Fundamentos de Transformadores y Máquinas Eléctricas

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Clase 5

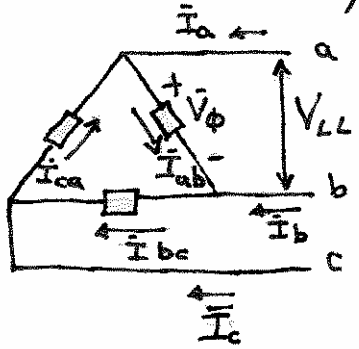
Circuitos Trifásicos Balanceados

$\therefore$ En conexión Y

$$\bar{V}_{LL} = \sqrt{3}\,[\bar{V}_\phi]\,\angle +30^\circ$$
$$\bar{I}_L = \bar{I}_\phi$$

Esto Aplica a todas las 3 fases

II Conexión Delta, Δ ("Delta Conection")

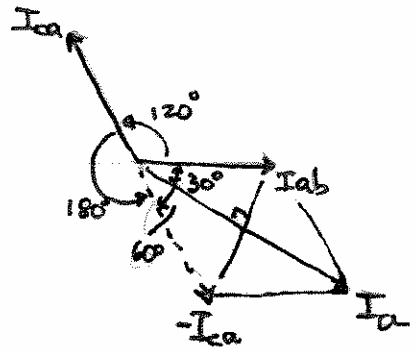


$$\bar{V}_{LL} = \bar{V}_\phi$$
$$\bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca}$$
$$\bar{I}_b = \bar{I}_{bc} - \bar{I}_{ab}$$
$$\bar{I}_c = \bar{I}_{ca} - \bar{I}_{bc}$$

$\bar{I}_{ab} = I_\phi\,\angle 0^\circ \quad \bar{I}_{bc} = \bar{I}_\phi\,\angle -120 \quad \bar{I}_{ca} = I_\phi\,\angle -240^\circ$

Circuitos Trifásicos Balanceados

$$\bar{I}_a = I_\phi \angle 0^\circ - I_\phi \angle +120^\circ$$



$$|I_a| = 2\,|\bar{I}_{ab}| \cos\theta$$

$$= 2\,|\bar{I}_{ab}| \frac{\sqrt{3}}{2}$$

$$|I_a| = \sqrt{3}\,|I_{ab}|$$

$$\bar{I}_a = \sqrt{3}\,[\bar{I}_{ab}] \angle -30^\circ$$

$\therefore$ En conexión Δ

$$\bar{V}_{LL} = \bar{V}_\phi$$

$$\bar{I}_L = \sqrt{3}\,[\bar{I}_\phi] \angle -30^\circ$$

Resumen

Conexión	Voltajes	Corrientes
Y	$\bar{V}_{LL} = \sqrt{3}\,[\bar{V}_\phi]\angle +30^\circ$ $\bar{V}_{ab} = \sqrt{3}\,[\bar{V}_{an}]\angle +30^\circ$ $\bar{V}_{bc} = \sqrt{3}\,[\bar{V}_{bn}]\angle +30^\circ$ $\bar{V}_{ca} = \sqrt{3}\,[\bar{V}_{cn}]\angle +30^\circ$	$\bar{I}_L = \bar{I}_\phi$
Δ	$\bar{V}_{LL} = \bar{V}_\phi$	$\bar{I}_L = \sqrt{3}\,[\bar{I}_\phi]\angle -30^\circ$ $\bar{I}_a = \sqrt{3}\,[\bar{I}_{ab}]\angle -30^\circ$ $\bar{I}_b = \sqrt{3}\,[\bar{I}_{bc}]\angle -30^\circ$ $\bar{I}_c = \sqrt{3}\,[\bar{I}_{ca}]\angle -30^\circ$

Potencia Trifásica

$$\bar{S}_\phi = \bar{V}_\phi \bar{I}_\phi^* \qquad \text{por FASE}$$

En Y: $\bar{S}_{3\phi} = 3\bar{S}_\phi = 3\bar{V}_\phi \bar{I}_\phi^* = \frac{\sqrt{3}\bar{V}_{LL}}{\sqrt{3}\angle 30^\circ}\bar{I}_L^* \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$$\bar{S}_{3\phi} = \sqrt{3}\,\bar{V}_{LL}\bar{I}_L^* \angle -30^\circ$$

