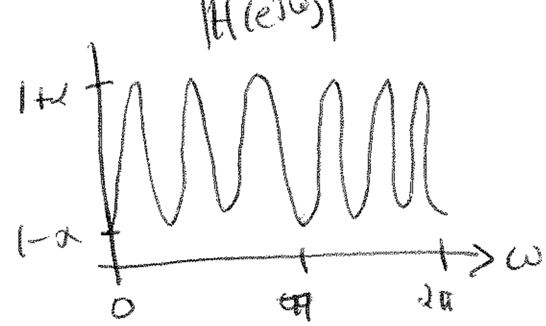
3.51 $h[n] = \delta[n] - \alpha\,\delta[n-R] \qquad |\alpha|<1$

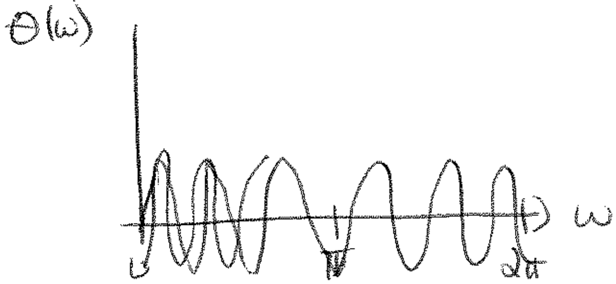
$H(z) = 1 - \alpha z^{-R}$ ROC whole z plane

$H(e^{j\omega}) = 1 - \alpha e^{-j\omega R}$

$R = 6$



6 peaks



6 peaks

3.56 $H_1(z) = 1 + a z^{-1} + b z^{-2}$

$H_2(z) = \dfrac{1}{1 - c z^{-1}} \qquad |z| > |c|$

$H_3(z) = \dfrac{1}{1 - d z^{-1}} \qquad |z| > |d|$

$$H_{eq}(z) = H_1(z)\,H_2(z)\,H_3(z) = \frac{1 + a z^{-1} + b z^{-2}}{(1 - c z^{-1})(1 - d z^{-1})}$$

$$= \frac{1 + a z^{-1} + b z^{-2}}{1 - (c+d) z^{-1} + cd\, z^{-2}}$$

$$H_{eq}(e^{j\omega}) = \frac{1 + a e^{-j\omega} + b e^{-j2\omega}}{1 - (c+d) e^{-j\omega} + cd\, e^{-j2\omega}}$$

for $|H_{eq}(e^{j\omega})| = 1$

$a = -c + d$

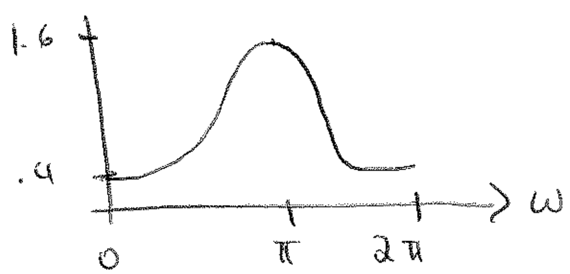
$b = cd$

3.54 $h[n] = a_1\,\delta[n] + a_2\,\delta[n-1] + a_3\,\delta[n-2] + a_4\,\delta[n-3]$
$+ a_5\,\delta[n-4]$

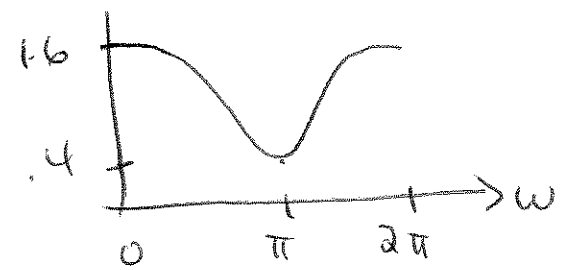
To have linear phase $\quad h[n] = h[2\tau_0 - n]$
(if $h(n)$ is real)

so $a_1 = a_5$
$a_2 = a_4$

3.75 a. $|H_A(e^{j\omega})|$



$|H_B(e^{j\omega})|$



b. $h_c[n] = (-1)^n h_A[n] = e^{j\pi n} h_A[n]$

$$H_c(e^{j\omega}) = \sum_{n=-\infty}^{\infty} e^{j\pi n} h_A[n]\, e^{-j\omega n} = \sum_{n=-\infty}^{\infty} h_A[n]\, e^{-j(\omega-\pi)n}$$

$= H_A(e^{j(\omega-\pi)})$ — H_c is H_A shifted by π.

