GATE-LEVEL MINIMIZATION

INEL 4205 - Spring 2012

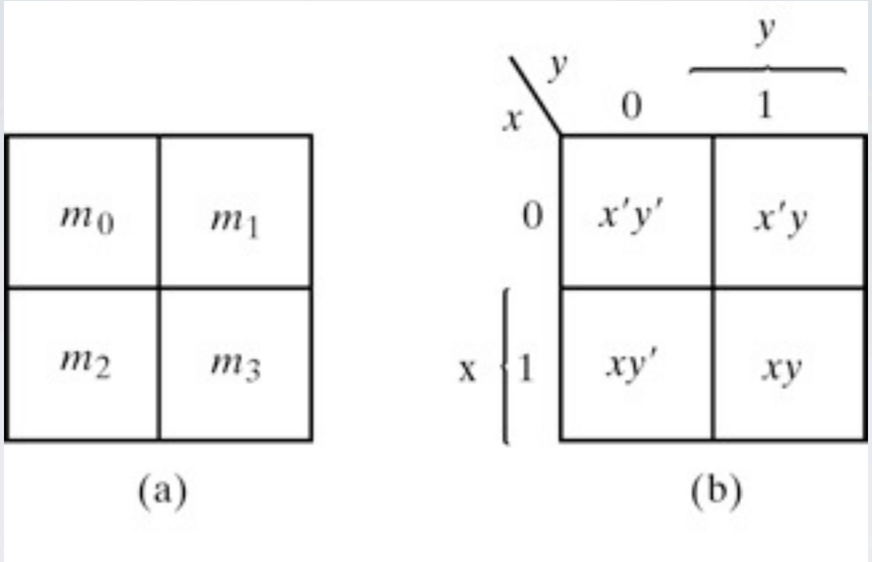


Fig. 3-1 Two-variable Map

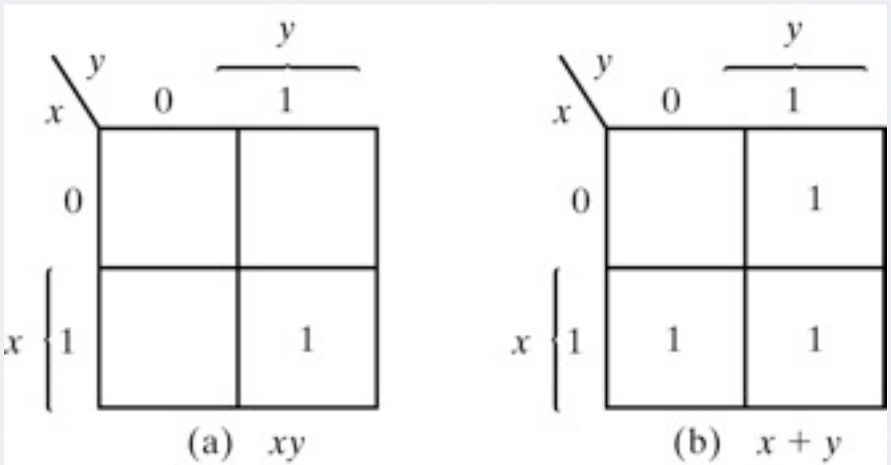


Fig. 3-2 Representation of Functions in the Map

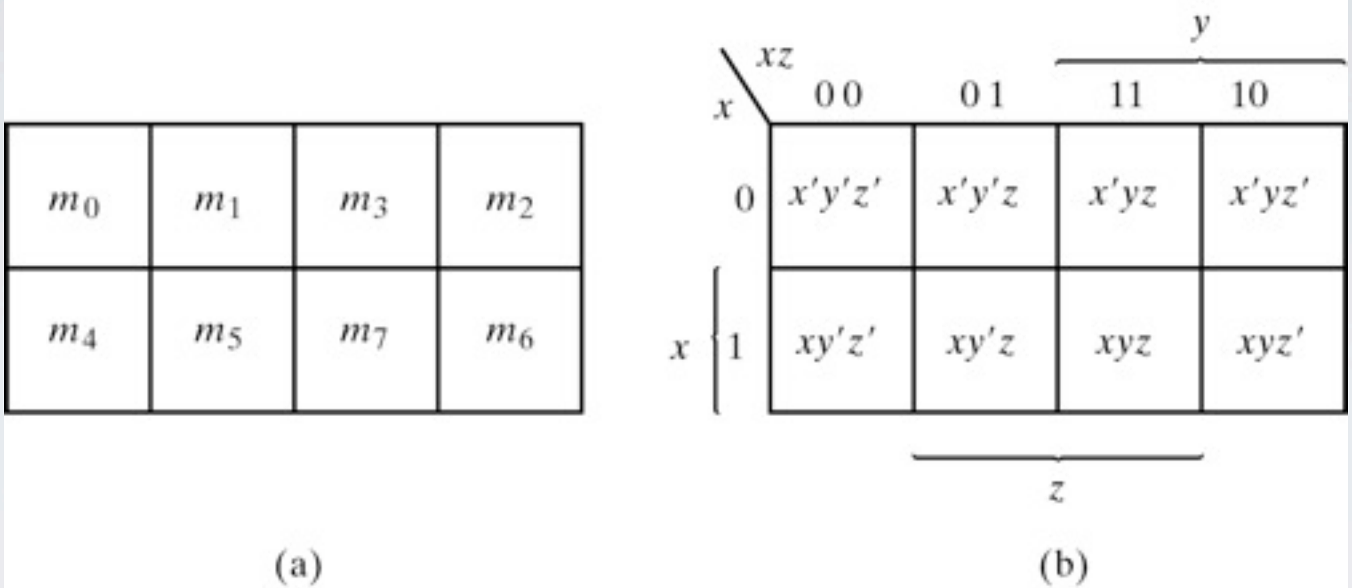


Fig. 3-3 Three-variable Map

