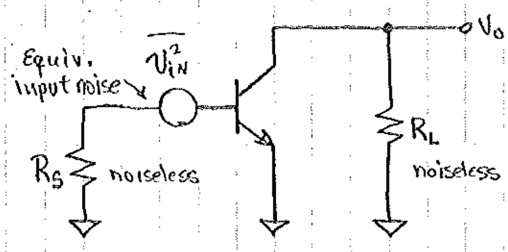
Equivalent Noise & Minimum Detectable signal

- Noise is affected by the circuit gain
- For meaningful comparisons, sources are expressed in terms of an equivalent input noise signal
- This is a source that would produce the same output noise as the circuit in consideration.



$$\overline{V_o^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2} \overline{V_{IN}^2}$$

Therefore

$$V_{IN}^2 = 4KT(R_S + r_b) + (R_S + r_b)^2\, 2q I_B + \frac{1}{g_m^2 R_L^2} \frac{|Z + r_b + R_S|^2}{|Z|^2} R_L^2 \left(4KT\frac{1}{R_L} + 2q I_C\right)$$

Example: Input noise in circuit CE. for 0-1MHz b.w.

Using the total output noise

$$A_v = \frac{r_\pi}{r_b + r_\pi + R_S} g_m R_L = \frac{26000}{200 + 26000 + 500} \cdot \frac{5000}{260} = 18.7$$

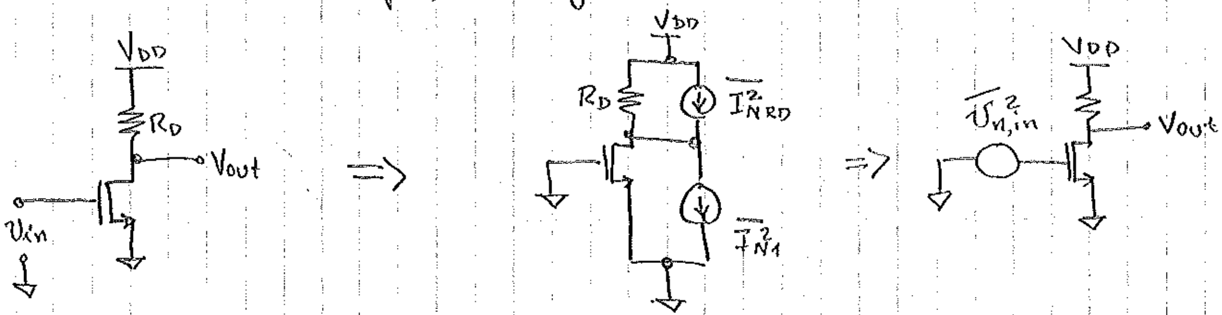
Flat spectrum @ 1MHz $\overline{v_{in}^2} = \frac{\overline{V_{oT}^2}}{A_v^2} = \frac{5 \times 10^{9}}{(18.7)^2} = 14.3 \times 10^{-12}\, V^2$

$$v_{inT_{RMS}} = \sqrt{\overline{v_{inT}^2}} = 3.78\, \mu V$$

This level is the minimum detectable signal for the circuit

Another example:

A common source amplifier stage



$$\underbrace{\overline{I^2_{NRD}} = 4KT/R_D}_{\text{Thermal } R_D} \qquad \overline{I^2_{N1}} = \underbrace{4KT\left(\tfrac{2}{3}\right)g_m}_{\text{channel Thermal}} + \underbrace{K_1 g_m^2 \frac{1}{C_{ox}WLf}}_{\text{Flicker}}$$

$$\overline{V^2_{n\,out}} = \left(4KT(2/3)g_m + \frac{K_1}{C_{ox}WL}\cdot\frac{1}{f}g_m^2 + \frac{4TK}{R_D}\right)R_D^2$$

The equivalent input noise: From 3rd circuit $\overline{V^2_{n,out}} = \overline{V^2_{n,in}} \cdot A_v^2$

$$\overline{V^2_{n,in}} = \frac{\overline{V^2_{n\,out}}}{A_v^2}$$

$A_v = g_m R_D$ (neglecting r_o) $\Rightarrow$

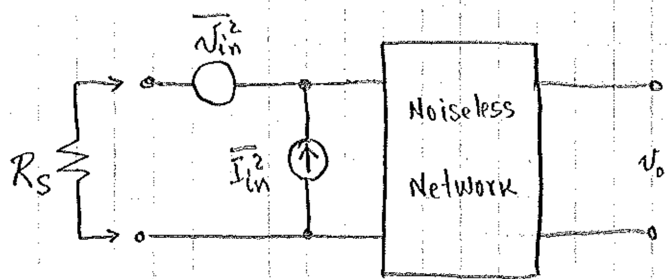
$$\overline{V^2_{n\,in}} = \left[4KT\left(\tfrac{2}{3}\right)g_m + \frac{K_1}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4KT}{R_D}\right)R_D^2 \cdot \frac{1}{g_m^2 R_D^2}$$

$$= \frac{8KT}{3g_m} + \frac{K_1}{C_{ox}WL\,f} + \frac{4KT}{g_m^2 R_D} = 4KT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K_1}{C_{ox}WLf}$$

NOTES:

- The equivalent input noise represents the minimum detectable signal of a circuit for an acceptable SNR
- The EIN is a mathematical model. Cannot be measured
- Modeling the EIN using a single voltage source produces an incomplete model. It would imply that noise is reduced if the source or input impedances are increased: wrong!
- A better model uses a series voltage source $\overline{V^2_{in}}$ and a parallel current source $\overline{I^2_{in}}$ in the input port, considering the correlation between both.

