Noise in Digital Circuits:

Elements:
- Noise source
- Coupling medium
- Receiver

Common sources for NDG:

1) Transmission line reflections: Ringing & overshoot in unterminated transmission lines

2) Crosstalk: Coupling through mutual inductance & capacitance

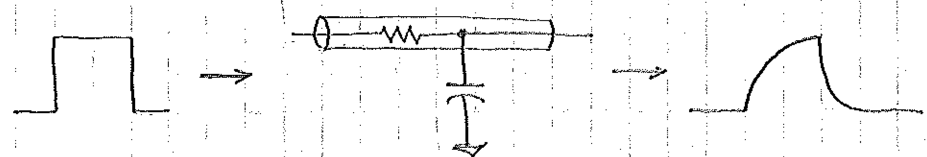
3) Dirty power supply: Current spikes caused by load switching & power line noise

4) Electromagnetic Interference: EMI = Energy radiated by other circuits

Electronic noise does not represent a problem for digital circuits due to built-in noise margins

Parameters in High-speed Digital Lines

- Attenuation: Edge rate degradation & amplitude loss due to line impedance.



- Propagation delays: Limit system speed. Unequal delays may lead to timing errors

- Reflections: Caused by mismatched impedances

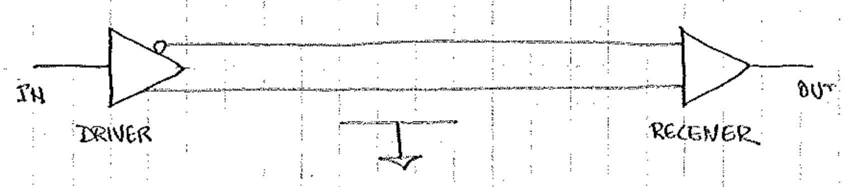
- Crosstalk: Coupling of a signal to a nearby path

Line Types:

Single-ended Lines: Single wire per signal:

- Affected by line noise
- Requires alternating signal-ground for high-speed parallel buses
- Low cost?

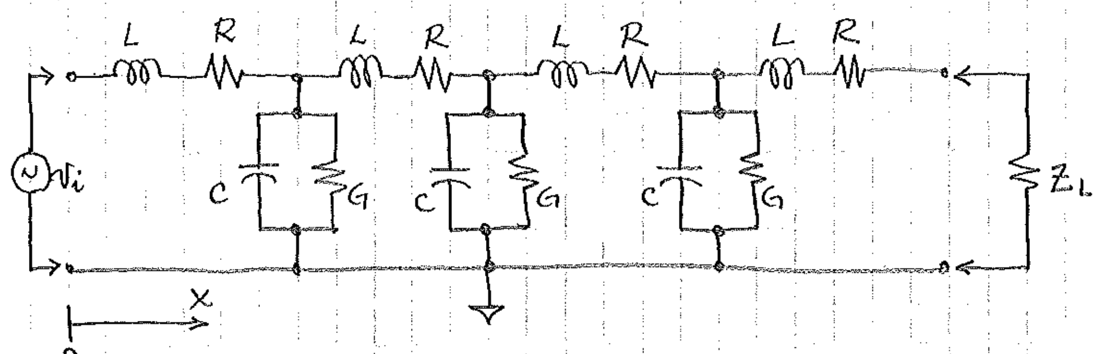
Differential Lines



- Use complementary signals in two different lines
- Highest noise immunity: Noise is equally coupled on both lines
  Enters in common mode (CMMRR)

Digital Transmission Lines

- A transmission line is a signal path that exhibits a characteristic impedance Z_0.
- Transmission line effects begin to manifest when the signal rise & fall times are comparable to the propagation delay of the line.
- When transmission times are long w.r.t. propagation delay, the line can be represented by lumped-circuit elements or distributed parameters as shown below:



$G = \frac{\sigma}{\varepsilon} C$

$$\frac{\partial v(x,t)}{\partial x} = L\frac{\partial i(x,t)}{\partial t} + R\,i(x,t) \quad (1)$$

$$\frac{\partial i(x,t)}{\partial x} = C\frac{\partial v(x,t)}{\partial t} + G\,v(x,t) \quad (2)$$