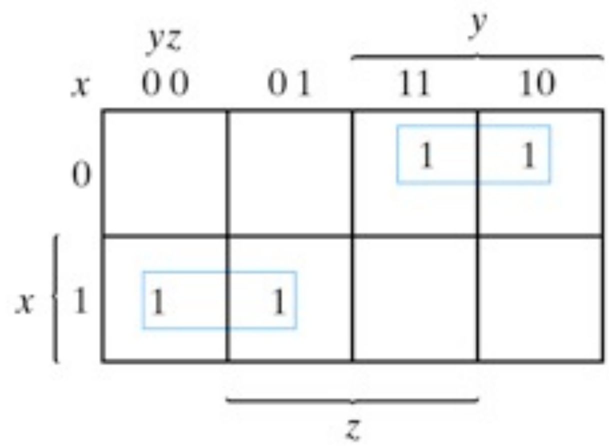


Fig. 3-4 Map for Example 3-1; $F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$

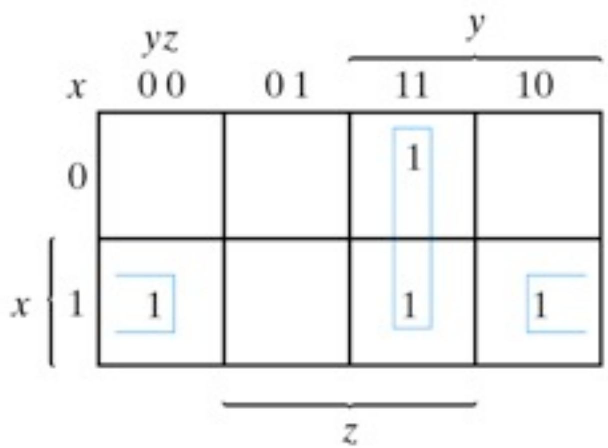


Fig. 3-5 Map for Example 3-2; $F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$