3.83 $h[n] = (-.5)^n \mu[n]$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} = \frac{1}{1+.5e^{-j\omega}}$$

$$\left|H(e^{j\frac{\pi}{5}})\right| = .6969 \qquad \theta\left(\frac{\pi}{5}\right) = .2063$$

$$x[n] = \sin\left(\frac{\pi}{5}n\right)\mu[n] \qquad y[n] = .6969 \sin\left(\frac{\pi}{5}n + .2063\right)\mu[n]$$

1. $h_1[n] = .3\delta[n] + .8\delta[n-1] + .3\delta[n-2]$

a. $H_1(e^{j\omega}) = .3 + .8e^{-j\omega} + .3e^{-j2\omega}$

$$= e^{-j\omega}\left[.3e^{j\omega} + .8 + .3e^{-j\omega}\right] = \left[.8 + .6\cos(\omega)\right]e^{-j\omega}$$

2. $h_2[n] = .2\delta[n] + .8(.5)^n \mu[n]$

$$H_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_2[n]e^{-j\omega n} = .2 + .8\sum_{n=0}^{\infty}(.5)^n e^{-j\omega n}$$

$$= .2 + .8 \frac{1}{1-.5e^{-j\omega}}$$

$$\left|H_2(e^{j\omega})\right| = \frac{\sqrt{[1-.1\cos(\omega)]^2 + [.1\sin(\omega)]^2}}{\sqrt{[1-.5\cos(\omega)]^2 + [.5\sin(\omega)]^2}}$$

$$\theta_2(\omega) = \tan^{-1}\left(\frac{.1\sin(\omega)}{1-.1\cos(\omega)}\right) - \tan^{-1}\left(\frac{.5\sin(\omega)}{1-.5\sin(\omega)}\right)$$

1. a. linear phase

   b. not linear phase

   c. not linear phase

   d. linear phase

2. a. $\theta(\omega) = -\omega \qquad \tau(\omega) = 1$

   b. $\theta(\omega) = -\tan^{-1}\left(\frac{.3\sin(\omega)}{1-.3\sin(\omega)}\right)$

   $$\tau(\omega) = \frac{.3\cos(\omega)\left[-1-.3\cos(\omega)\right] + .3^2\sin^2(\omega)}{\left[1-.3\cos(\omega)\right]^2 + \left[.3\sin(\omega)\right]^2}$$

   c. $H(e^{j\omega}) = .2 - .3e^{-j\omega} - .2e^{-j2\omega}$

   $$= e^{-j\omega}\left(.2e^{j\omega} - .3 - .2e^{-j\omega}\right) = \left(j2(.2)\sin(\omega) - .3\right)e^{-j\omega}$$

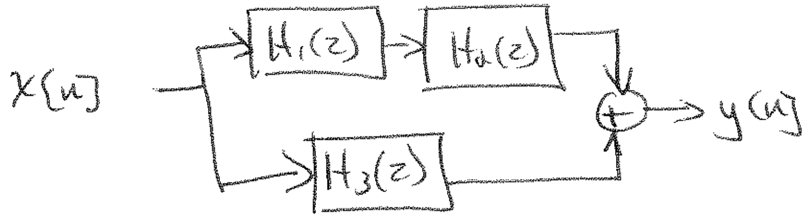
   $$= e^{-j\omega}\left(j(.4)\sin(\omega) - .3\right)$$

   $$\theta(\omega) = -\omega + \tan^{-1}\left(\frac{.4\sin(\omega)}{-.3}\right)$$

   d. $\theta(\omega) = -1.5\omega$

   $\tau(\omega) = 1.5$

6.37



$$H_{eq}(z) = \left[H_1(z) H_2(z)\right] + H_3(z)$$

$$= (1.2 + 3.3z^{-1} + .7z^{-2})(-4.1 - 2.5z^{-1} + .9z^{-2}) + (2.3 + 4.3z^{-1} + .8z^{-2})$$

