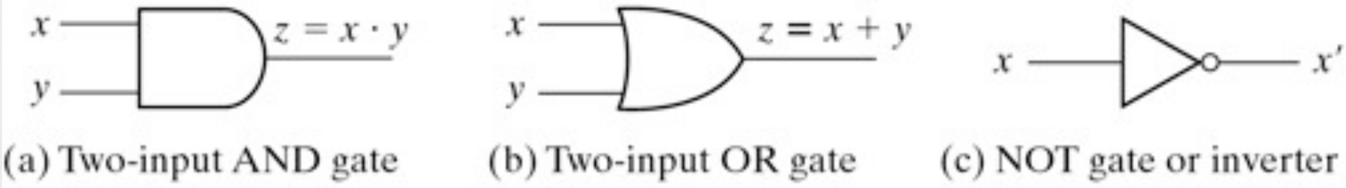
BOOLEAN ALGEBRA

INEL4205 - Spring 2012

OPERACIONES BÁSICAS

- AND - Salida es “1” si todas las entradas son “1”
- OR - salida es “1” si alguna entrada es “1”
- Not (complemento) - salida es “1” si la entrada es “0”, “0” si la entrada es “1”



(a) Two-input AND gate (b) Two-input OR gate (c) NOT gate or inverter

Fig. 1-4 Symbols for digital logic circuits

TABLAS DE VERDAD

- la tala de verdad presenta la salida para todas las combinaciones posibles de las entradas
- si hay "n" entradas, el total de combinaciones únicas es 2^N

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

AND

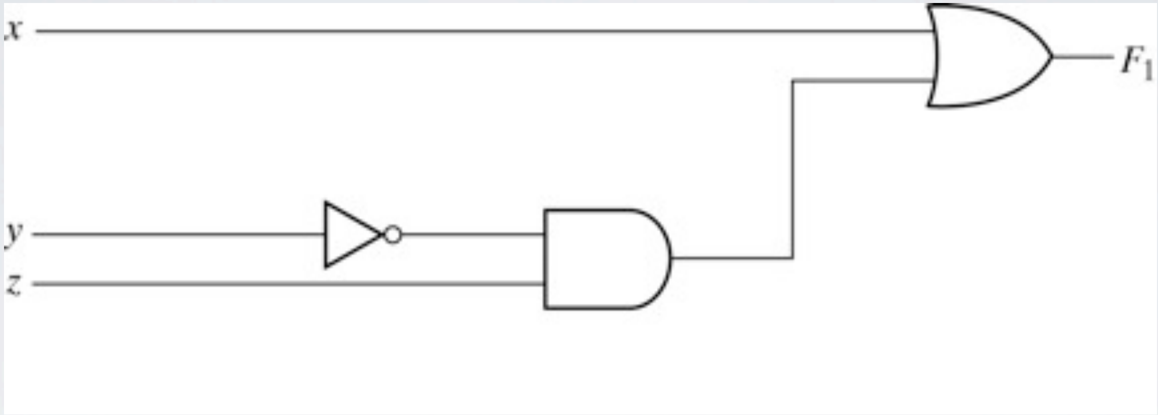
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

OR

A	Y
0	1
1	0

NOT

EXPRESIÓN BOOLEANA



Escriba la tabla de verdad y
la expresión booleana que representa
la salida "F_1" en términos de las entradas
"x", "y" y "z"

