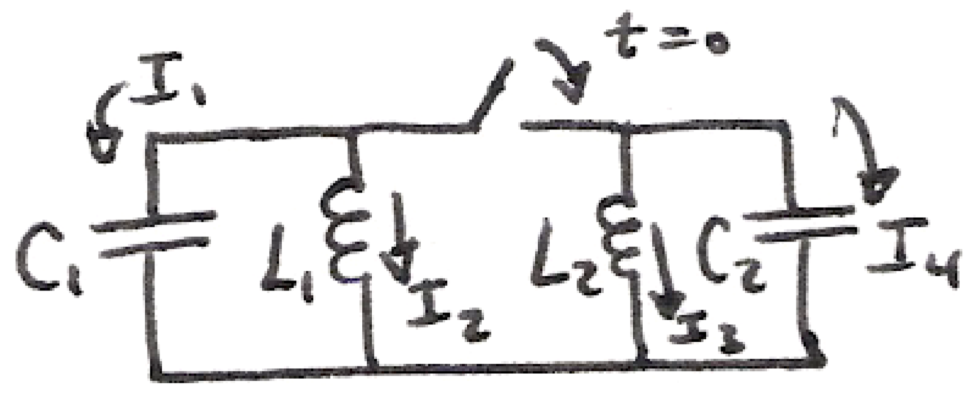
P 3.12



$C_1 = 100\,\mu F \qquad L_1 = 10\,mH$

$C_2 = 40\,\mu F \qquad L_2 = 20\,mH$

When switch closes all elements have common voltage

$$V = \frac{1}{C_1}\int I_1\,dt = L_1\frac{dI_2}{dt} = L_2\frac{dI_3}{dt} = \frac{1}{C_2}\int I_4\,dt \quad \text{and}$$

$I_1 + I_2 + I_3 + I_4 = 0$ therefore,

$$C_1\frac{d^2}{dt}V + \frac{V}{L_1} + \frac{V}{L_2} + C_2\frac{d^2}{dt}V = 0$$

$$(C_1+C_2)\,V'' + \left(\frac{1}{L_1}+\frac{1}{L_2}\right)V = 0 \quad ; \quad \frac{L_1+L_2}{L_1L_2} = \frac{1}{L_1}+\frac{1}{L_2}$$

$$\mathcal{L}\left\{V'' + \left(\frac{L_1+L_2}{L_1L_2(C_1+C_2)}\right)V = 0\right\} \quad ; \quad \omega_0^2 = \frac{L_1+L_2}{L_1L_2(C_1+C_2)}$$

$$s^2 v(s) - s\,V(0) - V'(0) + \omega_0^2\,v(s) = 0$$

$V(0) = 0$ as established by the problem.

$-I_2(0) = (C_1+C_2)\frac{dV}{dt}$ then $V'(0) = -\frac{I_2(0)}{C_1+C_2}$

then

$$v(s) = -\frac{I_2(0)}{C_1+C_2}\cdot\frac{1}{s^2+\omega_0^2}$$

that gives, $V(t) = \frac{-I_2(0)}{\omega_0(C_1+C_2)}\sin\omega_0 t \quad ; \quad \omega_0 = \sqrt{\frac{L_1+L_2}{L_1L_2(C_1+C_2)}}$

$= 1035$ rad/s

(a) $V_{max} = 3450\,V$

$$I_3 = \frac{1}{L_2}\int V\,dt = \frac{1}{L_2}\left(\frac{-I_2(0)}{\omega_0^2(C_1+C_2)}\right)\cos\omega_0 t + \text{const.}$$

$t=0 \quad I_3 = 0$ then $I_3 = \frac{-I_2(0)}{L_2\omega_0^2(C_1+C_2)}(\cos\omega_0 t - 1)$

$I_{peak} = (166\,A)\,2$, when $\omega_0 t = \pi$

(b) $I_{peak} = 332\,A$

(c) The circuit has only on frecuency: $f_0 = \frac{\omega_0}{2\pi} = 164.7\,Hz$

(d) Before $t=0$, $Z_0 = \left(\frac{10m}{100\mu}\right)^{1/2} = 10\,\Omega \quad V_{peak} = 500\,(10) = 5\,KV$