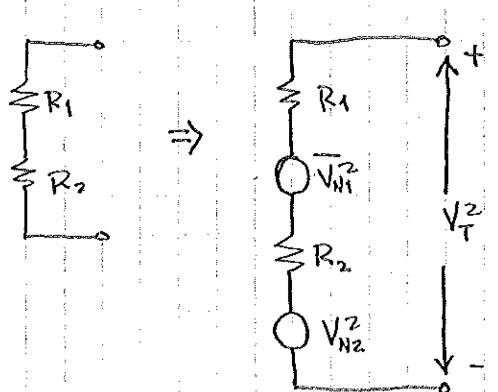
## Circuit Noise Calculations

Given a noise source with PSD $\overline{I_N^2} = S_x(f)\,\Delta f$

The RMS value of the noise would be $I_{RMS} = \sqrt{S_x(f)\,\Delta f}$

- Noise source in BW $\Delta f$ can be approximated by a sinusoidal generator of value $i_{RMS}$
- Analysis can be therefore carried using conventional circuit techniques for sinusoidal sources
- When sources are considered independent (in most cases they are) superposition can be applied

Ex: Two series resistors $R_1$, $R_2$




$$V_T = V_1 + V_2$$

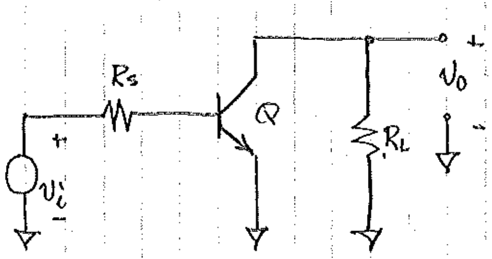
$$\Rightarrow \overline{V_T^2} = \overline{\left[V_{N1}^2 + V_{N2}^2\right]}$$

$$= \overline{V_{N1}^2} + \overline{V_{N2}^2} + \underbrace{\overline{2V_{N1}V_{N2}}}_{\text{Zero for uncorrelated sources}}$$

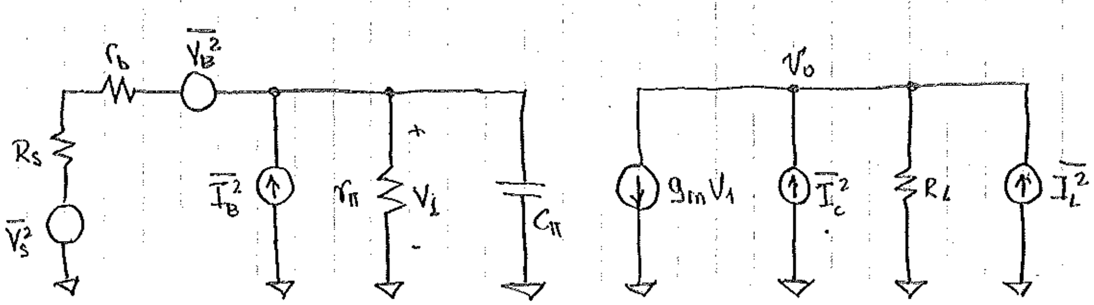
$$\overline{V_T^2} = \overline{V_{N1}^2} + \overline{V_{N2}^2}$$

$$\overline{V_{TN}^2} = 4KTR_1 + 4KTR_2 \quad \Rightarrow \quad \overline{V_{TN}^2} = 4KT\,(R_1 + R_2)$$

## Performance of a CE circuit:




- Input sources are shorted ($v_i = 0$)
- $C_\mu$ & $r_o$ are neglected
- Sources considered independent



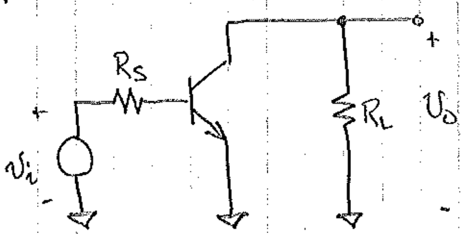
## Circuit Analysis w/noise generators:

Given a noise source with PSD $\overline{I_N^2} = S_x(f)$, ← Mean sq. value of $I_n$