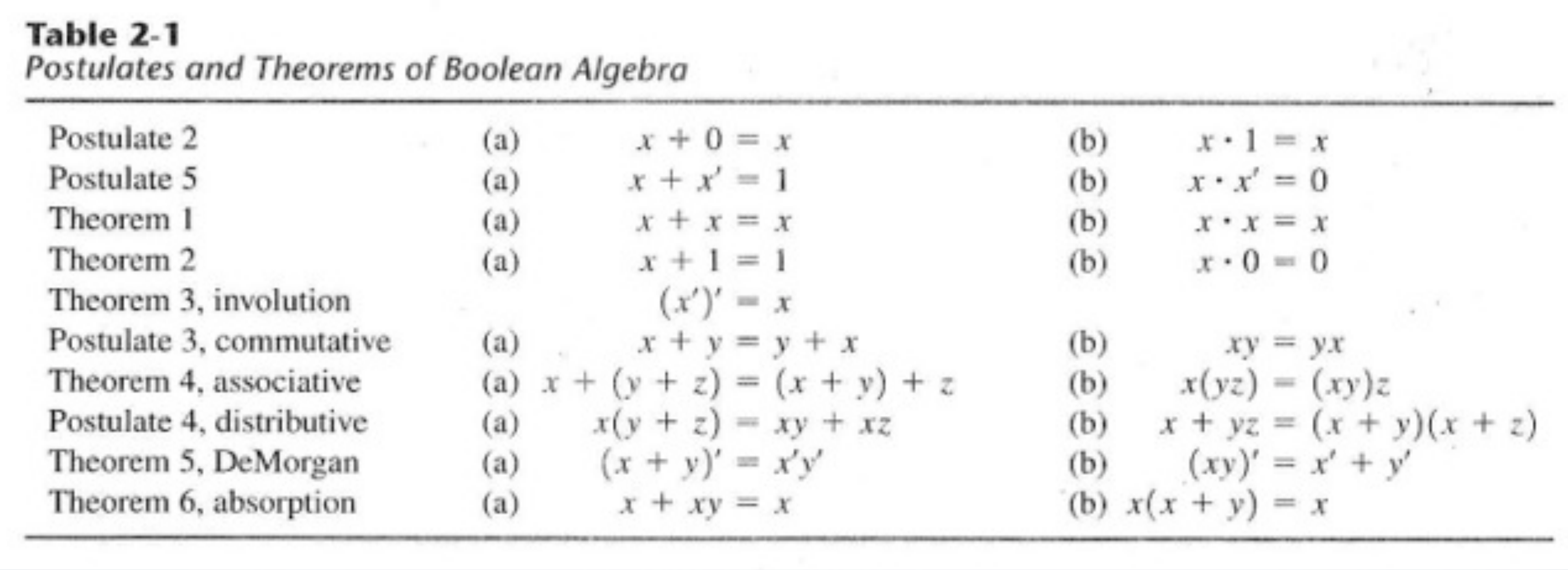


Table 2-1
Postulates and Theorems of Boolean Algebra

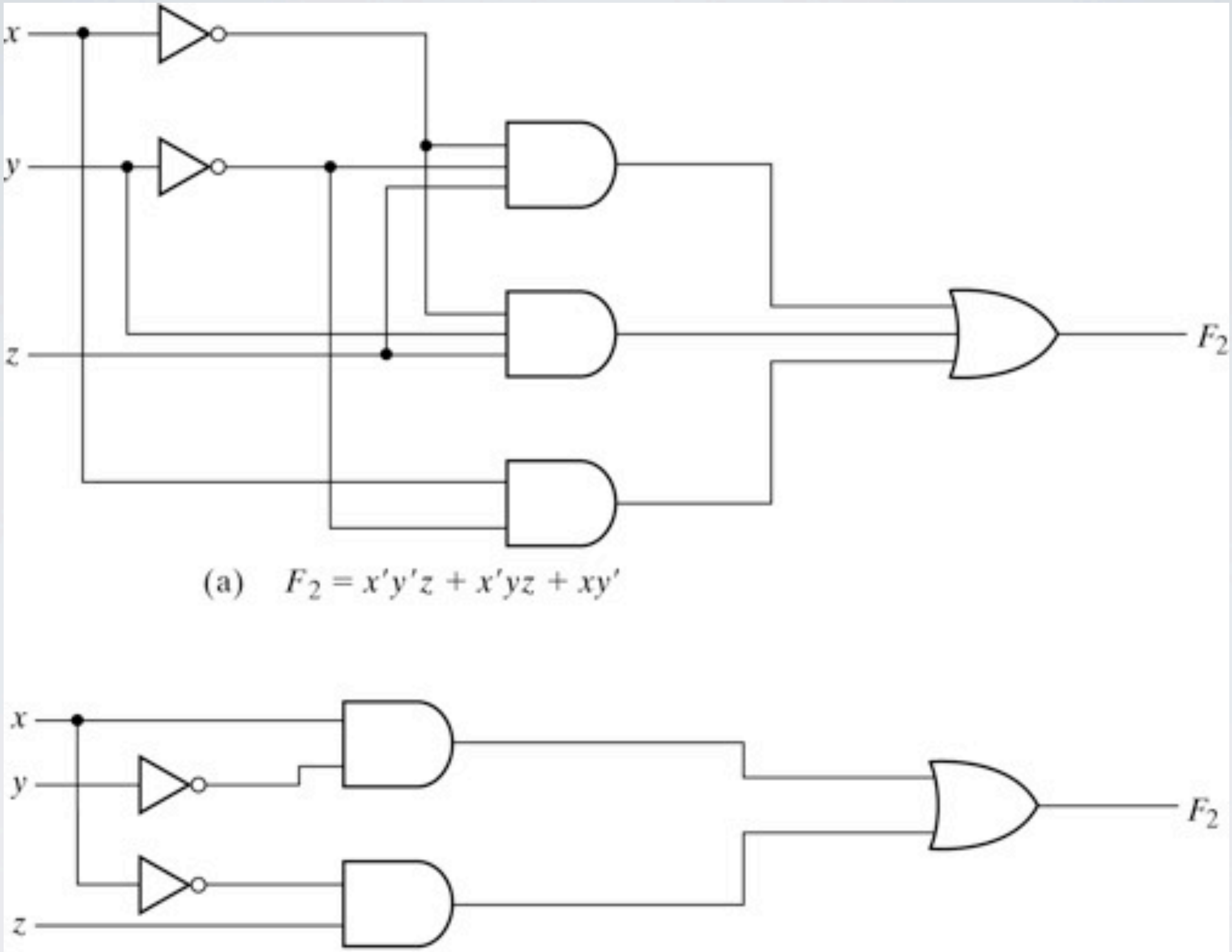
Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

