Deformation

• Under external forces a solid deforms:
  — By recovering their original shape after the forces are removed → Elastic deformations
  — By keeping its deformation after the forces are removed → Plastic deformations
• Most materials start behaving elastically until the applied forces transcend certain limiting value and then they behave plastically.

Elastic Deformation

Engineering Strain

The magnitude of $\varepsilon$ depends on the force but also on the material.

$$\varepsilon = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$
Stress

Stress: Force per unit area

- Normal Stress: Perpendicular to the area where it is applied \( \sigma = \frac{F}{A} \)
- Shear Stress: Parallel to the area where it is applied \( \tau = \frac{F}{A} \)

Units?

Each type of stress generates different types of deformation

Engineering Normal Stress

\[
\sigma = \frac{F}{A_0}
\]

\( A_0 \): Initial cross sectional area

Any problem with this definition?

Normal and Shear Stresses and the Resulting Elastic Deformations

For small normal stresses: proportional elastic longitudinal deformations

\[
\sigma \propto \varepsilon \Rightarrow \sigma = E \cdot \varepsilon
\]

This is called Hooke's law

For small shear stresses: proportional elastic angular deformations

\[
\tau \propto \gamma \Rightarrow \tau = G \cdot \gamma = G \cdot \tan \theta = G \cdot \frac{a}{h}
\]

\( E \): Longitudinal Elastic Modulus or Young's Modulus

\( G \): Transversal Elastic Modulus

\[
G = \frac{E}{2(1 + \nu)}
\]
Tensile Testing Machine

Notice that the machine does not apply a load. So where does the “load” come from?

The displacement of the crosshead is very slow to simulate a “static load” condition.

Different models have different position of the moving crosshead and the load cell.

We can also use this machine to test materials under compression (e.g. concrete in Civil Eng.)

INSE 4001 Engineering Materials

Tensile Test Extensometers

Extensometers are used to accurately measure very small deformations as those encountered during the elastic period in metallic materials.

The axial displacement of the clips creates an electrical signal that a calibrated board translates into a measurement.

Extensometers are one example of the application of strain gages (small resistance arrays used to measure small deformations).

Homework: Find the resolution of most common extensometers used in tensile tests of metallic materials.
Tensile Test Specimens

- Need standards to make tests comparable to each other
- Different types of specimens for different materials and for different applications.

Examples:

ASTM E 8 Metals

- 0.375
- 1.800
- 0.080
- 0.060
- 0.100

ASTM D 638 Polymers

Tensile Test Results

- Engineering stress vs. engineering strain for a 6061-T6 aluminum alloy
- In rose: the elastic region
- In blue: the plastic region

A36 steel HDPE

At the microscale, study of copper foil

At the nanoscale:
Behavior of Materials Undergoing Tensile Test

- Ultimate Tensile Strength (UTS)
- Proportionality region
- Young’s modulus
- Elastic limit
- Necking
- Elongation to failure
- Modulus of resilience $U_r$
- “Toughness” or absorbed energy to failure

Limit of the Elastic Period

For the same 6061-T6 alloy

- Elastic Limit or Yield Strength (YS):
  - Practical Elastic Limit or Proof Stress $\sigma_{0.2}$
  - Proportional Limit

Sometimes the proportional region is not well-defined (cast irons, some copper alloys, etc.) or the yield point is “doubled” (low alloy steels). The standards provide solutions to deal with most materials!
The Elastic Period (cont.)

Elastic Modulus $E = \frac{\sigma}{\varepsilon}$

- A measurement of the atomic bonding strength
- Remember:

What is the effect of small chemical composition changes on $E$?

Some Values of $E$

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum alloys</td>
<td>70</td>
</tr>
<tr>
<td>Steels</td>
<td>200</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>45</td>
</tr>
<tr>
<td>NaCl</td>
<td>40</td>
</tr>
<tr>
<td>Diamond</td>
<td>~1,000</td>
</tr>
<tr>
<td>Alumina fiber</td>
<td>400</td>
</tr>
<tr>
<td>Boron fiber</td>
<td>400</td>
</tr>
<tr>
<td>Glass</td>
<td>70</td>
</tr>
<tr>
<td>Basalt fiber</td>
<td>89</td>
</tr>
</tbody>
</table>
Necking

Localized plastic deformation after the UTS is reached

The tensile test was stopped after the onset of the necking but before fracture occurred.

Necking simulation

The tensile test was conducted until the specimen fractures.

Necking onset represents a plastic instability that only grows once it occurs. Since the cross section is smaller, the overall load drops but the local stress (in the neck) is higher.

Summary: plastic deformation is not uniform along the specimen

Fracture (just an intro…)

- Percent elongation to fracture or ductility

\[
\% \text{ elongation} = \delta = \frac{\text{final length} - \text{initial length}}{\text{initial length}} \cdot 100 = \frac{l_f - l_0}{l_0} \cdot 100
\]

- Percent reduction in area

\[
\% \text{ reduction} = \psi = \frac{\text{initial area} - \text{final area}}{\text{initial area}} \cdot 100 = \frac{A_0 - A_f}{A_0} \cdot 100
\]
Fracture (cont.)

