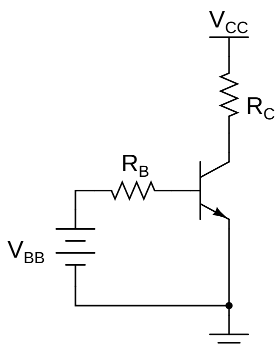
# Common Emitter Amplifier

M. Toledo - INEL 4076

# 1. DC Analysis

## 1. Basic Circuit




Let $V_{BE} = 0.7V$.

$$
\begin{aligned}
i_B &= \frac{V_{BB} - V_{BE}}{R_B} \\
i_C &= \beta i_B = \beta \frac{V_{BB} - V_{BE}}{R_B} \\
V_{CE} &= V_{CC} - R_C i_C = V_{CC} - R_C \times \beta \times \frac{V_{BB} - V_{BE}}{R_B}
\end{aligned}
$$

Since $\beta$ is not known precisely, there is considerable uncertainty in the quiescent (operating) point (or Q-point), defined by the $(y, x)$ pair $(i_C, V_{CE})$.

### 1.1. Load line

Use

$$i_C = \frac{V_{CC} - V_{CE}}{R_C}$$

to draw load line. Saturation current (y-intercept) is

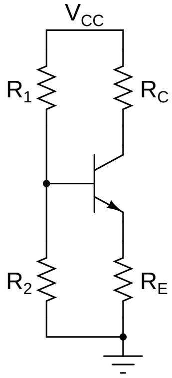
$$i_{C,sat} = i_C(V_{CE} = 0) = V_{CC}/R_C$$

and x-intercept is

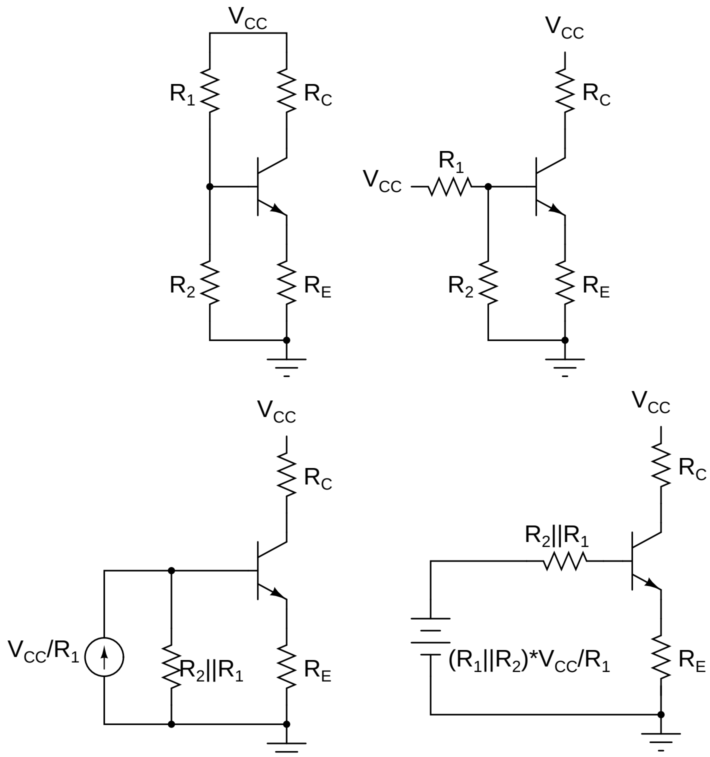
$$V_{CE}(i_C = 0) = V_{CC}$$

The Q-point must be in the load one.