postulado 4b forma 2	$X + X'Y = X + Y$
Teorema del Consenso	$XY+X'Z+YZ = XY+X'Z$

SIMPLIFICACIÓN

- Simplifique la siguiente expresión booleana

$$F_2=x'y'z+x'yz+xy'$$

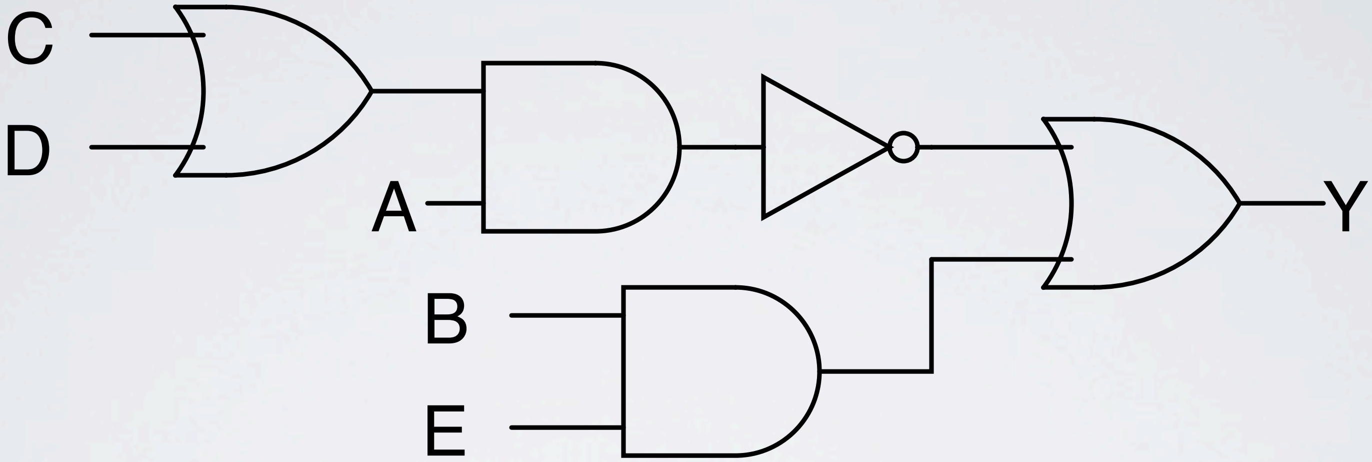


(a) $F_2 = x'y'z + x'yz + xy'$

(b) $F_2 = xy' + x'z$

Fig. 2-2 Implementation of Boolean function F_2 with gates

EJEMPLO



- Escriba la expresión booleana y la tabla de verdad para la "Y" del diagrama de arriba

- dibuje el diagrama para un circuito que produzca la siguiente salida

ACF+Dc'f'

EJEMPLOS

1. determine el complemento de la siguiente expresión booleana:

$$(a'+b)c'$$

2. Demuestre que:
   - $[(ab'+c)d'+e]' = [(a'+b)c'+d]e'$
   - $x+xy=x$

EJEMPLOS

simplifique:

