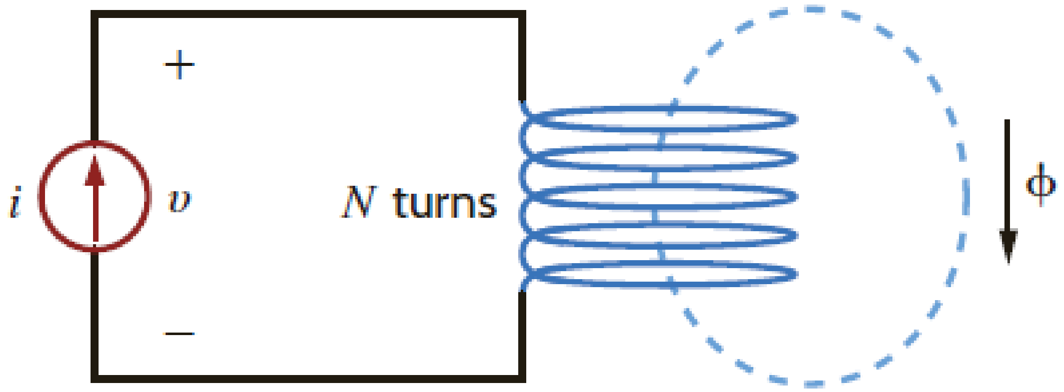
Magnetically Coupled Networks → Chapter #10

- **Mutual Inductance / Coefficient of Coupling / Turns Ration**
- **Circuit Analysis with Mutual Inductance**
- **Circuit Analysis with Ideal Transformers**

Single Coil Behavior



Flux Linkage → λ

coil turns

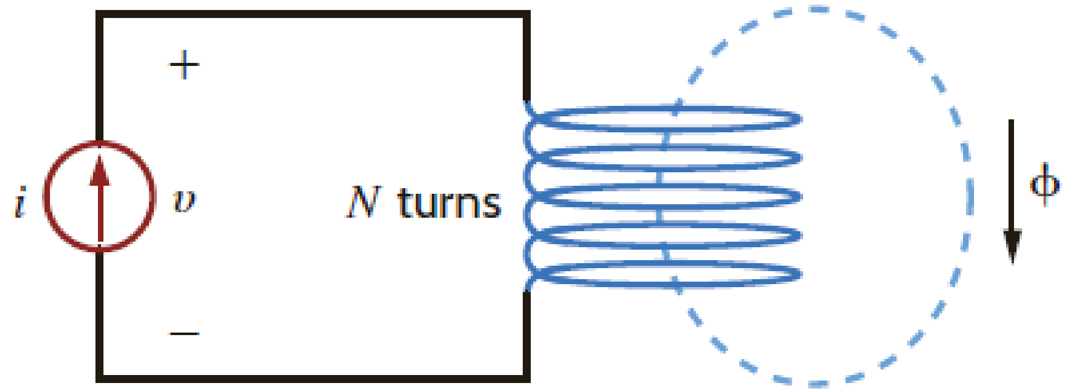
$$\lambda = \boxed{N} \cdot \boxed{\phi}$$