- “Toughness” or absorbed energy to fracture

\[ \text{Absorbed Energy} = \int_{0}^{\varepsilon_f} \sigma \cdot d\varepsilon \]

In reality it's the absorbed energy per unit volume. **Check units!**

- Larger areas define **ductile materials**
- Smaller areas define **brittle materials**

Compare not only the curve shapes and heights but also the areas under the curves. You can distinguish now clearly the concepts of **strength**, **ductility**, and **toughness** and use them properly to describe a material!
The Plastic Region

True Stress

Remember the definition of engineering stress

\[ \sigma = \frac{F}{A_0} \quad A_0: \text{initial cross section area} \]

We will now correct the error by considering the real (true) area:

So the true stress is calculated as:

\[ \sigma_{\text{true}} = \frac{F}{A_i} \]

True Strain

Engineering stresses were calculated as:

\[ \varepsilon = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0} \]

However, we can estimate the instantaneous (true) strain at any moment as:

\[ \frac{d\varepsilon}{l} \rightarrow \varepsilon_{\text{true}} = \int_{l_0}^{l} \frac{dl}{l} = \ln \left( \frac{l}{l_0} \right) \]

True Stress vs. True Strain

True stress-true strain and engineering curves for commercially pure Ti

The elastic deformations are very small (compared to the plastic ones) and cannot be seen at the x-scale used.

The curve can be sometimes described by:

\[ \sigma_i = K \cdot \varepsilon_i^n \] where \( n \) is called the strain rate coefficient.

Important: upon plastic deformation, we can assume that the sample volume remains constant: \( V_0 = A_0 \cdot l_0 = A \cdot l \)

Homework: With this premise, then:

- Develop an equation to calculate \( \sigma_{\text{true}} \) and \( \varepsilon_{\text{true}} \) based on the engineering stress \( \sigma \) and engineering strain \( \varepsilon \).
- Develop an equation to calculate \( \sigma \) and \( \varepsilon \) when \( \sigma_{\text{true}} \) and \( \varepsilon_{\text{true}} \) are known.
A Comment on Fracture

The results of the tensile test (curve shape) is corroborated by the analysis of fracture (fractography).

Effect of Temperature on UTS and YS

Commercial Mg Alloys

Pure Iridium
Tensile Properties of Ceramics

This is just a brief introduction to the subject. Some issues related to fracture will be clarified in the next chapter.

Ceramic materials are inherently brittle, with little or no plastic deformation. Therefore, they require special testing methods and fabrication techniques.

These are results for Si₃N₄.
The distribution is not symmetric but slanted towards the limit value (something like the theoretical strength of the ceramic).

This is a histogram of the measured strength. A histogram shows us how many times a certain range of results are measured.

The Weibull distribution describes best the variability in strength observed in ceramics.

Homework: Find a good definition for a Weibull distribution.

The three-point bending test

Main advantage:
We don't need grips.

Flexural strength:

\[ \sigma_{fs} = \frac{F_f \cdot L}{8 \cdot I_z \cdot h} \]

\( F_f \) is the load measured at the moment when the material fails.

\[ M = \frac{F \cdot L}{4} \]
\[ c = \frac{d}{2} \]
\[ I = \frac{bd^3}{12} \]
\[ \sigma = \frac{3FL}{2bd^2} \]

Rectangular

Circular

Possible cross sections

Support

Rectangular

Circular

\[ R = \frac{h}{2} \]

F
Elastic Behavior of Ceramics

Any difference with a metallic material?

E varies between 70-500GPa

Don't forget about Weibull!

So what was the reason for the lack of plasticity?

Powder pressing is one method to process ceramic parts. It explains the presence of fabrication flaws.

During sintering powder touches, forms neck, gradually neck thickens.

- add processing aids to help form neck
- little or no plastic deformation

Pressing can be:

- Uniaxial compression - compacted in single direction
- Isostatic (hydrostatic) compression - pressure applied by fluid - powder in rubber envelope
- Hot pressing - pressure + heat
Most ceramics are sintered from powders:

a) powder is compacted,
b) "green" compact is heated,
c) sintering occurs,
d) densification

Therefore, pores are quite common.

\[
\frac{\sigma_{fs}}{\sigma_0} = e^{-nP}
\]

according to Ryschkewitch, where \( n \) and \( \sigma_0 \) are experimental constants.

Now the problem with low \( K_{IC} \) in ceramics is evident.
Hardness

Measures the resistance of a material to be scratched or penetrated by other.

- Scratching method: Mohs’ scale (relative mineral hardness scale)
  1. Talc
  2. Gypsum
  3. Calcite
  4. Fluorite
  5. Apatite
  6. Feldspar
  7. Quartz
  8. Topaz
  9. Corundum
  10. Diamond

This is an arbitrary scale and not useful for engineering materials in general.

- Indentation/penetration methods
  - based on a specific force applied onto an indenter.
  - Measurements of hardness are based on depth of indentation or exposed area of indentation

- Correlation of hardness and UTS or YS

There are two ways of measuring the deformation imposed by this hard material on the testpiece:
  - Depth of indent
  - Visible size of the indent
There are two main ways of evaluating hardness using indentation:
- Measuring the depth of the indentation
- Measuring the visible size of the indentation
In most metallic materials there is a direct correlation.

Be careful when using this information!

Other metallic alloys use a different plot.

Keep in mind: hardness tests are inexpensive quality control tools in a plant, e.g. heat treating company.