- $x(x'+y)$
- $x+x'y$
- $(x+y)(x+y')$
- $xy+x'z+yz$
- $(x+y)(x'+z)(y+z)$

Práctica: probs. 2.2, 2.3 y 2.4

TEOREMAS ADICIONALES

Aplicando el postulado 4b

$$X + X'Y = X + Y$$

llamemoslo postulado 4b forma 2

Teorema del Consenso

$$XY+X'Z+YZ = XY+X'Z+YZ(X+X')$$

$$XY+X'Z+YZ = XY(1+Z)+X'Z(1+Y)$$

$$\boxed{XY+X'Z+YZ = XY+X'Z}$$

Simplifique

$$abcd' + a'b'cd + cd'$$

$$abcd' + a'b'cd + cd'$$

$$(ab+1)cd' + a'b'CD$$

$$c(d' + a'b'd)$$

Por el postulado 4b forma 2

$$C(D' + A'B')$$

simplifique

$ab'c' + cd' + bc'd'$

$$ab'c +cd' + bc'd'$$

$$ab'c + (c + bc')d'$$

postulado 4b forma 2

$$ab'c + cd' + bd'$$

simplifique

$$(a + b')(a' + b' + d)(b' + c + d')$$

$(a + b')(a' + b' + d)(b' + c + d')$

dualidad

$ab' + a'b'd + b'cd'$

$(a + a'd + cd')b'$

$(a + d + cd')B'$

$(a + d + c)b'$

dualidad

$acd + b'$