magnetic flux

$$\lambda = L \cdot i$$

$$\phi = \frac{L}{N} i$$

Single Coil Behavior



Flux Linkage → λ

coil turns

$$\lambda = N \cdot \phi$$

magnetic flux

$$\lambda = L \cdot i$$

↳ $$\phi = \frac{L}{N} i$$

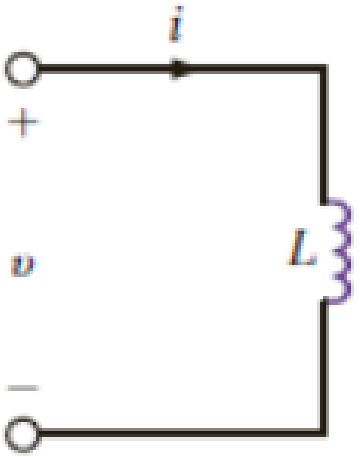
Faraday's Law → $v = f(\lambda)$

$$v = \frac{d\lambda}{dt}$$

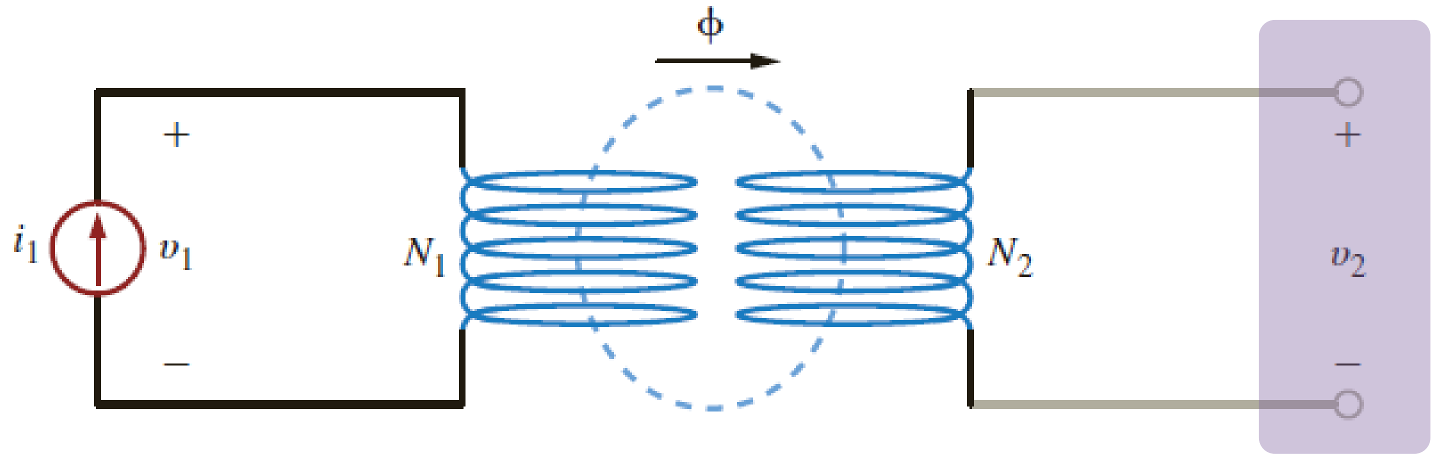
$$= L\frac{di}{dt} + i\frac{dL}{dt}$$

$L \neq f(t)$ ⇓

$$v = L\frac{di}{dt} \Longrightarrow$$



Two Coils → Magnetically Coupled



No Load!
→ $i_2 = 0$

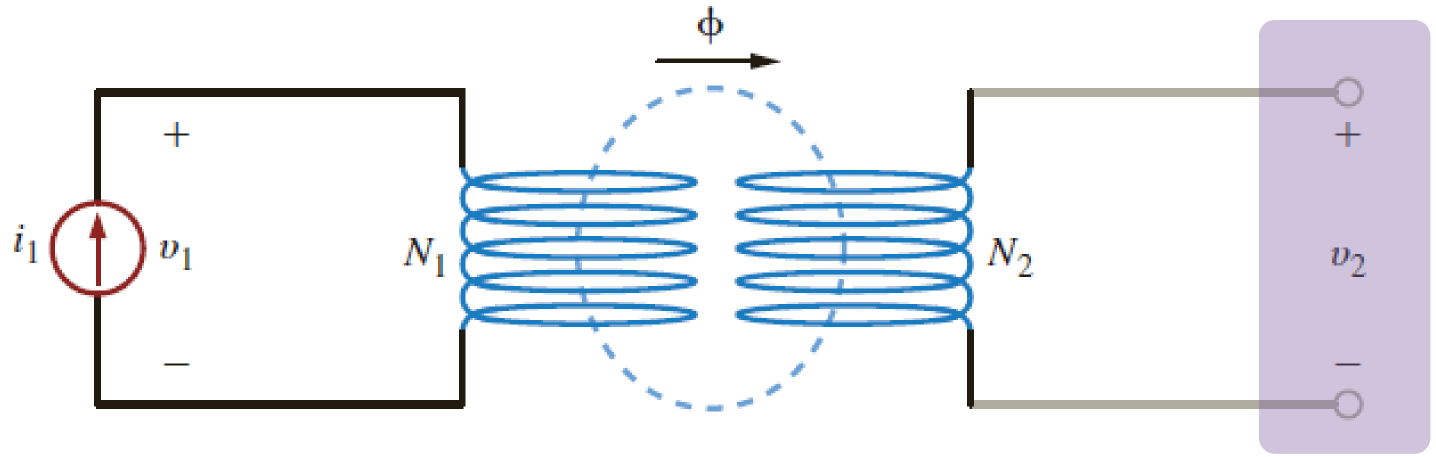
$$\lambda_1 = N_1 \cdot \phi = L_1 \cdot i_1$$

