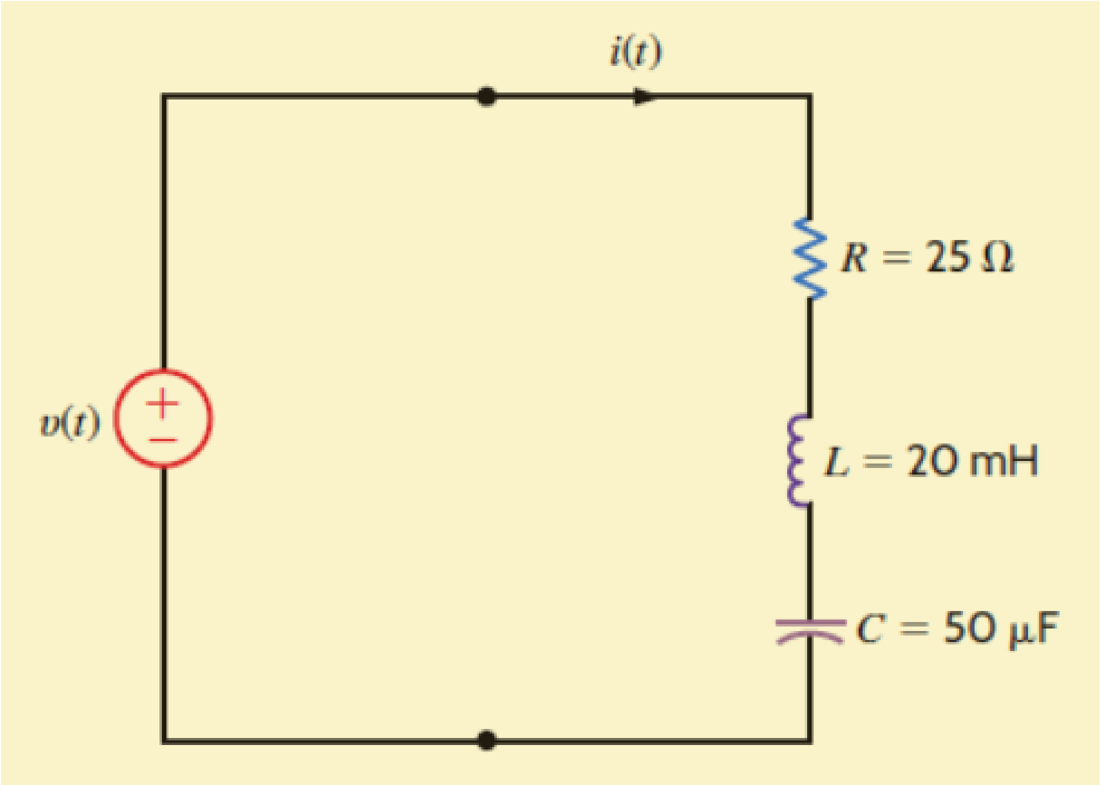
Parallel → Equivalent Impedance

$$\frac{1}{Z_p} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

KVL & KCL are valid in the frequency domain!

Example 8.9

Determine the equivalent impedance of the network provided if the frequency is f=60Hz. Then compute the current i(t) if the voltage source is $v(t) = 50\cos(\omega t + 30^o)\ V.$



Admittance

The ratio of the phasor current I to the phasor voltage V.

$$Y = \frac{I}{V} = \frac{1}{Z} \; [Siemens]$$

$$Y \angle \theta_y = G + jB$$

Conductance (G) *Susceptance* (B)