6.38 a. $y[n] - .1y[n-1] + .14\,y[n-2] + .49\,y[n-3] = 5x[n] + 9.5x[n-1] + 1.4x[n-2] - 24x[n-3]$

$$H(z) = \frac{5 + 9.5z^{-1} + 1.4z^{-2} - 24z^{-3}}{1 - .1z^{-1} + .14z^{-2} + .49z^{-3}}$$

$$= \frac{(5 + 9.5z^{-1} + 1.4z^{-2} - 24z^{-3})\,z^3}{(z + .7)(z - .4 - j.7348)(z - .4 + j.7348)}$$

all poles in unit circle, stable.

b. $y[n] - .5y[n-1] + .1y[n-2] + .3y[n-3] - .0936\,y[n-4] = 5x[n] + 16.5x[n-1] + 14.7x[n-2] - 22.04x[n-3] - 33.6x[n-4]$

$$H(z) = \frac{5 + 16.5z^{-1} + 14.7z^{-2} - 22.04z^{-3} - 33.6z^{-4}}{1 - .5z^{-1} + .1z^{-2} + .3z^{-3} - .0936z^{-4}}$$

$$= \frac{(5 + 16.5z^{-1} + 14.7z^{-2} - 22.04z^{-3} - 33.6z^{-4})\,z^4}{(z + .6)(z - .3)(z - .4 - j.6)(z - .4 + j.6)}$$

all poles inside unit circle, so stable.

6.40 $H(z) = \dfrac{1-3.3z^{-1}+.36z^{-2}}{1+.3z^{-1}-.18z^{-2}}$ causal

$$= \frac{z^2 - 3.3z + .36}{(z+.6)(z-.3)} = A_0 + \frac{A_1 z}{z+.6} + \frac{A_2 z}{z-.3}$$

$A_0 = 0$

$$A_1 = H(z)\left|\frac{z+.6}{z}\right|_{z=-.6} = \left.\frac{z^2-3.3z+.36}{(z-.3)}\right|_{z=-.6} = -3$$

$$A_2 = H(z)\left.\frac{z-.3}{z}\right|_{z=.3} = \left.\frac{z^2-3.3z+.36}{(z+.6)}\right|_{z=.3} = -.6$$

$$H(z) = -3\frac{1}{1+.6z^{-1}} - .6\frac{1}{1-.3z^{-1}} \qquad |z| > .6$$

$$h[n] = -3(-.6)^n \mu[n] - .6(.3)^n \mu[n]$$

b. $x[n] = 2.1(.4)^n \mu[n] + .3(-.3)^n \mu[n]$

$$X(z) = 2.1\frac{1}{1-.4z^{-1}} + .3\frac{1}{1+.3z^{-1}} \qquad |z| > .4$$

$$= \frac{2.1z}{z-.4} + \frac{.3z}{z+.3} = \frac{2.1z(z+.3) + .3z(z-.4)}{(z-.4)(z+.3)}$$

$$= \frac{2.1z^2 + 6.3z + .3z^2 - .12z}{(z-.4)(z+.3)} = \frac{2.4z^2 + 6.18z}{(z-.4)(z+.3)}$$

$$Y(z) = X(z)\,H(z)$$

$$= \frac{z(2.4z + 6.18)}{(z-.4)(z+.3)} \cdot \frac{z^2 - 3.3z + .36}{(z+.6)(z-.3)}$$

$$= A_0 + \frac{A_1 z}{z-.4} + \frac{A_2 z}{z+.3} + \frac{A_3 z}{z+.6} + \frac{A_4 z}{z-.3} \qquad |z|>.6$$

$A_0 = 0$

$$A_1 = H(z)\frac{z-.4}{z}\Big|_{z=.4} = \frac{z(2.4z+6.18)(z^2-3.3z+.36)}{(z+.3)(z+.6)(z-.3)}\Big|_{z=.4}$$

$= 32.64$

$$A_1 = H(z)\frac{z+.3}{z}\Big|_{z=-.3} = -18.72$$

$$A_2 = H(z)\frac{z+.6}{z}\Big|_{z=-.6} = 28.44$$

$$A_3 = H(z)\frac{z-.3}{z}\Big|_{z=.3} = 20.7$$

$$Y(z) = 32.64\frac{1}{1-.4z^{-1}} - 18.72\frac{1}{1+.3z^{-1}} + 28.44\frac{1}{1+.6z^{-1}} + 20.7\frac{1}{1-.3z^{-1}}$$

$|z|>.6$

$$y[n] = 32.64(.4)^n\mu[n] - 18.72(-.3)^n\mu[n] + 28.44(-.6)^n\mu(n) + 20.7(.3)^n\mu[n]$$