$$v_1 = \frac{d\lambda_1}{dt}$$

$$\lambda_2 = N_2 \cdot \phi$$

$$v_2 = \frac{d\lambda_2}{dt}$$

Two Coils → Magnetically Coupled



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$$v_1 = \frac{d\lambda_1}{dt}$$

$$\lambda_2 = N_2 \cdot \phi$$

$$v_2 = \frac{d\lambda_2}{dt}$$

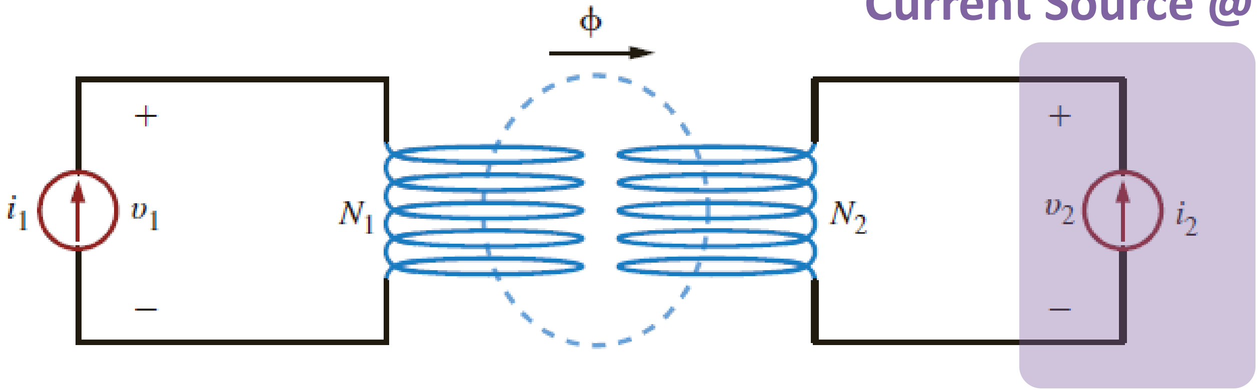
L_{21} → Mutual Inductance

$$v_2 = \frac{d\lambda_2}{dt} = \frac{d}{dt}(N_2 \cdot \phi) = \frac{d}{dt}\left(\frac{N_2}{N_1} \cdot \lambda_1\right) = \frac{d}{dt}\left(\frac{N_2}{N_1} \cdot L_1 \cdot i_1\right) = \boxed{\frac{N_2}{N_1} L_1} \frac{di_1}{dt}$$

Magnetically Coupled Coils

Using Superposition

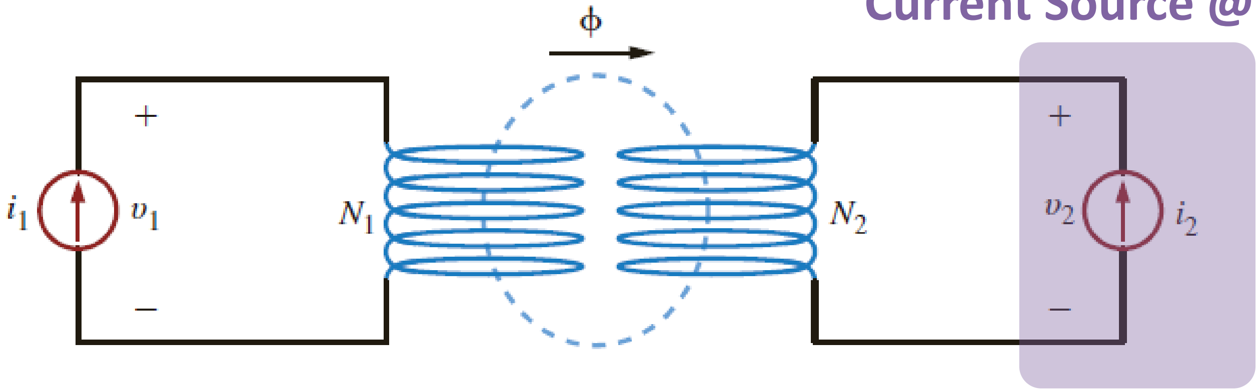
Current Source @ L_2!