## 2. Four-R Circuit




Use source transformation to transform the circuit as follows:




Observe that

$$R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

Now invoke KVL to write the equation for the base-emitter loop:

$$\begin{aligned}
\frac{V_{CC}}{\cancel{R_1}} \frac{\cancel{R_1} R_2}{R_1 + R_2} &= i_B \frac{R_1 R_2}{R_1 + R_2} + V_{BE} + R_E i_E \\
i_B &= \frac{i_E}{\beta + 1} \\
V_{CC} \frac{R_2}{R_1 + R_2} &= \frac{i_E}{\beta + 1} \frac{R_1 R_2}{R_1 + R_2} + V_{BE} + R_E i_E \\
V_{CC} \frac{R_2}{R_1 + R_2} &= \left( \frac{1}{\beta + 1} \frac{R_1 R_2}{R_1 + R_2} + R_E \right) i_E + V_{BE} \\
i_E &= \frac{V_{CC} \frac{R_2}{R_1 + R_2} - V_{BE}}{\frac{1}{\beta + 1} \frac{R_1 R_2}{R_1 + R_2} + R_E}
\end{aligned}$$

Invoking KVL on the collector-emitter loop gives

$$V_{CE} = V_{CC} - i_C \times R_C - i_E \times R_E$$

where $i_C = \frac{\beta}{\beta+1} i_E$ (often $\beta >> 1$ and $i_C \simeq i_E$).

### 2.1. Load line

Use

$$i_C = \frac{V_{CC} - V_{CE}}{R_C + R_E}$$

(assuming $i_C \simeq i_E$) to draw load line. Saturation current (y-intercept) is

$$i_{C,sat} = i_C(V_{CE} = 0) = \frac{V_{CC}}{R_C + R_E}$$

and x-intercept is

$$V_{CE}(i_C = 0) = V_{CC}$$

The Q-point must be in the load one.

**Example**

Find $i_C$ if $V_{CC} = 9V$, $\beta = 100$, $R_1 = 100k\Omega$, $R_2 = 47k\Omega$, $R_E = 3.9k\Omega$ and $R_C = 6.8k\Omega$.

$$\begin{aligned} i_C &= \frac{\beta}{\beta+1} i_E = \frac{\beta}{\beta+1} \frac{V_{CC}\frac{R_2}{R_1+R_2} - V_{BE}}{\frac{1}{\beta+1}\frac{R_1R_2}{R_1+R_2} + R_E} \\ &= \frac{100}{101} \frac{9V\frac{47}{147} - 0.7V}{\frac{1}{101}\frac{100k\Omega \times 47k\Omega}{100k\Omega + 47k\Omega} + 3.9k\Omega} = \boxed{0.51mA} \end{aligned}$$

### 2.2. Approximation for large $\beta$

If $i_B << i_{R_2}$, we can neglectt $i_B$ and apply the voltage divider rule to $R_1$ and $R_2$ to obtain

$$V_B \simeq V_{CC} \frac{R_2}{R_1 + R_2}$$

so that

$$\begin{aligned} V_E &= V_B - V_{BE} = V_{CC} \frac{R_2}{R_1 + R_2} - V_{BE} \\ i_E &= \frac{V_E}{R_E} = \frac{V_{CC}\frac{R_2}{R_1+R_2} - V_{BE}}{R_E} \end{aligned}$$

which is equivalent to neglecting the term $\frac{1}{\beta+1}\frac{R_1R_2}{R_1+R_2}$ in our previous result for $i_E$.

**Example**

Find $i_C$ if $V_{CC} = 9V$, $\beta = \infty$, $R_1 = 100k\Omega$, $R_2 = 47k\Omega$, $R_E = 3.9k\Omega$ and $R_C = 6.8k\Omega$.

$$i_C = i_E = \frac{9V\frac{47}{147} - 0.7V}{3.9k\Omega} = \boxed{0.56mA}$$

**Design example**

Problem statement: Design a four resistor biasing network to obtain a Q-point current $i_{CQ} = 10mA$ with $V_{CE,Q} = 4V$ using a 12V battery. Assume $V_{BE} = 0.7V$. Neglect $i_B$.