Parallel → Equivalent Admittance

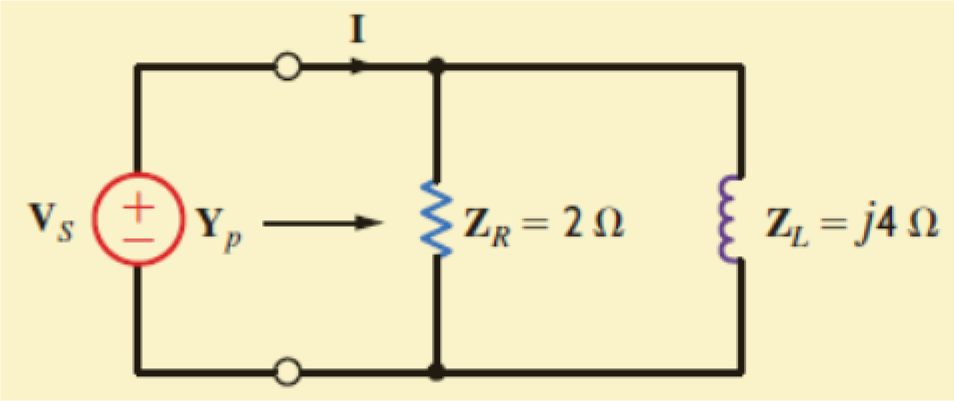
$$Y_p = Y_1 + Y_2 + \cdots + Y_n$$

Series → Equivalent Admittance

$$\frac{1}{Y_s} = \frac{1}{Y_1} + \frac{1}{Y_2} + \cdots + \frac{1}{Y_n}$$

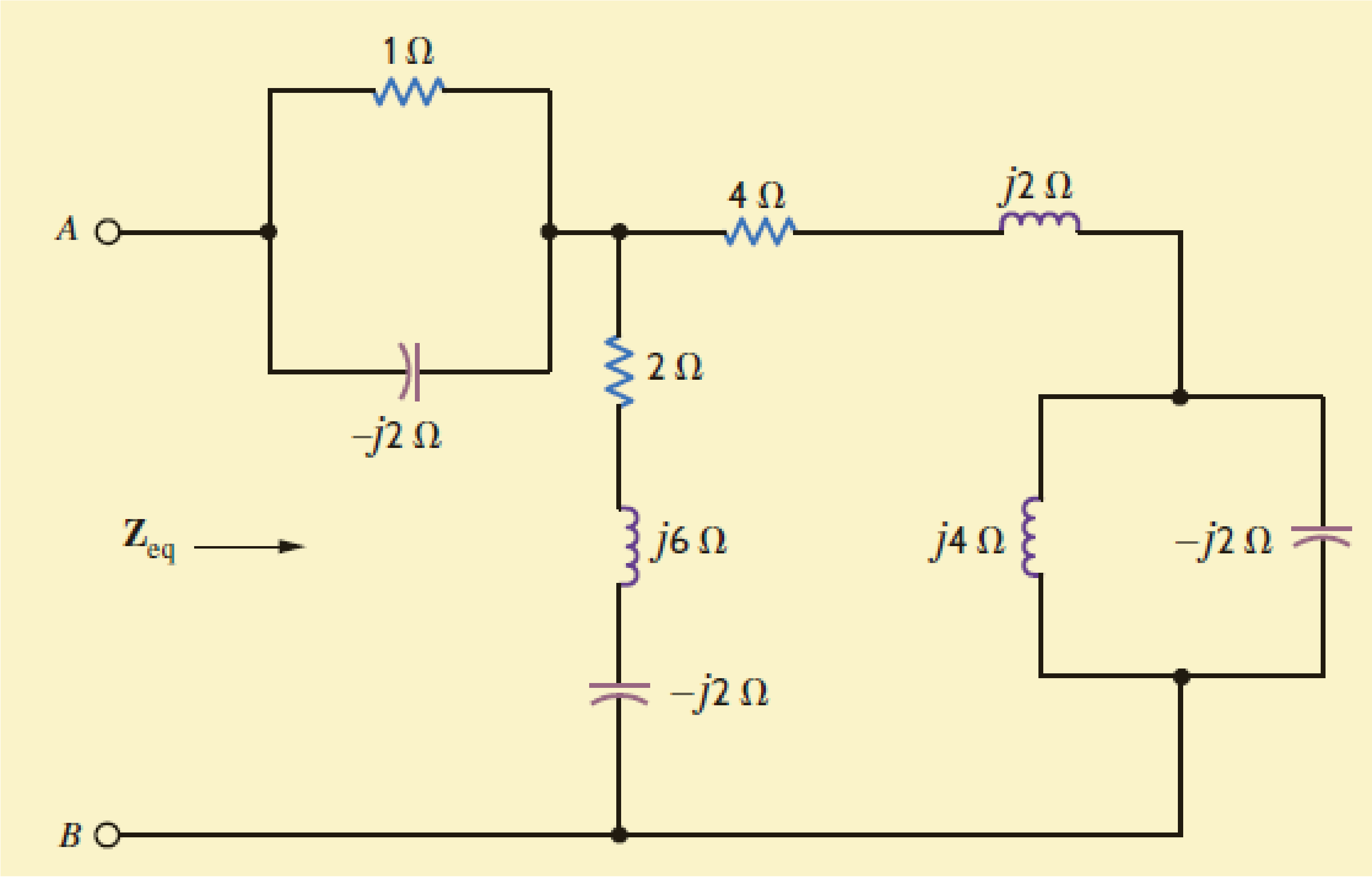
Example 8.10

Calculate the equivalent admittance Y_p for the network provided and use it to determine the current I if $V_s = 60\langle 45^o$.