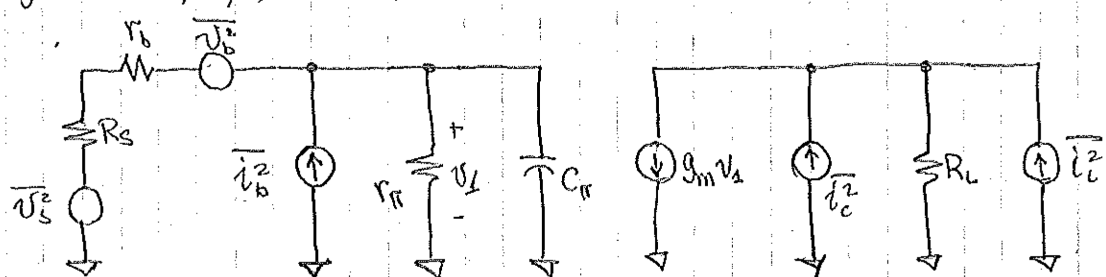
$I_N = \sqrt{S_x(f)}$ ← RMS value of $I_n$

- Circuit analysis can be performed by replacing $I_n$ by a sinusoidal generator and performing sinusoidal circuit analysis.
- In most cases, multiple noise sources are uncorrelated, making it possible to apply superposition.

Example: CE Amplifier circuit




The small signal equivalent circuit with noise sources (Neglects $v_i$, $C_\mu$, and $r_o$)




Noise sources

$\overline{v_s^2} = 4KTR_S$ ← Thermal noise in $R_S$

$\overline{v_b^2} = 4KTr_b$ ← " " " $r_b$

$\overline{i_b^2} = 2qI_B + K_1 \frac{I_B^a}{f} + \cancel{K_2 \frac{I_B^c}{1+(f/f_c)^2}} \approx 0$ ← Shot + Flicker + Burst (neglect)

$\overline{i_c^2} = 2qI_C$ ← Shot

$\overline{i_l^2} = 4KT\frac{1}{R_L}$ ← Thermal in $R_L$

The total output noise can be found by superpositoon

$$\overline{V_s^2} = 4KTR_s$$

$$\overline{I_L^2} = 4KT\frac{1}{R_L}$$

Applyin superposition

- Effect of $\overline{V_s^2}$ $\quad V_1 = \frac{Z}{Z + r_b + R_s} V_s \qquad Z = r_\pi \parallel \frac{1}{sC} \qquad s = j\omega$

$$Z = \frac{r_\pi \cdot \frac{1}{sC}}{r_\pi + \frac{1}{sC}}$$

$$Z = \frac{r_\pi}{1 + s r_\pi C} \qquad V_1 = \frac{\frac{r_\pi}{1+s r_\pi C}}{\frac{r_\pi}{1+s r_\pi C} + r_b + R_s} V_s$$

$$V_1 = \frac{r_\pi}{r_\pi + (r_b + R_s)(1 + s r_\pi C)} V_s$$

$$V_{o1} = -g_m R_L V_1 = \frac{-g_m R_L r_\pi}{r_\pi + (r_b + R_s)(1 + s r_\pi C)}$$

$$v_{o1} = -g_m R_L \frac{Z}{Z + r_b + R_s} v_s$$

$$\overline{v_{o1}^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_s|^2} \overline{v_s^2} \quad \leftarrow \text{Effect of } \overline{V_s^2}$$

$$\overline{v_{o2}^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_s|^2} \overline{V_b^2} \quad \leftarrow \text{Effect of } \overline{V_b^2}$$

$$\overline{v_{o3}^2} = g_m^2 R_L^2 \frac{(R_s + r_b)^2 |Z|^2}{|Z + r_b + R_s|^2} \overline{i_b^2} \quad \leftarrow \text{Effect of } \overline{I_B^2}$$