ANSWER: Since $V_{CE,Q} = 4V$ and $V_{CC} = 12V$, the combined voltage drop in $R_E$ and $R_C$ is $8V$. Assign voltage drops of $2V$ and $6V$ across $R_E$ and $R_C$, respectively. Since $i_C \simeq i_E$, this implies that $R_C \simeq 3R_E$. Because $i_{CQ} = 10mA$, we have

$$8V = 10mA\,(R_C + R_E) = 10mA \times 4R_E$$

$$\boxed{R_E = 200\Omega}$$

$$\boxed{R_C = 600\Omega}$$

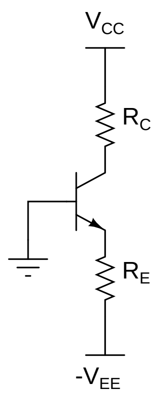
Since $V_E = 200\Omega \times 10mA = 2V$,

$$V_B = V_E + V_{BE} = 2.7V = 12V\frac{R_1}{R_1 + R_2} = \frac{12V}{1 + R_2/R_1}$$

$$\frac{R_2}{R_1} = \frac{12V}{2.7V} - 1 = 3.444$$

To make sure $i_{R_1} \gg i_B$ (although we neglected $i_B$ we know that $\beta \neq \infty$ but about 100 so $i_B \simeq 10mA/100 = 0.1mA$), select $\boxed{R_2 = 10R_E = 2k\Omega}$ (other choices are possible, but this gives $i_{R_2} = 2.7V/2k\Omega = 1.35mA \gg i_B$) and $\boxed{R_1 = 6890\Omega}$.

**Design example**




For the following circuit, find the values of $R_E$ and $R_C$ that will produce a collector current $i_C = 1mA$ and a collector -to-emitter voltage $V_{CE} = 5V$. Assume $V_{CC} = V_{EE} = 9V$, $V_{BE} = 0.7V$ and $\beta = 10$.

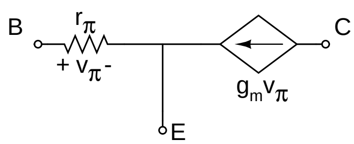
$$\begin{aligned}
i_E &= \frac{\beta+1}{\beta} i_C = 1.1mA = \frac{0V - 0.7V - (-9V)}{R_E} = \frac{8.3V}{R_E} \\
R_E &= \frac{8.3V}{1.1mA} = \boxed{7546\Omega} \\
V_{CE} &= V_{CC} - R_C \times i_C - R_C \times i_E - (-V_{EE}) \\
&= 18V - 1mA \times R_C - 1.1mA \times 7546\Omega = 9.7V - 1mA \times R_C = 5V \\
R_C &= \frac{9.7V - 5V}{1mA} = \boxed{4.7k\Omega}
\end{aligned}$$

# Small-signal analysis

General method:

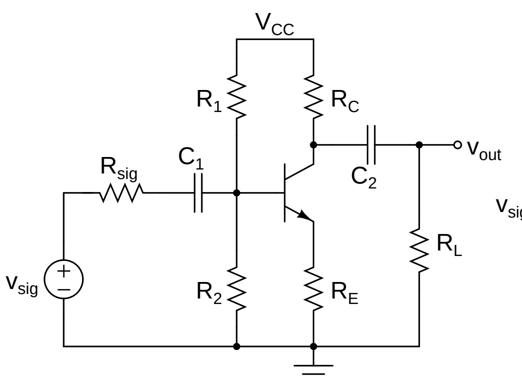
1. Replace DC voltage sources by short circuits.
2. Replace DC current sources by open circuits.
3. Replace coupling and bypass capacitors with short circuits.
4. Redraw diagram to show the AC equivalent circuit.
5. Replace transistor by its model.
6. Use KVL, KCL and Ohm's law to analyze the circuit and find the desired quantities.