$$xy + x'yz' + yz$$

$$xy + x'yz' + yz$$

$$xy + x'yz' + yz(x + x')$$

$$xy(1 + z) + x'y(z' + z)$$

$$xy + x'y$$

$$Y$$

$(xy' + z)(x + y')\ z$

$$(xy' + z)(x + y')\ z$$

$$(xy' + zx + zY')z$$

$$xy'z + xz + y'z$$

$$(x + 1)y'z + xz$$

$$(x + y')z$$

$$xy' + z + (x' + y)z'$$

$$xy' + z + (x' + y)z'$$

postulado 4b forma 2

$$xy' + x' + Y + z$$

postulado 4b forma 2

$$x' + Y' + Y + z$$

$$\boxed{1}$$

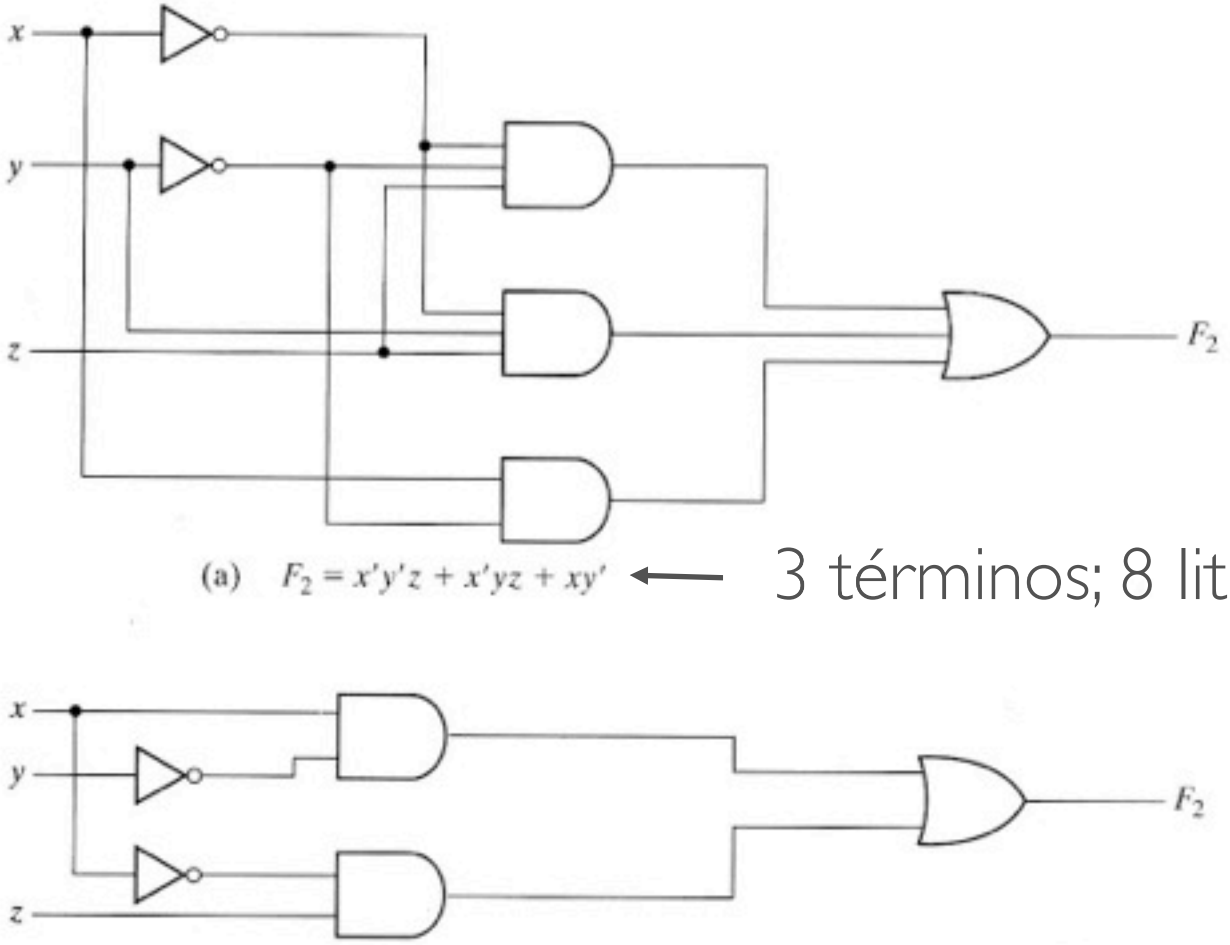
otra forma:

$$(xy' + z)' = (x' + y)z'$$

así que la expresión tiene la forma

$$A + A'$$

y por lo tanto siempre es 1



(a) $F_2 = x'y'z + x'yz + xy'$ ← 3 términos; 8 literales

(b) $F_2 = xy' + x'z$ ← 2 términos; 4 literales

FIGURE 2-2
Implementation of Boolean function F_2 with gates

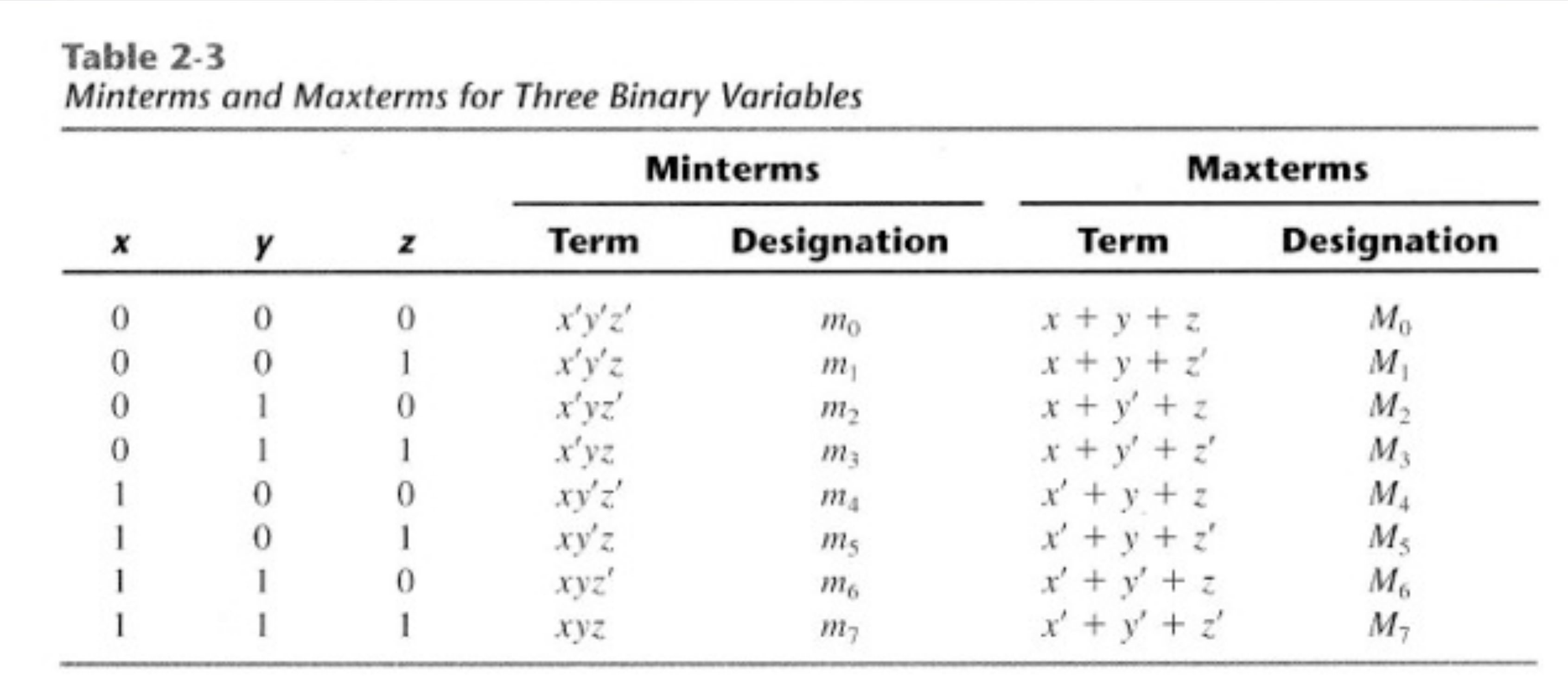
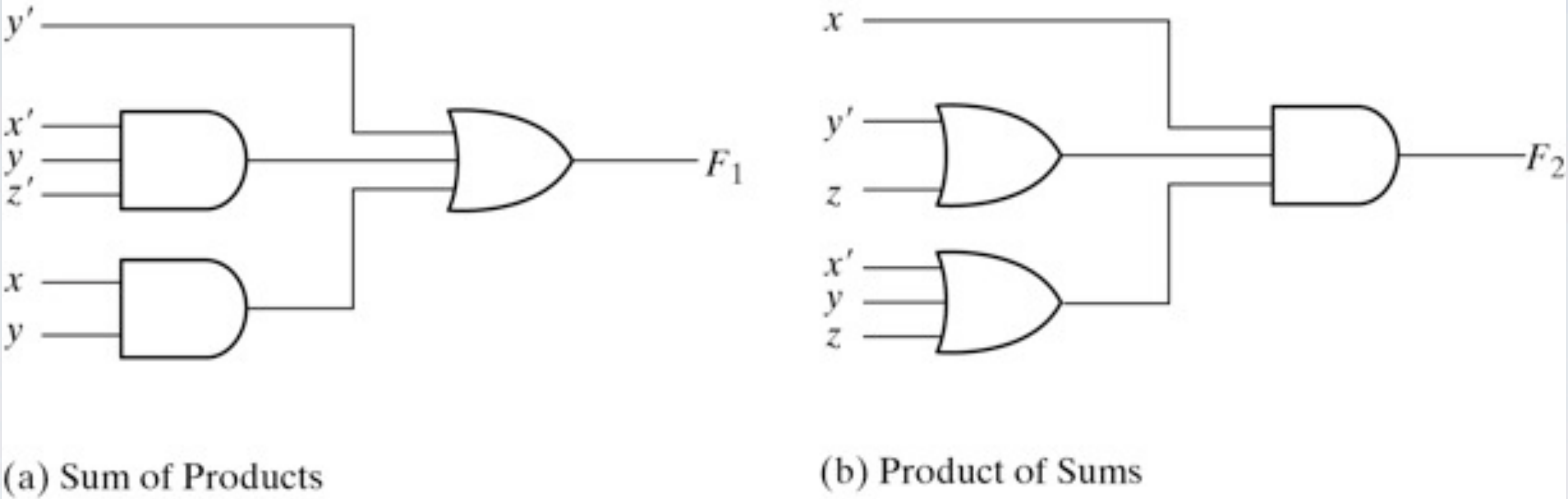