$$v_1 = L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + L_{21} \frac{di_1}{dt}$$

Magnetically Coupled Coils



Current Source @ L_2!

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$$v_1 = L_1\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

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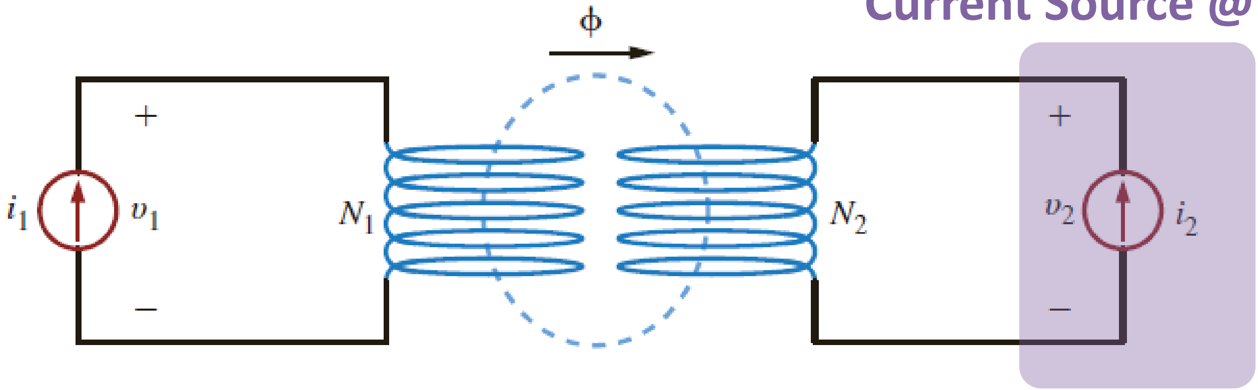
Self Term

Mutual Term

$$L_{12} = L_{21} = M$$

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad v_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Magnetically Coupled Coils