$v(x,t)$ = Potential of a trace w.r.t ground (V)
$i(x,t)$ = Current, positive in the x direction (A)
L = Inductance per unit length (H/m)
R = Resistance per " " (Ω/m)
C = Capacitance " " " (F/m)
G = Conductance " " " ($\mho$/m)

(L, R, C, G): Transmission line parameters

Wave Equations

Multiplying ② by $\partial x/\partial t$

$$\frac{\partial i}{\partial t} = C\frac{\partial v}{\partial t}\cdot\frac{\partial x}{\partial t} + Gv\frac{\partial x}{\partial t} \quad ③$$

Replacing ③ into ①

$$\frac{\partial v}{\partial x} = LC\frac{\partial v}{\partial t}\cdot\frac{\partial x}{\partial t} + LG\,v\frac{\partial x}{\partial t} + Ri$$

Taking $\partial/\partial x$

$$\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2} + LG\frac{\partial v}{\partial t} + R\frac{\partial i}{\partial x} \quad ④$$

Replacing ② into ④ and re-arranging

$$\boxed{\frac{\partial^2 v(x,t)}{\partial x^2} - LC\frac{\partial^2 v(x,t)}{\partial t^2} = (LG+RC)\frac{\partial v(x,t)}{\partial t} + RG\,v(x,t)} \quad ⑤$$

Equation ⑤ is the wave equation for the voltage in a transmission line.

In an analogous way, the wave equation for the current results

$$\boxed{\frac{\partial^2 i(x,t)}{\partial x^2} - CL\frac{\partial^2 i(x,t)}{\partial t^2} = (CR+GL)\frac{\partial i(x,t)}{\partial t} + GR\,i(x,t)} \quad ⑥$$

In the case of a single wire ⑤ and ⑥ are scalar equations. Multi-wire lines have R, G, L, and C as matrices

Definition: Lossless transmission line: $R=0$ and $G=0$

Definition: Speed of propagation of a lossless line

$$c = \frac{1}{\sqrt{LC}}$$

Definition: Characteristic impedance of a lossless line

$$Z_0 = \sqrt{L/C}$$

The wave equation of a lossless line reduces to

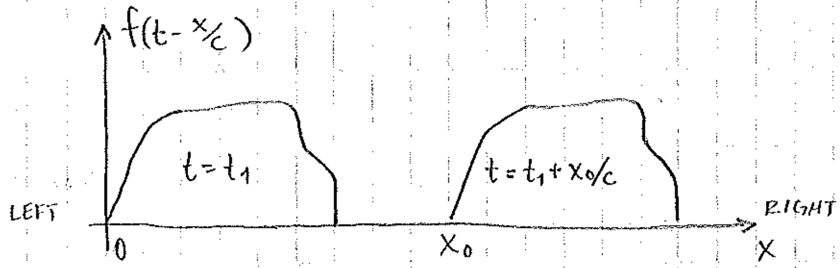
$$\frac{\partial^2 v(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \quad (7)$$

$$\frac{\partial^2 i(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 i(x,t)}{\partial t^2} = 0 \quad (8)$$

Note that (7) and (8) are valid for any arbitrary function of $(t - x/c)$ or $(t + x/c)$

$f(t - x/c) \rightsquigarrow$ Forward moving signal: remains constant along path $(t - x/c)$ From $x=0$ to $x>0$

$f(t + x/c) \rightsquigarrow$ Backward moving signal From $x>0$ to $x=0$



Forward moving signal at two time instants separated x_0/c

Thus, forward and backward voltage and current signals:

$$v_F(t - x/c) = Z_0\, i_F(t - x/c) \quad (9)$$

$$v_B(t + x/c) = -Z_0\, i_B(t + x/c) \quad (10)$$

The general solution for a lossless line can be written as:

$$v(x,t) = v_F(t - x/c) + v_B(t + x/c) \quad (11)$$

$$i(x,t) = i_F(t - x/c) + i_B(t + x/c) \quad (12)$$

- When a transmission line is terminated with an impedance $Z_L = Z_0$, the load appears as an infinite extension to the line. So there will be no reflexion when the wave front reaches the termination