En Δ: $\bar{S}_{3\phi} = 3\bar{S}_\phi = 3\bar{V}_\phi \bar{I}_\phi^* = 3\bar{V}_{LL}\left(\frac{\bar{I}_L}{\sqrt{3}\angle -30^\circ}\right)^* = \frac{\sqrt{3}\bar{V}_{LL}\bar{I}_L^*}{\sqrt{3}\angle 30^\circ}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$$\bar{S}_{3\phi} = \sqrt{3}\,\bar{V}_{LL}\bar{I}_L^* \angle -30^\circ = 3\bar{V}_\phi \bar{I}_\phi^*$$

Resumen

En Resumen:

Potencia Trifásica

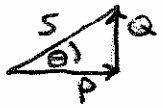
$$\bar{S}_{3\phi} = \sqrt{3}\,\bar{V}_{LL}\,\bar{I}_L^{*}\,\angle -30^\circ = 3\,\bar{V}_\phi\,\bar{I}_\phi^{*} \qquad \text{para } \Delta \text{ y } Y$$

de aqui,

$$|\bar{S}_{3\phi}| = \sqrt{3}\,|\bar{V}_{LL}||\bar{I}_L|$$

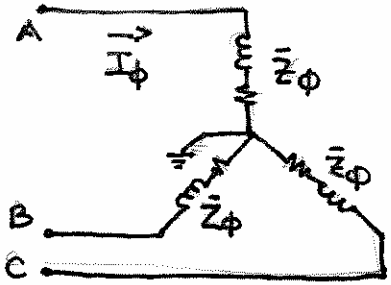
$$P_{3\phi} = \sqrt{3}\,|\bar{V}_{LL}||\bar{I}_L|\cos\theta$$

$$Q_{3\phi} = \sqrt{3}\,|\bar{V}_{LL}||\bar{I}_L|\sin\theta$$



$$P.F. = \cos\theta = \frac{P_{3\phi}}{S_{3\phi}} = \frac{\cancel{\sqrt{3}V_{LL}I_L}\cos\theta}{\cancel{\sqrt{3}V_{LL}I_L}}$$

Ejemplo:



$|\bar{V}_{AB}| = 208\,V$

$\bar{Z}_\phi = 4 + j3\,\Omega = 5\angle 36.9^\circ\ \Omega$

$\bar{S}_{3\phi} = ?$

$$\bar{S}_{3\phi} = 3\bar{V}_\phi \bar{I}_\phi^*$$

$$\bar{V}_{an} = \frac{208}{\sqrt{3}}\angle 0^\circ \qquad \bar{I}_\phi = \frac{\bar{V}_{an}}{\bar{Z}_\phi} = \frac{208\angle 0^\circ}{\sqrt{3}(5\angle 36.9^\circ)} = 24\angle -36.9^\circ\,A$$

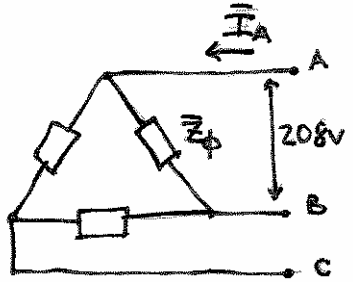
$$\bar{S}_{3\phi} = 3\,(120\angle 0^\circ_V)(24\angle -36.9^\circ_A)^* = 8632.8\angle +36.9^\circ\ VA$$

$$\bar{S}_{3\phi} = (6919.5 + j\,5195.3)\ VA$$

$P_{3\phi}$ $\quad$ $Q_{3\phi}$

$$P.F. = \cos(36.9^\circ)\ lag. = 80\%\ lag.$$

Ejemplo:



$\bar{Z}_\phi = 12 + j9\ \Omega = 15\angle 36.9^\circ\ \Omega$

$\bar{V}_\phi = \bar{V}_{LL} = 208\angle 0^\circ\ V$

$$\bar{I}_\phi = \frac{\bar{V}_\phi}{\bar{Z}_\phi} = \frac{208\angle 0^\circ\ V}{15\angle 36.9^\circ\ \Omega} = 13.87\angle -36.9^\circ\ A$$

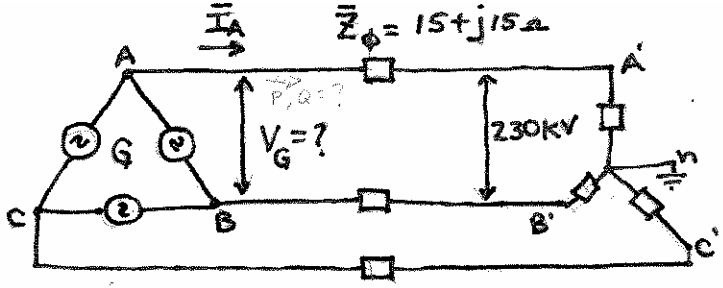
$$\bar{S}_{3\phi} = 3\ \bar{V}_\phi\ \bar{I}_\phi^* = 3\left(208\angle 0^\circ\ V\right)\left(13.87\angle -36.9^\circ A\right)^* = \left(8652.8\angle 36.9^\circ\ VA\right)$$

$\bar{S}_{3\phi} = (6919.5 + j\ 5195.3)\ VA$

P Q

PF = cos θ = cos (36.9°) lag. = 80% lag.

Ejemplo:



$P_L = 20MW$ @ .8 pf. lag

(3φ, si no indica lo contrario)

En la Carga:

I. $\bar{V}_{AB} = 230\angle 30°\ KV$

$\bar{V}_{An} = \frac{230}{\sqrt{3}}\angle 0°\ KV$

II. $\theta = \cos^{-1}(.8) = -36.9°$ negativo por ser lagging.

III. $P_{3\phi} = \sqrt{3}|\bar{V}_{LL}||\bar{I}_L|\cos\theta$

$$I_L = \frac{P_{3\phi}}{\sqrt{3}|\bar{V}_{LL}|\cos\theta} = \frac{20MW}{\sqrt{3}(230KV)(.8)} = 62.76\ A$$

$\bar{I}_{A'} = 62.76\angle -36.9°\ A$

Entre Fuente y Línea:

IV. $\bar{V}_{An} = \bar{V}_{A'n} + \bar{V}_{Line} = \bar{V}_{A'n} + \bar{I}_{A'} \bar{Z}_{\phi} = \left(230/\sqrt{3}\angle 0^\circ \text{ KV}\right) + \left(62.76\angle -36.9^\circ \text{ A}\right)(15+j15)$

$\quad = (132.8\angle 0^\circ \text{ KV}) + (1331.34\angle 8.1^\circ)$

$\bar{V}_{An} = 133.3\angle .081^\circ \text{ KV}$

En la Fuente

V. $\bar{V}_{AB} = \sqrt{3}\, \bar{V}_{An} \angle +30^\circ = 230.9\angle 30.08^\circ \text{ KV}$

$\bar{I}_{\phi G} = \dfrac{\bar{I}_A}{\sqrt{3}\angle -30^\circ} = \dfrac{62.76\angle -36.9^\circ \text{ A}}{\sqrt{3}\angle -30^\circ} = 36.23\angle -6.9^\circ \text{ A}$

VI. $\bar{S}_{3\phi} = 3\bar{V}_{\phi}\bar{I}_{\phi}^{*} = 3\left(230.9\angle 30.08^\circ \text{ KV}\right)\left(36.23\angle -6.9^\circ \text{ A}\right)^{*}$

$\quad = 25.10\angle 36.98^\circ \text{ MVA}$

$\bar{S}_{3\phi} = (20.05 + j\,15.1)\text{ MVA}$ $\qquad$ P.F. $= \dfrac{20.05}{25.1} = 0.7988$