Table 2-3
Minterms and Maxterms for Three Binary Variables

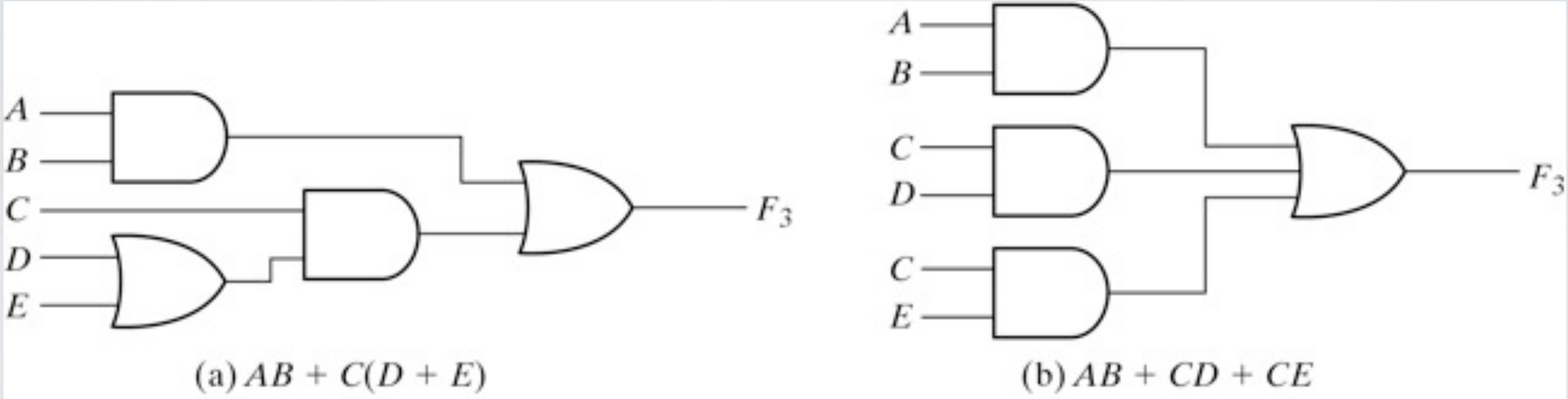
			Minterms		Maxterms	
x	**y**	**z**	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7