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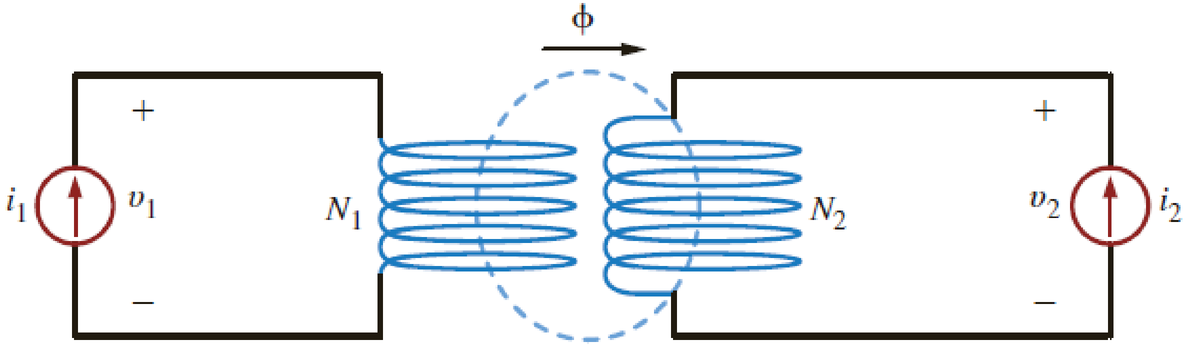
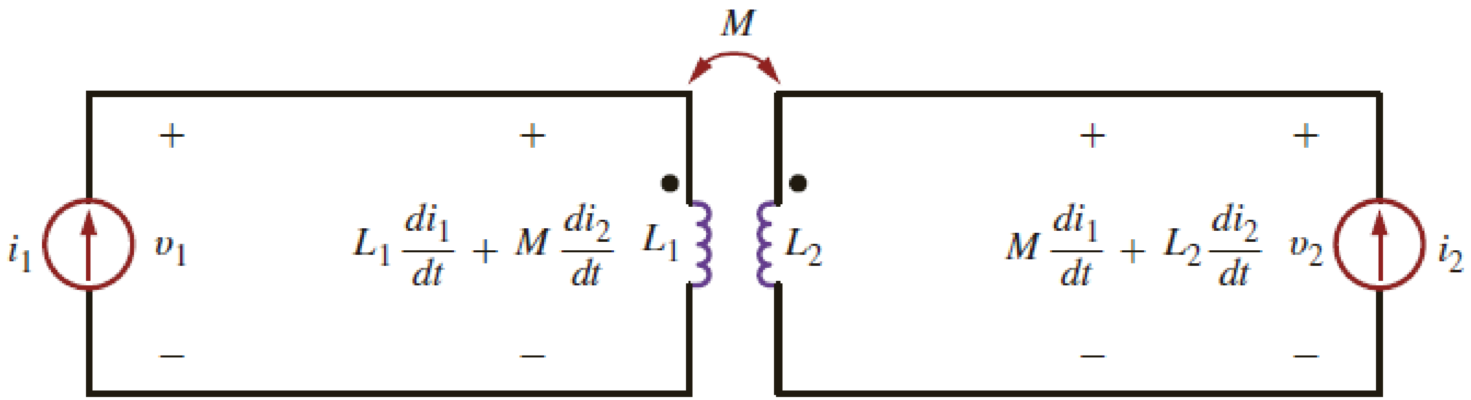
Self Term Mutual Term

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



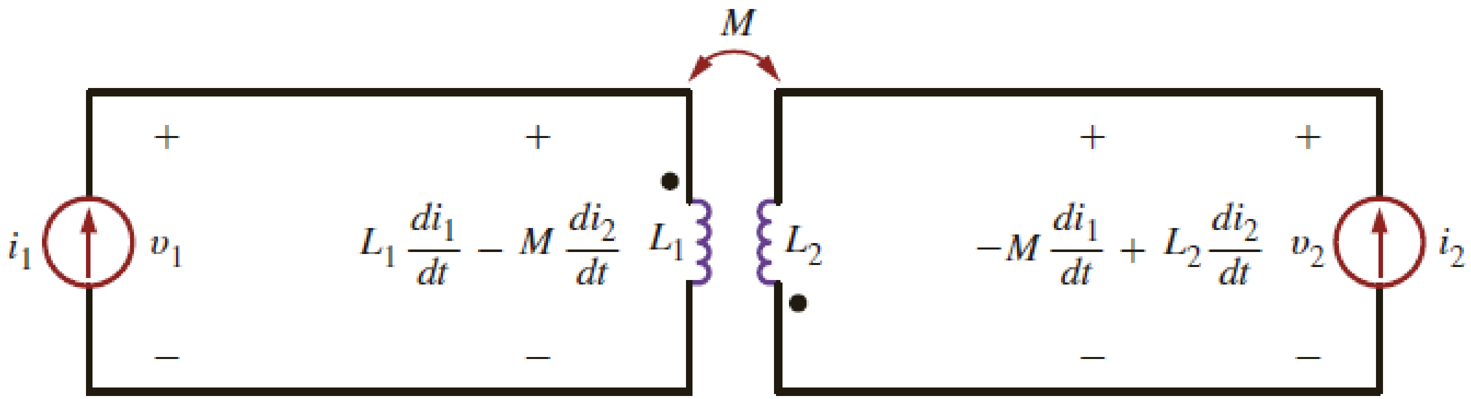
$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Magnetically Coupled Coils → Circuit Diagram



(a)