6.41 a. $h[n] = (-.4)^n\mu[n] \qquad x[n] = (.2)^n\mu[n]$

$$H(z) = \frac{1}{1+.4z^{-1}} \quad |z|>.4 \qquad X(z) = \frac{1}{1-.2z^{-1}} \quad |z|>.2$$

$$Y(z) = \frac{1}{(1+.4z^{-1})(1-.2z^{-1})} \quad |z|>.4$$

$$= \frac{z^2}{(z+.4)(z-.2)} = A_0 + \frac{A_1 z}{z+.4} + \frac{A_2 z}{z-.2}$$

$A_0 = H(z)\big|_{z=0} = 0$

$A_1 = H(z)\,\frac{z+.4}{z}\Big|_{z=-.4} = \frac{z}{z-.2}\Big|_{z=-.4} = \frac{2}{3}$

$A_2 = H(z)\,\frac{z\cancel{+}-.2}{z}\Big|_{z=.2} = \frac{z}{z+.4}\Big|_{z=.2} = \frac{1}{3}$

$Y(z) = \frac{2}{3}\,\frac{1}{1+.4z^{-1}} + \frac{1}{3}\,\frac{1}{1-.2z^{-1}} \qquad |z| > .4$

$y(n) = \frac{2}{3}(-.4)^n\mu[n] + \frac{1}{3}(.2)^n\mu[n]$

6.43

a. $y[n] - .2y(n-1] - .08y[n-2] = 2x[n]$

$H(z) = \frac{2}{1-.2z^{-1}-.08z^{-2}}$ causal

poles ~~roots~~ at $.4, -.2$ so ROC is $|z| > .4$

b. $H(z) = \frac{2}{(z-.4)(z+.2)} \qquad |z| > .4$

$= A_0 + A_1\frac{z}{z-.4} + A_2\frac{z}{z+.2}$

$A_0 = H(0) = -25$

$A_1 = H(z)\,\frac{z-.4}{z}\Big|_{z=.4} = \frac{2}{(z+.2)z}\Big|_{z=.4} = 8.333$

$A_2 = H(z)\,\frac{z+.2}{z}\Big|_{z=-.2} = \frac{2}{(z-.4)(z)}\Big|_{z=-.2} = 16.67$

$H(z) = -25 + 8.333\,\frac{1}{1-.4z^{-1}} + 16.67\,\frac{1}{1+.2z^{-1}} \qquad |z| > .4$

$h[n] = -.25\,\delta[n] + 8.33(.4)^n\mu[n] + 16.67(-.2)^n\mu[n]$

c. $s[n] = \mu[n] \Leftrightarrow S(z) = \dfrac{1}{1-z^{-1}} \quad |z|>1$

$$Y(z) = S(z)\,H(z) = \frac{2z}{(z-.4)(z+.2)(z-1)} \qquad |z|>1$$

$$= A_0 + A_1 \frac{z}{z-.4} + A_2 \frac{z}{z+.2} + A_3 \frac{z}{z-1}$$

$A_0 = 0$

$$A_1 = \cancel{H}(z)\left.\frac{z-.4}{z}\right|_{z=.4} = \frac{2}{(.6)(-.6)} = -5.556$$

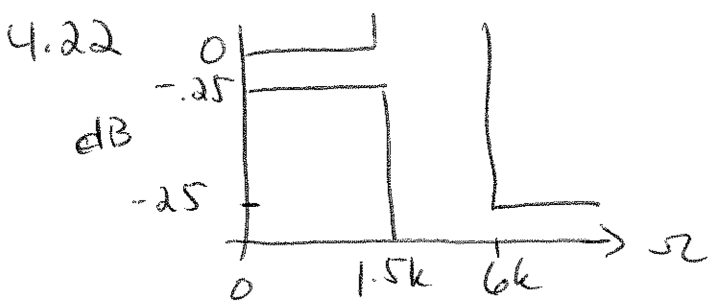
$$A_2 = Y(z)\left.\frac{z+.2}{z}\right|_{z=-.2} = \frac{2}{(-.6)(-1.2)} = 2.778$$

$$A_3 = Y(z)\left.\frac{z-1}{z}\right|_{z=1} = \frac{2}{(.6)(1.2)} = 2.778$$

$$Y(z) = -5.556\frac{1}{1-.4z^{-1}} + 2.778\frac{1}{1+.2z^{-1}} + 2.778\frac{1}{1-z^{-1}} \quad |z|>1$$