(a) Sum of Products

(b) Product of Sums

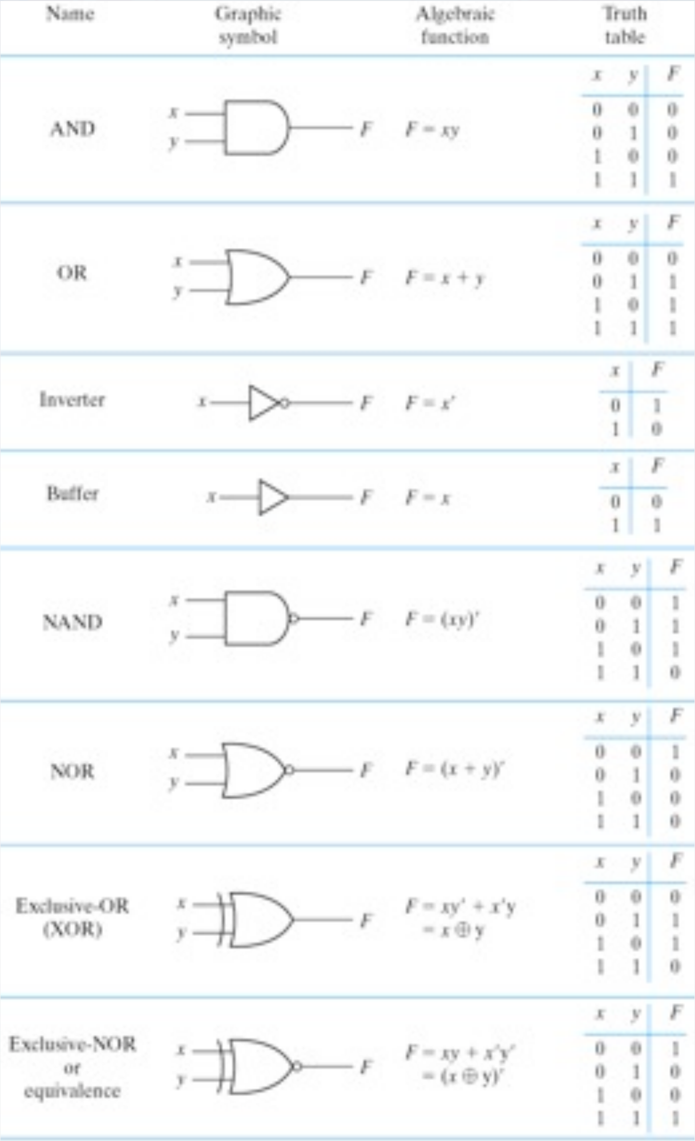
Fig. 2-3 Two-level implementation



(a) $AB + C(D + E)$

(b) $AB + CD + CE$

Fig. 2-4 Three- and Two-Level implementation



Name	Graphic symbol	Algebraic function	Truth table
AND		$F = xy$	x y | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = x + y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
Inverter		$F = x'$	x | F 0 | 1 1 | 0
Buffer		$F = x$	x | F 0 | 0 1 | 1
NAND		$F = (xy)'$	x y | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = (x + y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1

Fig. 2-5 Digital logic gates

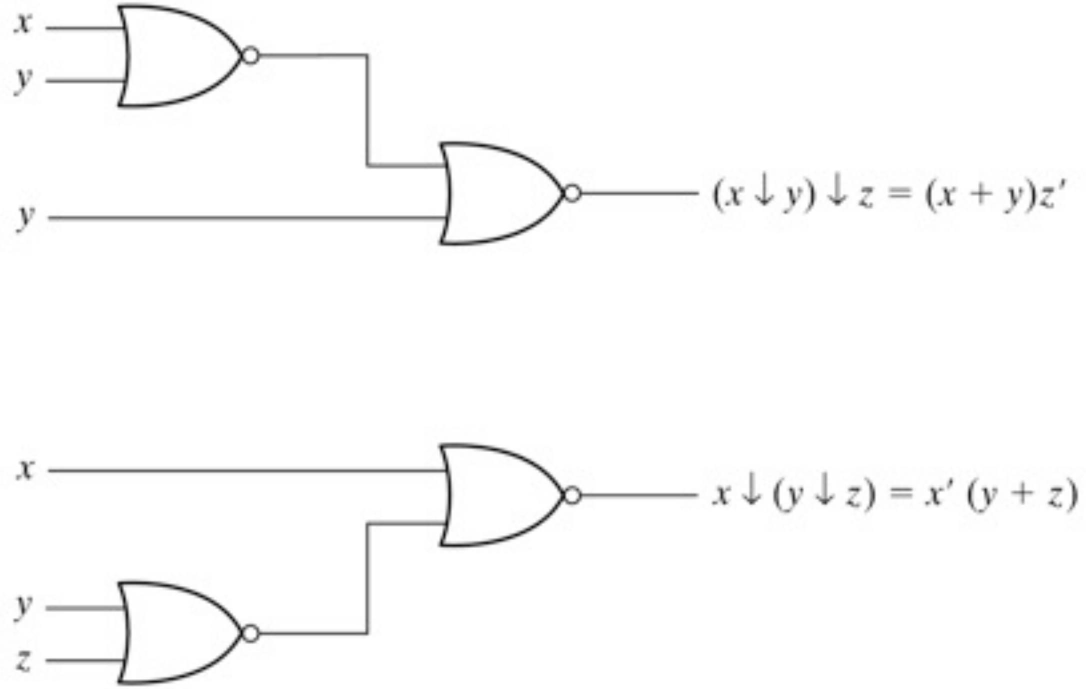
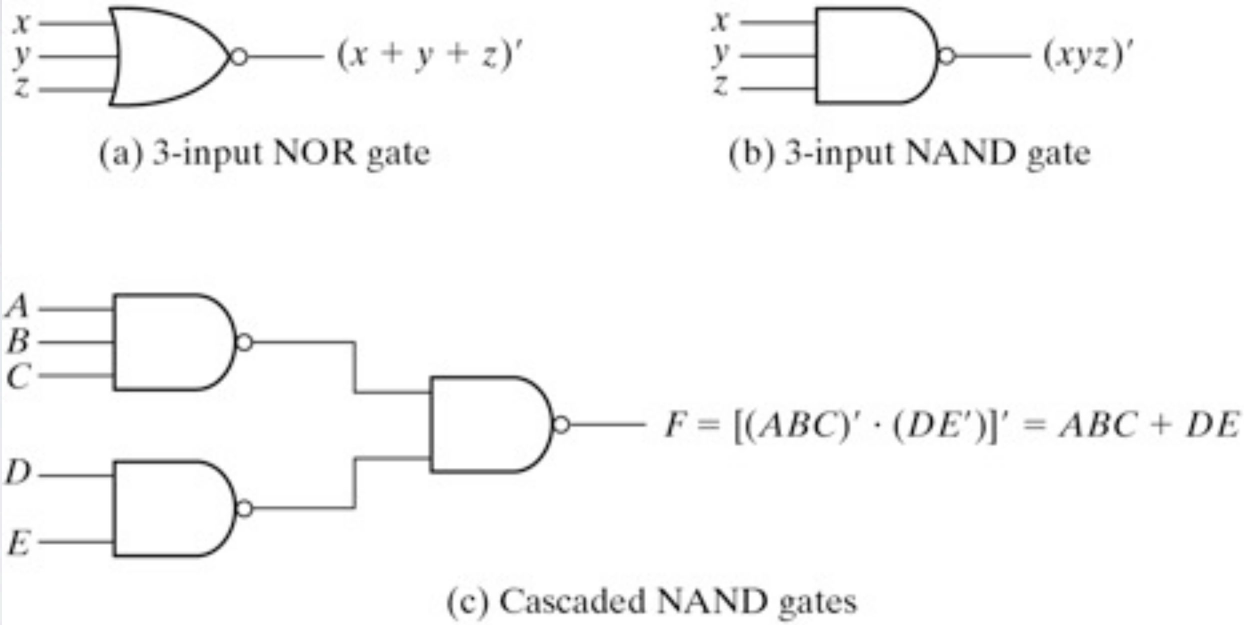