To compute $\overline{V_{nin}^2}$ and $\overline{I_{nin}^2}$ consider the extreme cases: $R_S = 0$ & $R_S = \infty$



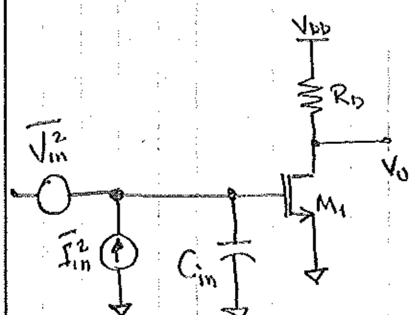
1) Represent the circuit with noise generators and EIG

2) $R_S = 0$: Short-circuit the input of both representations, compute v_o and equate to find v_{in}

3) $R_S = \infty$: Open-circuit both circuit inputs, compute v_o and equate to find i_{in}

- When $R_S = 0$, the short-ckt cancels $\overline{I_{in}^2}$ and only $\overline{V_{in}^2}$ contributes to $\overline{V_o^2}$
- When $R_S = \infty$, $\overline{V_{in}^2}$ has no effect and only $\overline{I_{in}^2}$ affects $\overline{V_o^2}$.

Example: Equivalent Noise Generators for CS amplifier with input impedance C_{in}



$$\boxed{\overline{V_{nin}^2} = 4KT\left(\frac{2}{3g_m}\right) + \frac{K_1}{C_{ox}WLf} + \frac{4KT}{g_m^2 R_D}}$$

(From previous example)

With $R_S = \infty$, $v_{gs} = I_{in} \cdot Z_c = I_{in}\left(\frac{1}{j\omega C}\right)$; $v_o = v_{gs}\, g_m R_D$

Thus $\overline{V_{nout}^2} = \overline{I_{in}^2}\left(\frac{1}{\omega C_{in}}\right)^2 g_m^2 R_D^2$

Equating the output noise $\qquad \overline{V_{in}^2} \cdot g_m^2 R_D^2 = \overline{V_o^2}$ ← Taken from previous example

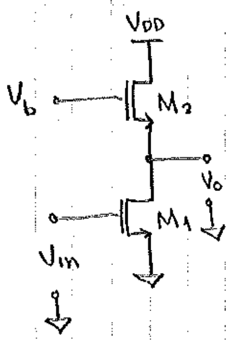
$$\overline{I_{in}^2}\left(\frac{1}{\omega C_{in}}\right)^2 g_m^2 \cancel{R_D^2} = \overbrace{\left(4KT\left(\frac{2}{3}g_m\right) + \frac{K_1}{C_{ox}WL}\cdot\frac{1}{f}\, g_m^2 + \frac{4KT}{R_D}\right)\cancel{R_D^2}}$$

$$\boxed{\overline{I_{in}^2} = \left(\frac{\omega C_{in}}{g_m}\right)^2\left[\frac{8KTg_m}{3} + \frac{K_1 g_m^2}{C_{ox}WLf} + \frac{4KT}{R_D}\right]}$$

The EING of a mosfet suggests that $g_m\uparrow$ reduces the noise in the output.
(Amplifier application)

Reducing $\overline{I_n^2}$ in the output achieved by $g_m\downarrow$ (Current source applic.)

Analyze this:



Considering only thermal noise.
$M_2 \rightarrow$ current source $\}$ Both work saturated
$M_1 \rightarrow$ Amplifier

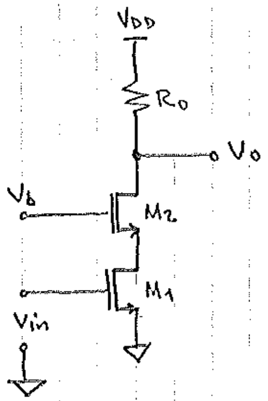
$$\overline{V^2_{n_{out}}} = 4KT\left(\tfrac{2}{3}g_{m1} + \tfrac{2}{3}g_{m2}\right)(r_{o1}\|r_{o2})^2$$

The equivalent input noise: (divide by $A_v^2 = g_{m1}^2(r_{o1}\|r_{o2})^2$)

$$\overline{V^2_{n_{in}}} = 4KT\left(\frac{2}{3g_{m1}} + \frac{2g_{m2}}{3g_{m1}^2}\right)$$