$$y[n] = -5.556\,(.4)^n\mu[n] + 2.778\,(-.2)^n\mu[n] + 2.778\,\mu[n]$$

4.22



0 dB ⇒ 1

-.25 dB ⇒ .9716

-25 dB ⇒ .05623

$K = 1$

$$\frac{K}{\sqrt{1+\varepsilon^2}} = .9716 \qquad \varepsilon^2 = .05925$$

$$\frac{K}{A} = .05623 \qquad A = 17.78$$

$$N \geq \frac{\log\left(\frac{A^2-1}{\varepsilon^2}\right)}{2\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\log\left(\frac{17.78^2-1}{\varepsilon^2}\right)}{2\left(\frac{6k}{1.5k}\right)} = 3.09$$

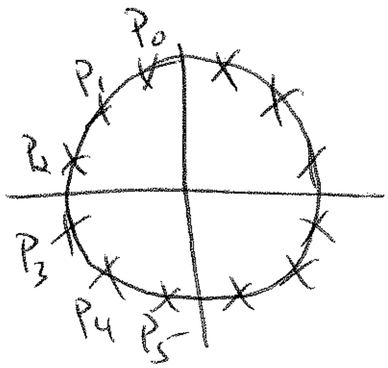
Choose $N=4$

4.23 unity 3 dB cutoff $(\Omega_c = 1)$

pole locations

$$P_k = \Omega_c e^{j\frac{(N+2l+1)}{2N}\pi} \qquad l = 0, \dots 2N-1$$

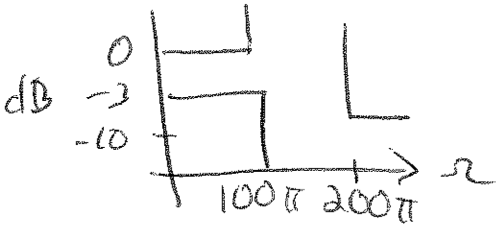
6^{th} order

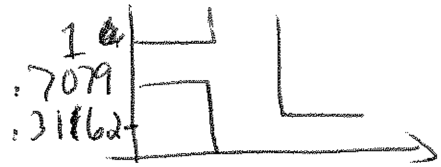


$P_0 = 1\, e^{j\frac{7}{12}\pi}$ $P_2 = 1\, e^{j\frac{11}{12}\pi}$ $P_4 = 1\, e^{j\frac{15}{12}\pi}$

$P_1 = 1\, e^{j\frac{9}{12}\pi}$ $P_3 = 1\, e^{j\frac{13}{12}\pi}$ $P_5 = 1\, e^{j\frac{17}{12}\pi}$

1. Design IIR Butterworth filter for





$K = 1$

$\frac{K}{\sqrt{1+\varepsilon^2}} = .7079 \qquad \varepsilon^2 = .9955$

$\frac{K}{A} = .3162 \qquad A = 3.163$

$N \geq \frac{\log\left(\frac{3.163^2-1}{.9955}\right)}{2\log\left(\frac{200\pi}{100\pi}\right)} = 1.588 \qquad \text{Choose } N = 2$

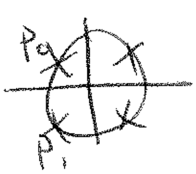
$\frac{100\pi}{(.9955)^{\frac{1}{4}}} \leq \Omega_c \leq \frac{200\pi}{(3.163^2-1)^{\frac{1}{4}}}$

$100\pi \leq \Omega_c \leq 115\pi$

Choose $\Omega_c = 110\pi$

$H_a'(s) = \frac{K\,\Omega_c^2}{(S-P_0)(S-P_1)} =$

Poles



$P_0 = \Omega_c e^{-j\frac{3}{4}\pi} = 110\pi e^{-j\frac{3\pi}{4}} = 110\pi\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$

$P_1 = 110\pi e^{-j\frac{5\pi}{4}} = 110\pi\left(-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$

$H_a(s) = \frac{1\,(110\pi)^2}{\left(S - 110\pi\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\right)\left(S - 110\pi\left(-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\right)}$

$= \frac{(110\pi)^2}{S^2 + (110\pi)\sqrt{2}S + (110\pi)^2}$