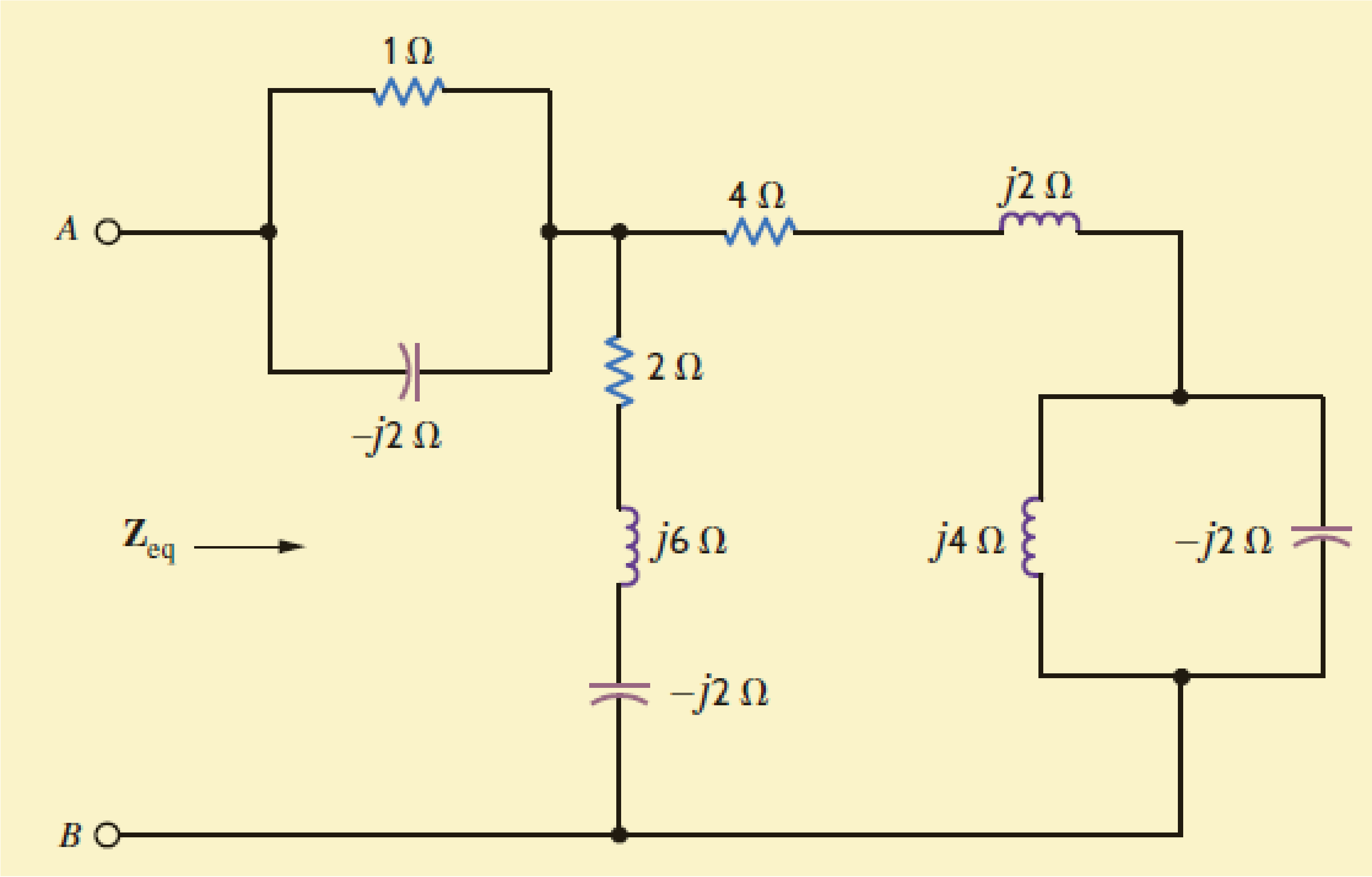
Example 8.11

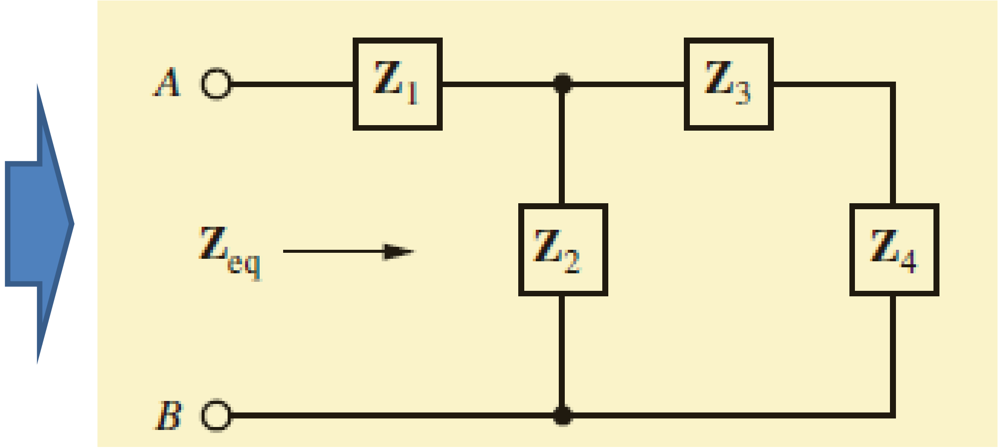
For the given circuit calculate the equivalent impedance Z_{eq} at terminals A-B.



Example 8.11

For the given circuit calculate the equivalent impedance Z_{eq} at terminals A-B.