The contribution of $\overline{I_L^2}$ and $\overline{I_c^2}$

$$\overline{V_{o4}^2} = \overline{I_L^2} R_L^2$$

$$\overline{V_{o5}^2} = \overline{I_c^2} R_L^2$$

Applyin superposition $\quad \overline{V_o^2} = \sum_{n=1}^{5} \overline{V_{on}^2} \quad \leftarrow$ *By independence*

$$\overline{V_o^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z+r_b+R_S|^2}\left(\overline{V_s^2}+\overline{V_b^2}+(R_S+r_b)^2\overline{i_b^2}\right)+R_L^2\left(\overline{I_L^2}+\overline{I_C^2}\right)$$

Replacing the shot noise sources (neglecting flicker)

$$\overline{V_o^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z+r_b+R_S|^2}\left[4KT(R_s+r_b)+(R_s+r_b)^2 2q I_B\right]+R_L^2\left[4KT\frac{1}{R_L}+2q I_C\right]$$

Replacing $Z = r_\pi \,\|\, \frac{1}{j2\pi f C}$

$$\overline{V_o^2} = g_m^2 R_L^2 \frac{r_\pi^2}{(r_\pi+R_S+r_b)^2}\cdot\frac{1}{1+\left(f/f_1\right)}\left[4KT(R_S+r_b)+(R_s+r_b)^2 2q I_B\right]$$

$$+\,R_L^2\left(4KT\frac{1}{R_L}+2q I_C\right), \quad \text{where } f_1 = \frac{1}{2\pi\left[r_\pi \| (R_S+r_b)\right]C_\pi}$$

Two terms:

① Frequency dependent part: Gain begins to fall for $f > f_1$
Therefore noise due to $\overline{V_S^2}$, $\overline{V_B^2}$, and $\overline{I_B^2}$ decays for high frequencies

② Constant term due to sources $\overline{I_L^2}$ and $\overline{I_C^2}$
- Actually these decay when $C_\mu$ not neglected

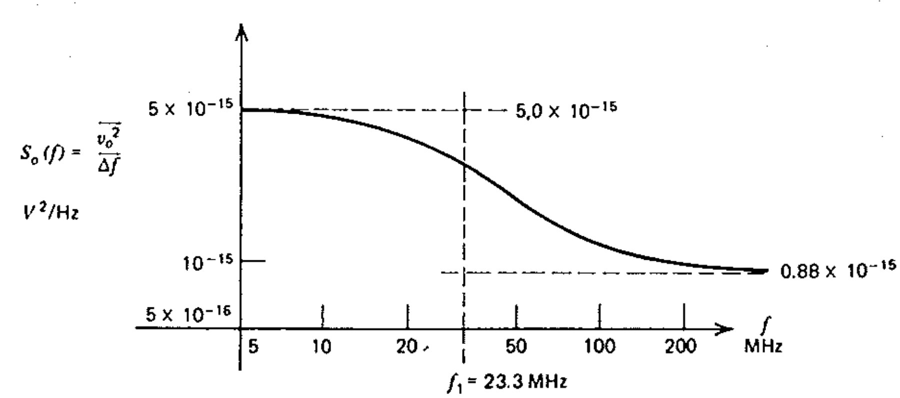
Assigning values: $r_\pi = \frac{\beta}{g_m}$

$I_C = 100\mu A$ $\quad$ $\beta = 100$
$R_S = 500\Omega$ $\quad$ $C_\pi = 10\,pF$
$R_L = 5K\Omega$ $\quad$ $r_b = 200\Omega$

$$\overline{V_o^2} = \frac{4.13\times10^{-15}}{1+\left(f/f_1\right)^2} + 0.88\times10^{-15}\ V^2/Hz$$

$f_1 = 23.3\,MHz$

$r_\pi = \frac{\beta}{g_m}$ $\quad$ $g_m = \frac{I_C}{\Phi_T}$ $\quad$ $\Phi_T = \frac{KT}{q} = 26mV$ @ $300°K$




In a 0–1MHz b.w there is no decay
$\Rightarrow \overline{V_n^2}$ constant at $5\times10^{-15}\ V^2/Hz$

$$V_o^2 = \int_0^{1MHz}\overline{V_n^2}\,df$$

$$= 5\times10^{-15}\times10^{6}$$

$$= 5\times10^{-9}\ V^2$$

$$V_{oT\,RMS} = \sqrt{V_o^2} = 71\mu V$$