## Transistor model

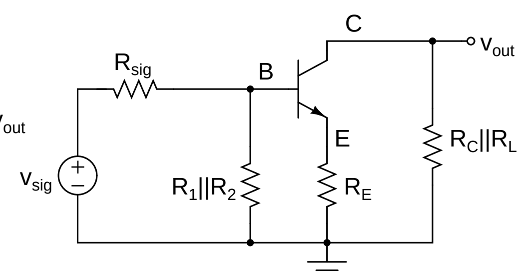



$$g_m = \frac{I_{CQ}}{V_T},\ r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_{CQ}}$$

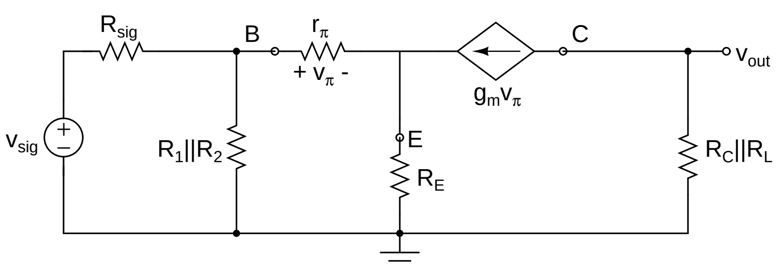
## CE Amplifier equivalent circuit




(a) Original circuit



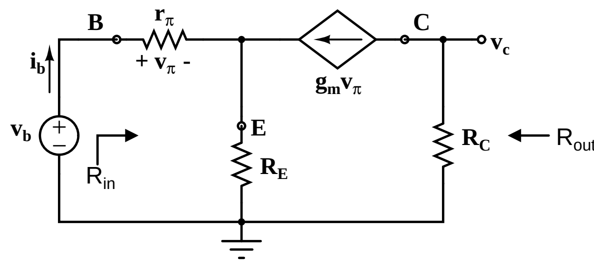

(b) AC equivalent circuit




(c) AC equivalent circuit with transistor's replaced by its model

## Voltage gain and equivalent resistances removing $R_{sig}$ and $R_L$

To simplify the algebra, remove the non-essential components $R_1 \| R_2$, $R_{sig}$, and $R_L$ and calculate the gain from base to collector. The simplified circuit is:




### Gain from collector to base

$$
\begin{aligned}
v_c &= -g_m v_\pi R_C \\
v_b &= v_\pi + R_E\left(i_b + g_m v_\pi\right) \\
v_\pi &= i_b \times r_\pi \\
\frac{v_c}{v_b} &= \frac{-g_m v_\pi R_C}{v_\pi + R_E\left(i_b + g_m v_\pi\right)} \\
&= \frac{-g_m \times \cancel{i_b} \times r_\pi \times R_C}{\cancel{i_b} \times r_\pi + R_E\left(\cancel{i_b} + g_m \times \cancel{i_b} \times r_\pi\right)} \\
&= \frac{-g_m \times r_\pi \times R_C}{r_\pi + R_E\left(1 + g_m \times r_\pi\right)} \\
g_m r_\pi &= \beta \\
\frac{v_c}{v_b} &= \boxed{\mu = \frac{-\beta \times R_C}{r_\pi + R_E\left(1+\beta\right)}}
\end{aligned}
$$

### Input resistance

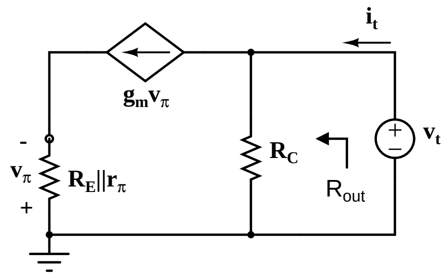
$$
\begin{aligned}
R_{in} &= \frac{v_b}{i_b} \\
v_c &= -g_m v_\pi R_C \\
v_b &= v_\pi + R_E\left(i_b + g_m v_\pi\right) \\
v_\pi &= i_b \times r_\pi \\
v_b &= i_b \times r_\pi + R_E\left(i_b + g_m \times i_b \times r_\pi\right) \\
R_{in} &= \frac{v_b}{i_b} = \frac{\cancel{i_b} \times r_\pi + R_E\left(\cancel{i_b} + g_m \times \cancel{i_b} \times r_\pi\right)}{\cancel{i_b}} \\
&\boxed{R_{in} = r_\pi + R_E\left(1+\beta\right)}
\end{aligned}
$$