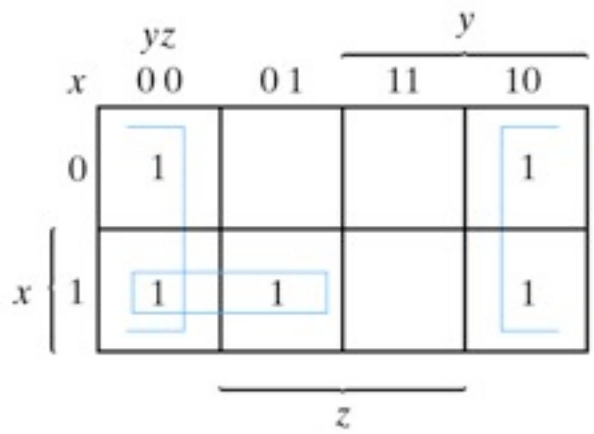


Fig. 3-6 Map for Example 3-3; $F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$

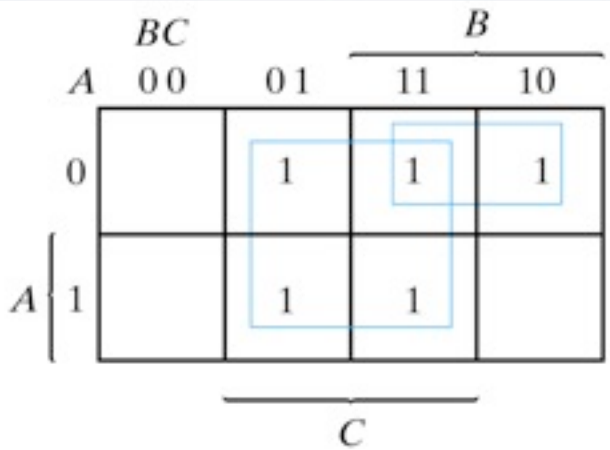


Fig. 3-7 Map for Example 3-4; $A'C + A'B + AB'C + BC = C + A'B$

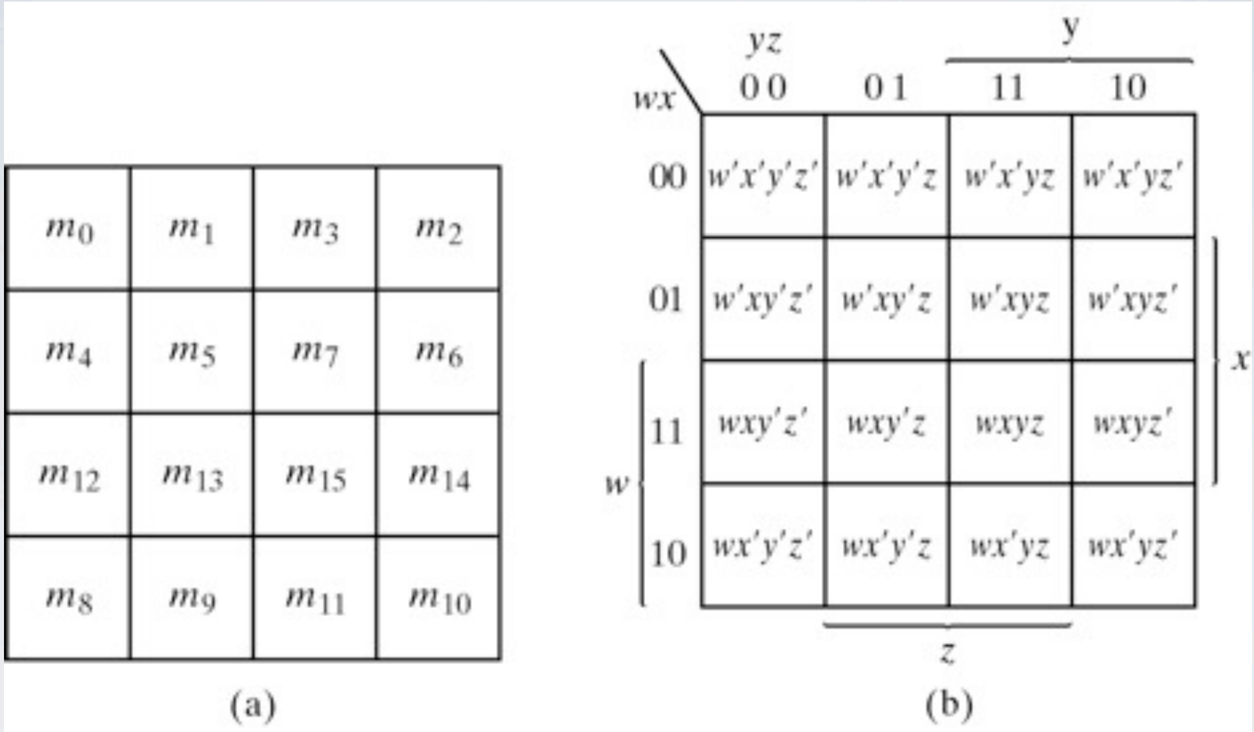