- **Current enters the dotted terminal → voltage at coupled coil is positive at the dotted terminal**

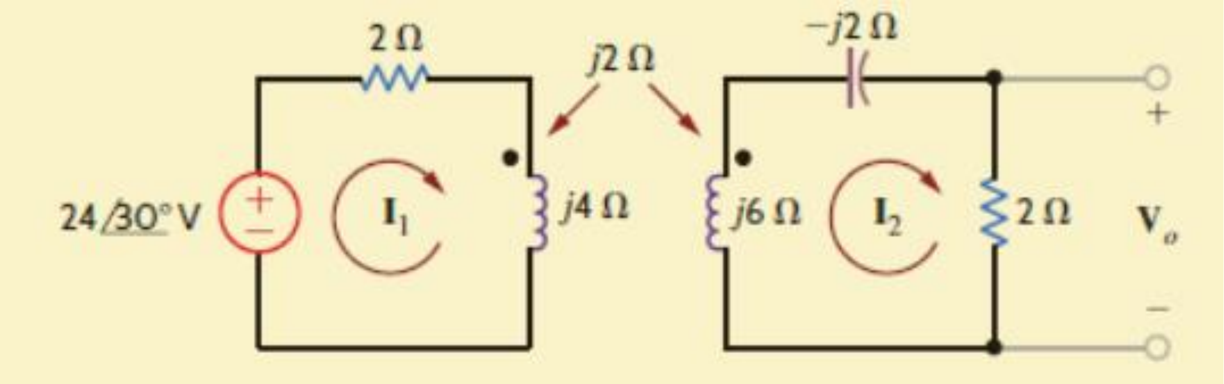


(b)

- **Current enters the undotted terminal → voltage at coupled coil is positive at the undotted terminal**

Example 10.4

Determine V_0 for the given circuit.



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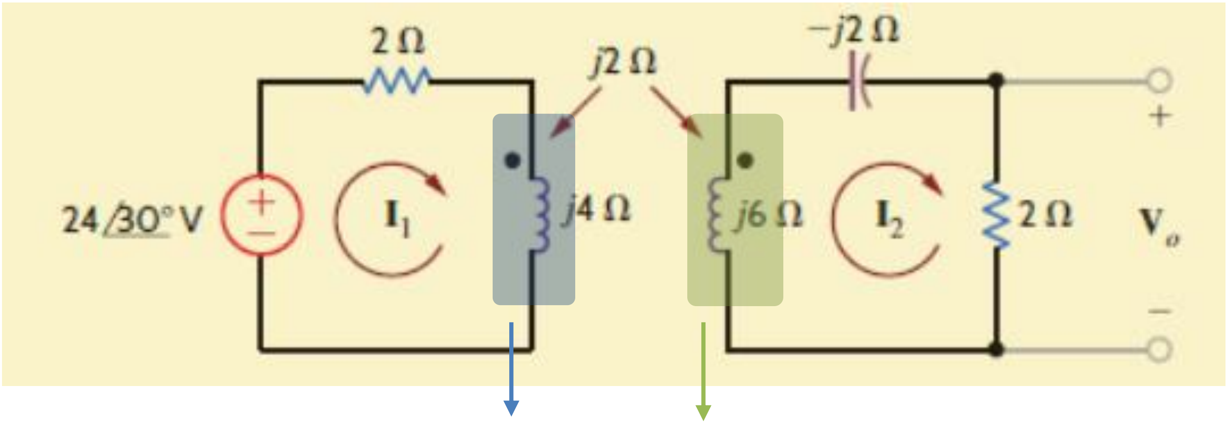
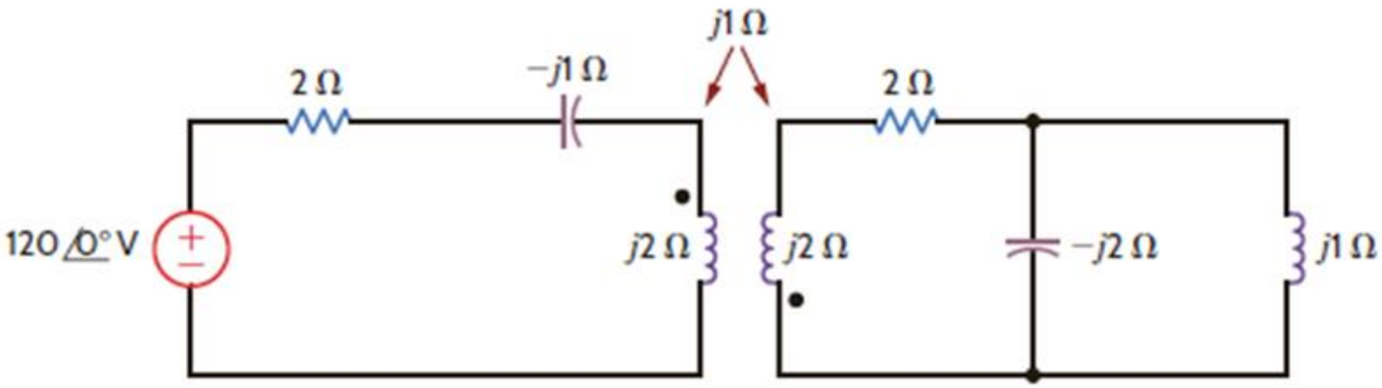