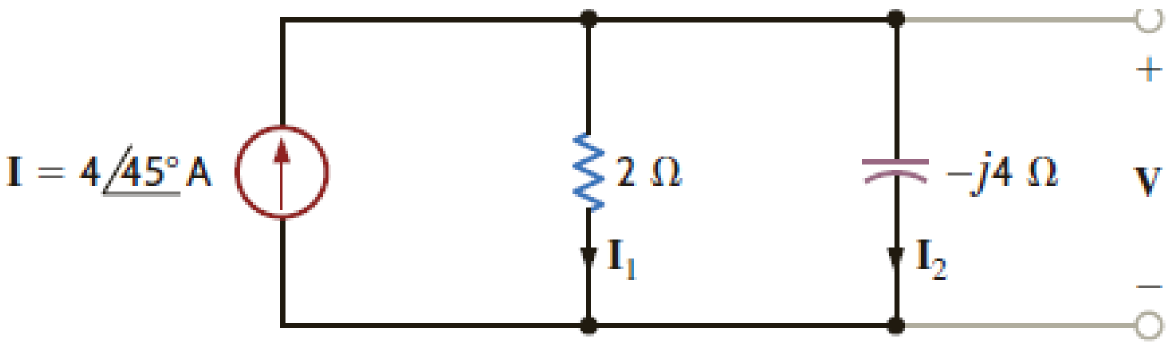


Technique for taking the reciprocal…

$$\frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$

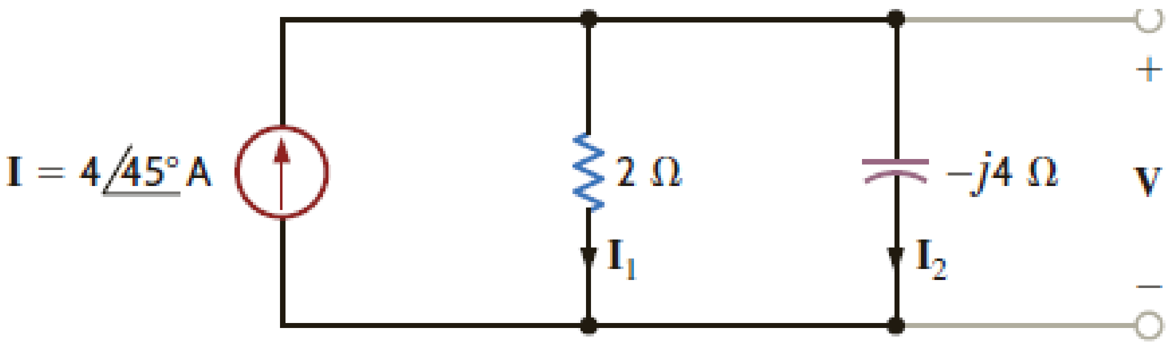
Learning Assessment E8.12

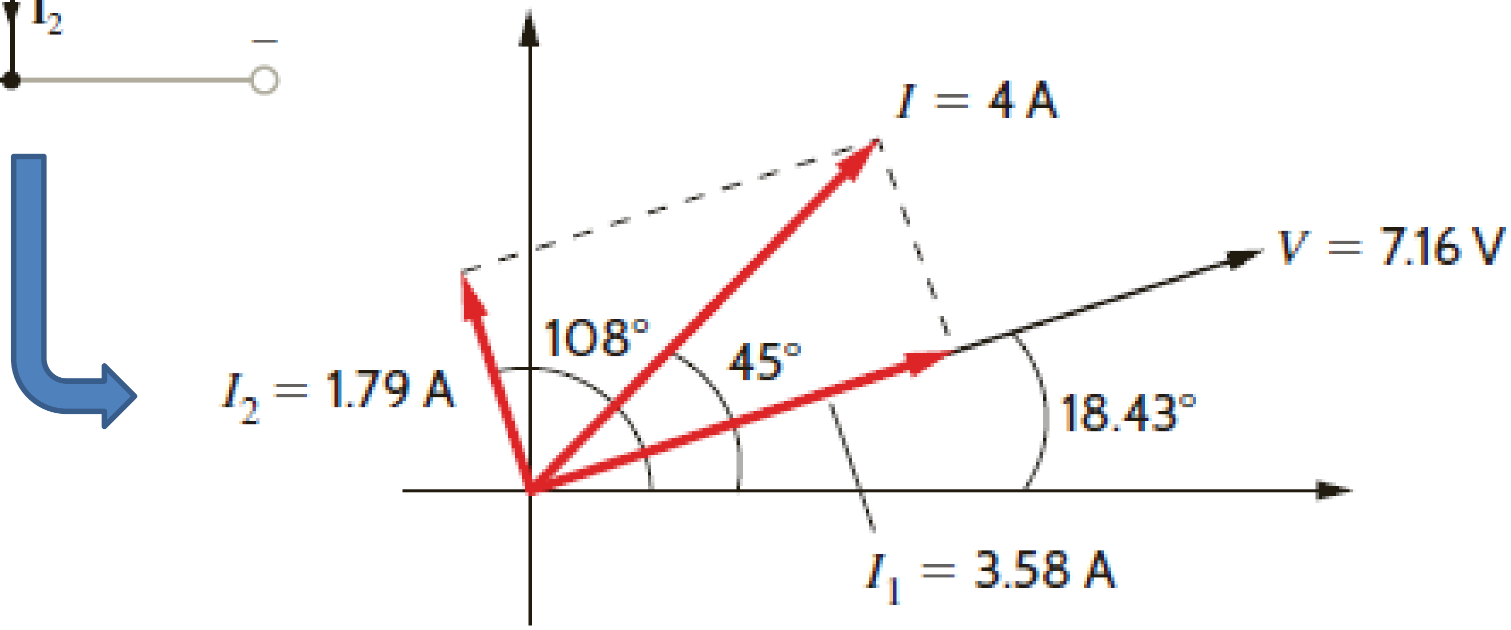
Draw the phasor diagram to illustrate all currents and voltages for the network provided.



Learning Assessment E8.12

Draw the phasor diagram to illustrate all currents and voltages for the network provided.





Example 8.15

For the given network determine I_0 using nodal analysis.