### Output resistance

General procedure to find equivalent resistance:

1. Set independent sources to zero
2. Apply a test voltage source $v_t$ across the terminals where the equivalent resistance is to be found.
3. Compute current $i_t$ provided by test source
4. Compute equivalent resistance from $v_t/i_t$

For the amplifier's output resistance, following these steps leads to the following circuit:

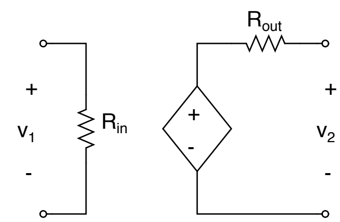



The dependent current source will cause $v_\pi$ to reverse polarity. This in turn will reverse the direction of the dependent source's current direction. The only consistent solution is $v_\pi = 0$, which leads to

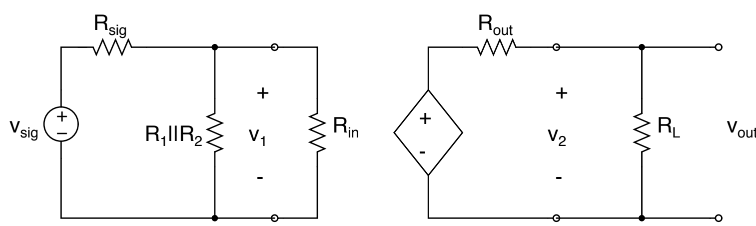
$$\boxed{R_{out} = R_C}$$

## Two-port network amplifier model

The amplifier can be represented by its voltage gain $\mu = v_c/v_b$, $R_{in}$ and $R_{out}$




Two-port network model of the amplifier.




Original circuit, including $R_{sig}$ and $R_L$, using the two-port network model.

To find the gain of the original circuit, include the components previously removed (namely, $R_1\|R_2$, $R_{sig}$, and $R_L$) as shown in the second diagram.

Now apply KVL at the two loops to find the voltage gain of the whole circuit.

$$\begin{aligned}
A_v &= \frac{v_{out}}{v_{sig}} \\
v_{out} &= \mu v_1 \frac{R_L}{R_{out} + R_L} \\
v_1 &= v_{sig} \frac{R_1\|R_2\|R_{in}}{R_1\|R_2\|R_{in} + R_{sig}} \\
v_{out} &= \mu \frac{R_L}{R_{out} + R_L} \frac{R_1\|R_2\|R_{in}}{R_1\|R_2\|R_{in} + R_{sig}} v_{sig}
\end{aligned}$$

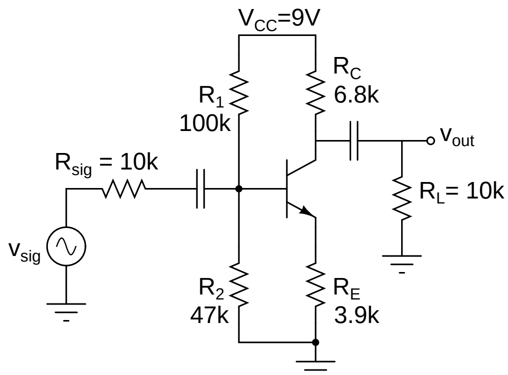
$$\boxed{A_v = \frac{v_{out}}{v_{sig}} = \mu \frac{R_L}{R_{out} + R_L} \frac{R_1\|R_2\|R_{in}}{R_1\|R_2\|R_{in} + R_{sig}}}$$

where

$$
\begin{aligned}
\mu &= \frac{-\beta \times R_C}{r_\pi + R_E\,(1+\beta)} \\
R_{in} &= r_\pi + R_E\,(1+\beta) \\
R_{out} &= R_C
\end{aligned}
$$

**Example**

Find the voltage gain $A_v = v_{out}/v_{sig}$ for the following amplifier. Assume $\beta = 100$. Assume the capacitors are short circuits at the frequency of operation.