$V_1 = jX_{L1}I_1 - jX_{LM}I_2$

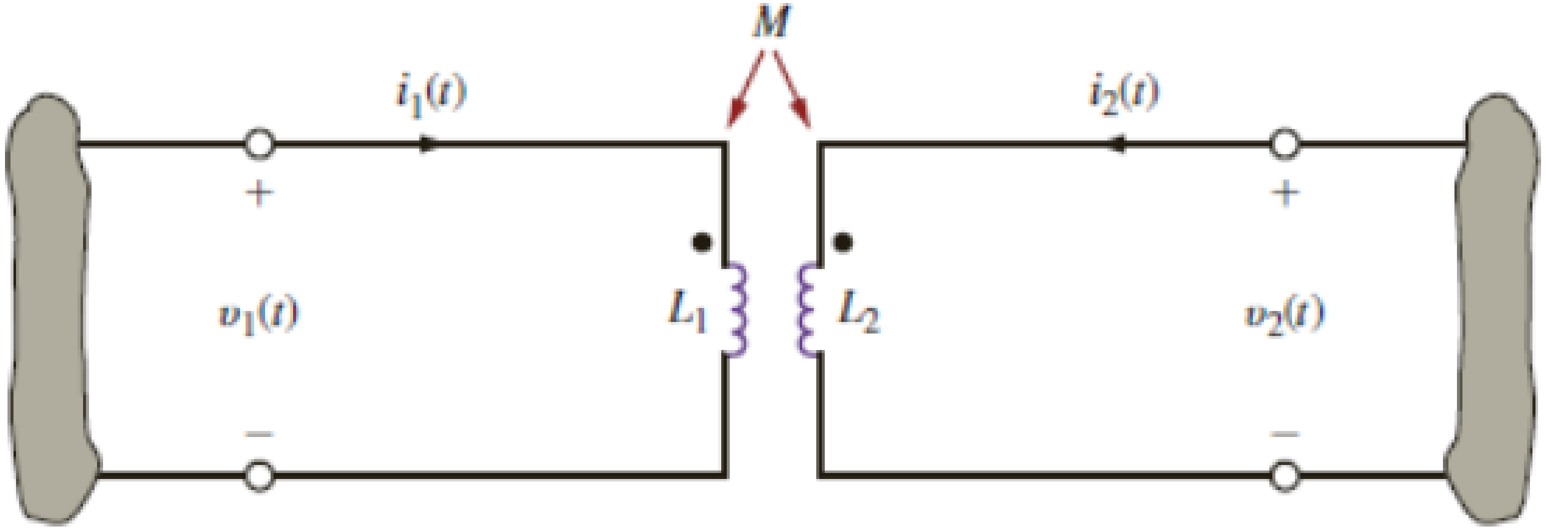
$V_2 = jX_{L2}I_2 - jX_{LM}I_1$

Learning Extension E10.7

Find the impedance seen by the source in the circuit below.