x
y
y
$(x \downarrow y) \downarrow z = (x + y)z'$
x
y
z
$x \downarrow (y \downarrow z) = x' (y + z)$

Fig. 2-6 Demonstrating the nonassociativity of the NOR operator; $(x \downarrow y) \downarrow z \neq x \downarrow (y \downarrow z)$

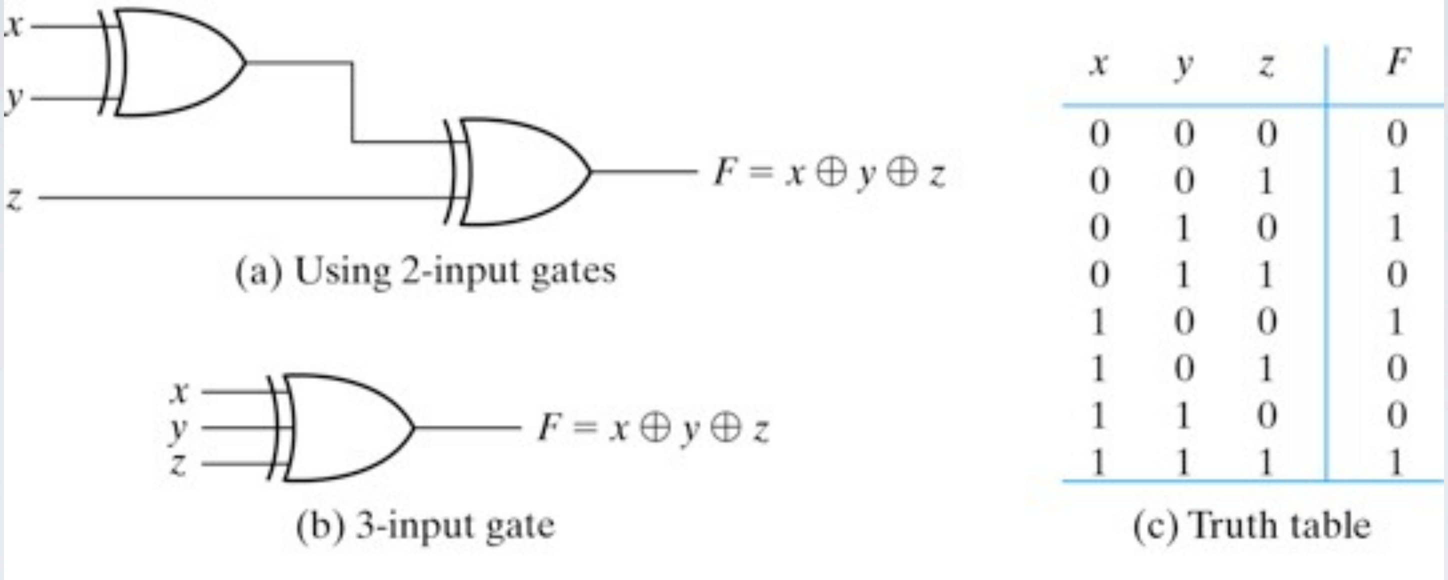


(a) 3-input NOR gate

(b) 3-input NAND gate

(c) Cascaded NAND gates

Fig. 2-7 Multiple-input and cascated NOR and NAND gates



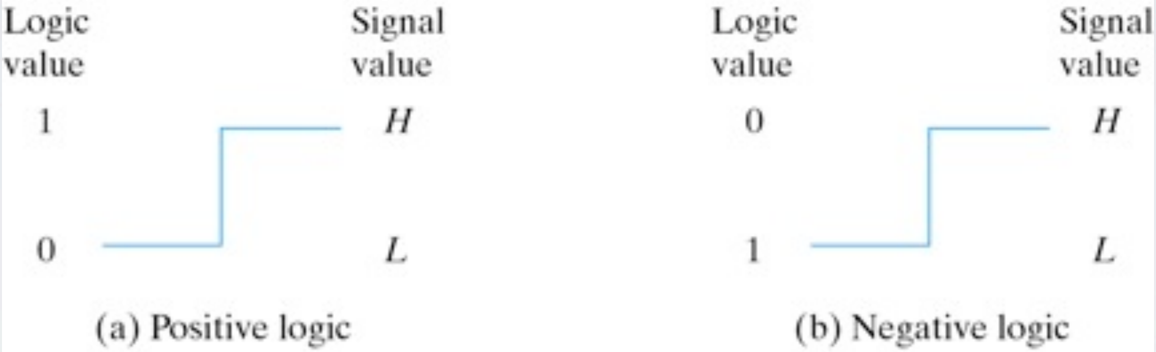
(a) Using 2-input gates

(b) 3-input gate

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(c) Truth table

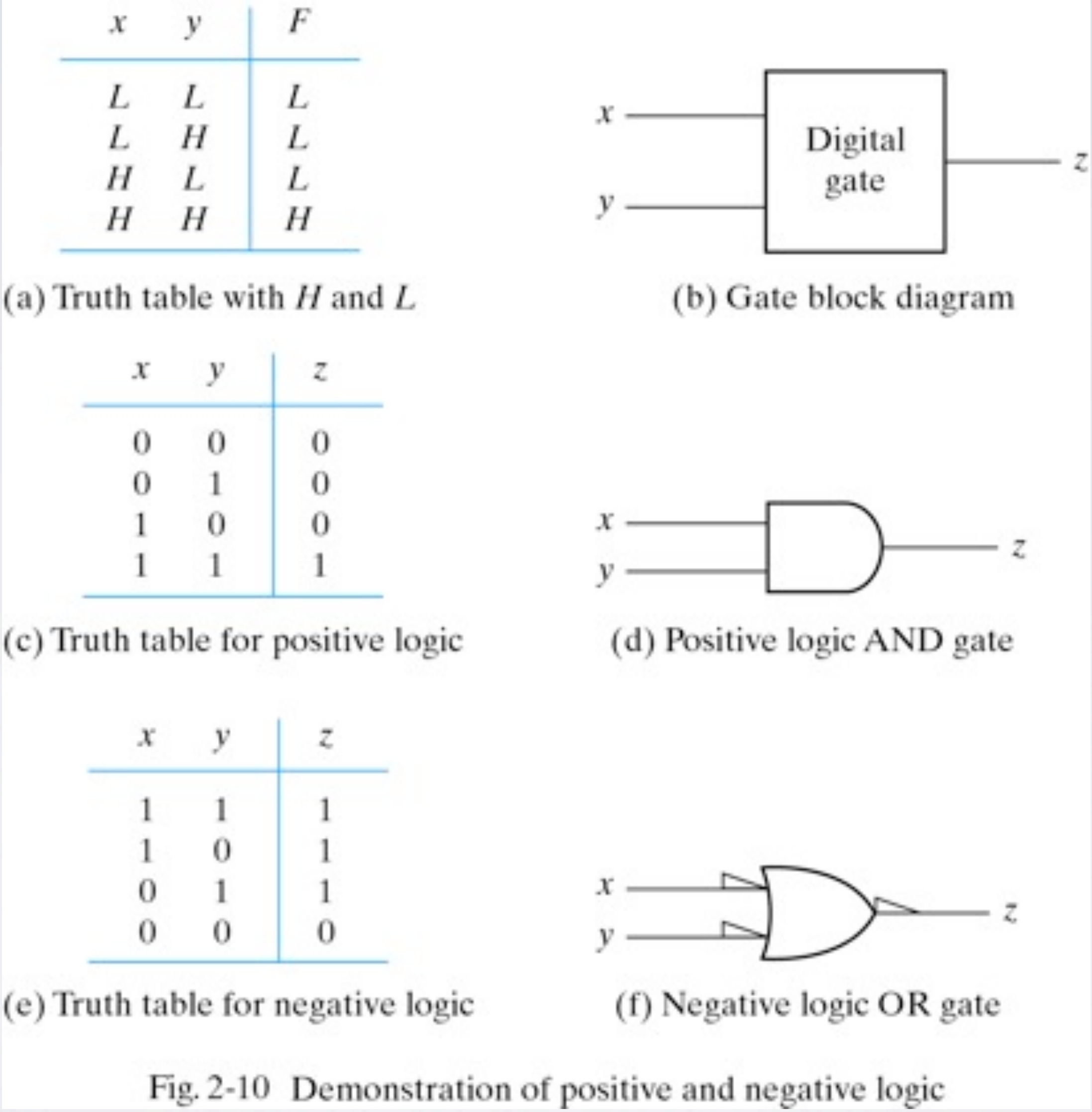
Fig. 2-8 3-input exclusive-OR gate



(a) Positive logic

(b) Negative logic

Fig. 2-9 signal assignment and logic polarity



x	*y*	*F*
L	*L*	*L*
L	*H*	*L*
H	*L*	*L*
H	*H*	*H*

(a) Truth table with *H* and *L*

(b) Gate block diagram

x	*y*	*z*
0	0	0
0	1	0
1	0	0
1	1	1

(c) Truth table for positive logic

(d) Positive logic AND gate

x	*y*	*z*
1	1	1
1	0	1
0	1	1
0	0	0

(e) Truth table for negative logic

(f) Negative logic OR gate

Fig. 2-10 Demonstration of positive and negative logic