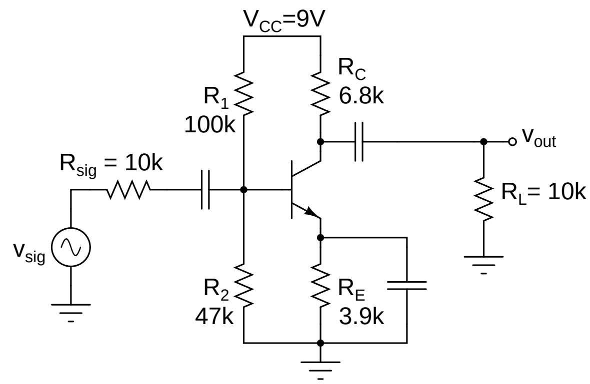



The dc analysis was performed in section 2.1 and $I_{CQ}$ was fount to be $0.51mA$.

$$
\begin{aligned}
g_m &= \frac{I_{CQ}}{V_T} = \frac{0.51mA}{0.025V} = 20.4mA \\
r_\pi &= \frac{\beta}{g_m} = \frac{100}{20.4mA} = 4.9k\Omega \\
\mu &= \frac{-\beta \times R_C}{r_\pi + R_E\,(1+\beta)} \\
&= \frac{-100 \times 6.8k\Omega}{4.9k\Omega + 101 \times 3.9k\Omega} = 1.7 \\
A_v &= \frac{v_{out}}{v_{sig}} = \mu \frac{R_L}{R_{out} + R_L} \times \frac{R_1 \| R_2 \| R_{in}}{R_1 \| R_2 \| R_{in} + R_{sig}} \\
R_{in} &= r_\pi + R_E\,(1+\beta) = 4.9k\Omega + 101 \times 3.9k\Omega = 399k\Omega \\
R_{out} &= R_C = 6.8k\Omega \\
&= -1.7V/V \frac{10k\Omega}{6.8k\Omega + 10k\Omega} \times \frac{32k\Omega \| 399k\Omega}{32k\Omega \| 399k\Omega + 10k\Omega} \\
&= -1.7V/V \frac{10}{16.8} \frac{29.6}{39.6} = -0.76V/V
\end{aligned}
$$

**Example**

We can increase the gain by including a bypass capacitor across $R_E$. The circuit becomes:




The dc analysis is not affected by the presence of the bypass capacitor, but for ac the bypass capacitor is a short circuit and $R_E$ becomes 0.

$$\begin{aligned}
\mu &= \frac{-\beta \times R_C}{r_\pi + R_E\,(1+\beta)} \\
&= \frac{-100 \times 6.8k\Omega}{4.9k\Omega + 101 \times 0} = \frac{-100 \times 6.8k\Omega}{4.9k\Omega} = -139V/V \\
A_v &= \frac{v_{out}}{v_{sig}} = \mu \frac{R_L}{R_{out} + R_L} \times \frac{R_1 \| R_2 \| R_{in}}{R_1 \| R_2 \| R_{in} + R_{sig}} \\
R_{in} &= r_\pi + R_E\,(1+\beta) = 4.9k\Omega + 101 \times 0 = 4.9k\Omega \\
R_{out} &= R_C = 6.8k\Omega \\
A_v &= -139V/V \frac{10k\Omega}{6.8k\Omega + 10k\Omega} \times \frac{32k\Omega \| 4.9k\Omega}{32k\Omega \| 4.9k\Omega + 10k\Omega} \\
&= -139V/V \frac{10}{16.8}\frac{4.25}{14.25} = -24.7V/V
\end{aligned}$$