Magnetically Coupled Coils → Energy

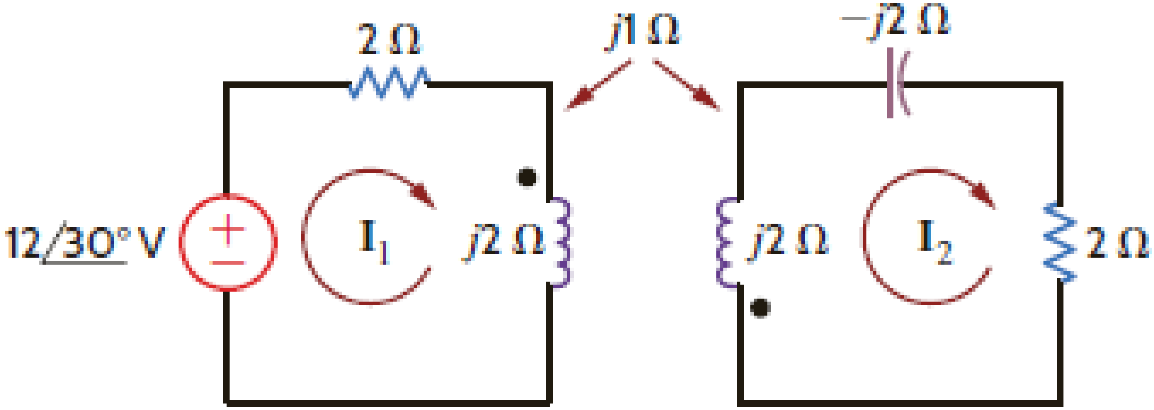


$$w(t)=\frac{1}{2}L_1[i_1(t)]^2+\frac{1}{2}L_2[i_2(t)]^2\pm Mi_1(t)i_2(t)$$

Coefficient of Coupling $k=\frac{M}{\sqrt{L_1L_2}}$ $0\le k\le 1$

Learning Assessment → E10.8

Assuming the network operates at 60Hz, compute the energy stored in the mutually coupled inductors at time t=10ms.



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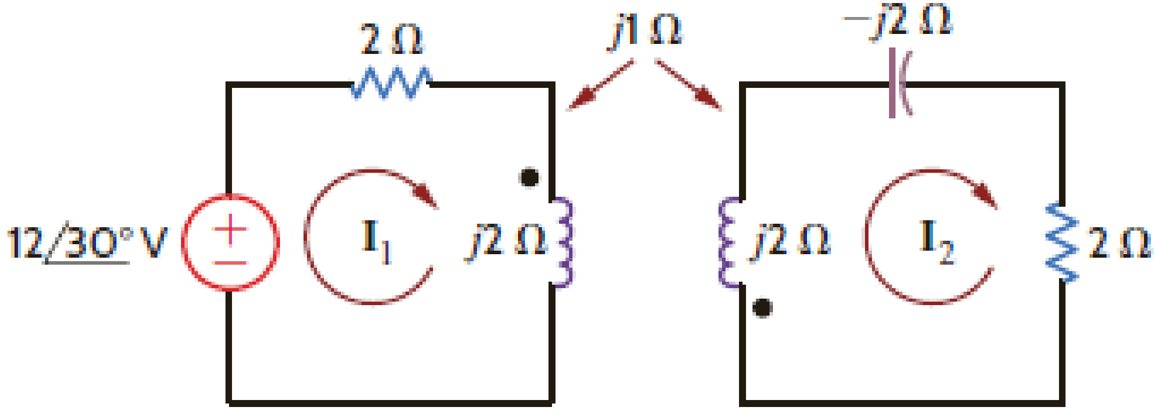

$$I_1, I_2 \rightarrow i_1(t), i_2(t)$$
$$\rightarrow i_1(t = 10ms), i_2(t = 10ms)$$
$$\rightarrow w(t = 10ms)$$

Problem → 10.41

Given the network shown below, determine the value of the capacitor C that will cause the impedance seen by the 24<0º V voltage source to be purely resistive, f=60Hz.