Note how to reduce $\overline{V^2_{n_{in}}}$

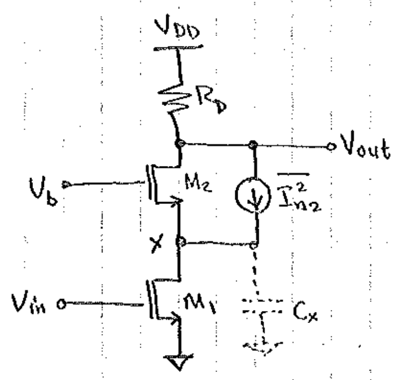
Consider a cascode connection:



$$\overline{V^2_{n_{in}}}\Big|_{M_1 R_D} = 4KT\left(\frac{2}{3g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right) \quad \text{(Ignoring } 1/f\text{)}$$

The noise effect of M_2 (thermal only)

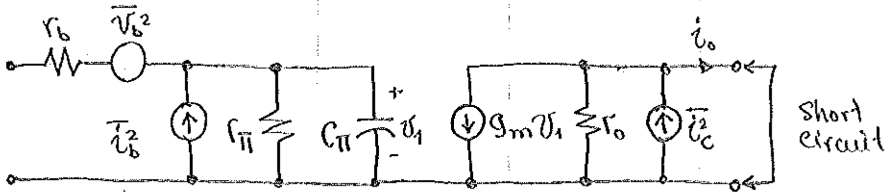
- At low frequencies is negligible



Note that for $V_{in}=0$
$I_{n2} + I_{D_2} = 0$
So M_2 has no effect on $V_{n,out}$

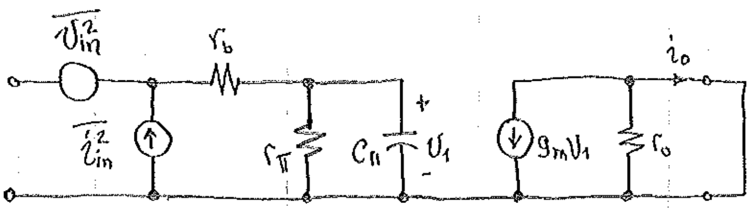
- At high frequencies we have to consider $C_X = C_{GS_2} + C_{GD_1}$
- The gain experienced by the noise: $\dfrac{V_{n\,out}}{V_{n2}} \simeq \dfrac{-R_D}{\frac{1}{g_{m2}} + \frac{1}{j\omega C_X}}$
- C_X wares multiplying the noise of M_2 and shunting the signal current of M_1

Consider a bipolar transistor:

I


Model neglects the effect of C_μ and Short circuits the output for max i_o

The equivalent EIG circuit

II


1) Short-circuiting input and equating i_o in both circuits

I: $i_o = g_m v_1 + i_c$ assuming $r_b \ll r_\pi$, $v_1 = v_b$

II: $i_o = g_m v_1$ assuming $r_b \ll r_\pi$ $v_1 = v_{in}$

$\Big\}\ g_m v_b + i_c = g_m v_{in}$

Thus $v_{in} \cong v_b + i_c/g_m$ ← Note that r_b small cancels $\overline{i_b^2}$

Assuming independence: $\overline{v_{in}^2} = \overline{v_b^2} + \overline{i_c^2}/g_m^2$ $\qquad I_c = \frac{2KT}{q} g_m$?

$\overline{v_b^2} = 4KTr_b$, and $\overline{i_c^2} = 2qI_c$ $\Rightarrow$ $\overline{v_{in}^2} = 4KT\left(r_b + \frac{1}{2g_m}\right)$

Term $(r_b + 1/2g_m)$ represents the "Equivalent input noise resistance"

r_b ← Physical base resistance; $1/2g_m$ ← Effect of i_c referred to the input.

2) Open-ckt input and equate i_o: $\beta(j\omega)\, i_i = i_c + \beta(j\omega)\, i_b$

So $i_i = i_b + \dfrac{i_c}{\beta(j\omega)}$ $\Rightarrow$ $\overline{i_i^2} = \overline{i_b^2} + \dfrac{\overline{i_c^2}}{|\beta(j\omega)|^2}$ (*)

$$\beta(j\omega) = \frac{\beta_o}{1 + j\frac{\omega}{\omega_\beta}}, \quad \omega_\beta = \frac{1}{\beta_o}\,\frac{g_m}{C_\pi + C_\mu}$$

Thus, replacing $\overline{i_b^2}$ & $\overline{i_c^2}$ into (*)

$$\overline{i_i^2} = 2q\left[I_B + K_1' \frac{I_B^a}{f} + \frac{I_c}{|\beta(j\omega)|^2}\right]$$

where $K_1' = K_1/2q$ and burst noise has been excluded

I_B → Base current; $K_1' \frac{I_B^a}{f}$ → flicker in I_B; $\frac{I_c}{|\beta(j\omega)|^2}$ → I_c noise referred to input.

See §11.5.1 for additional details