Fig. 3-8 Four-variable Map

Example: $f(w,x,y,z) = \sum(0,1,2,4,5,6,8,9,12,13,14)$

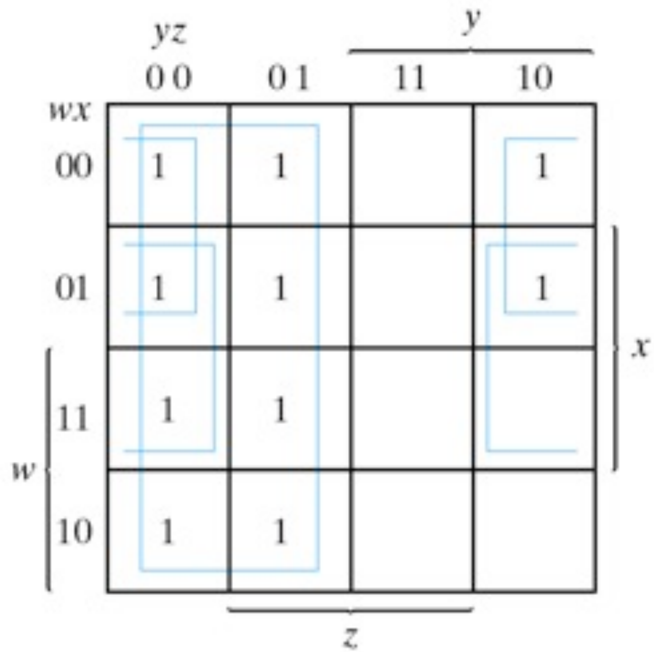


Fig. 3-9 Map for Example 3-5; $F(w, x, y, z)$
$= \Sigma\,(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) = y' + w'z' + xz'$

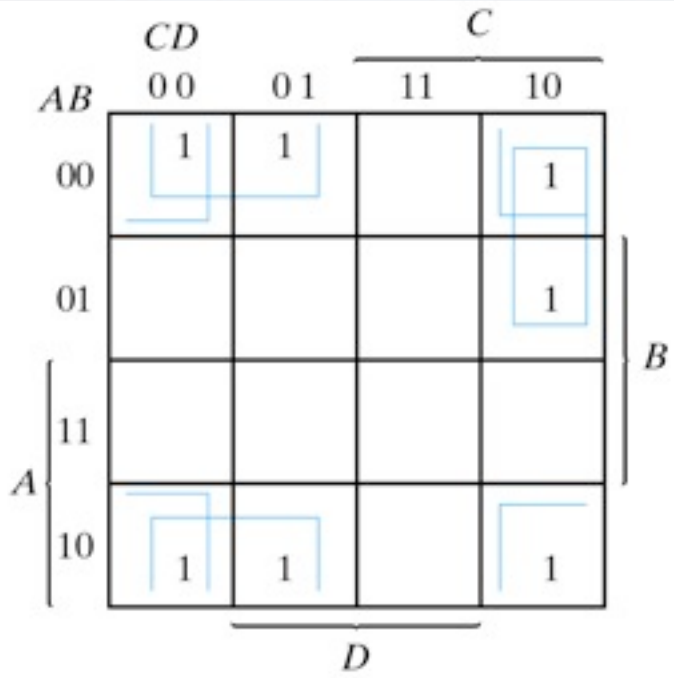
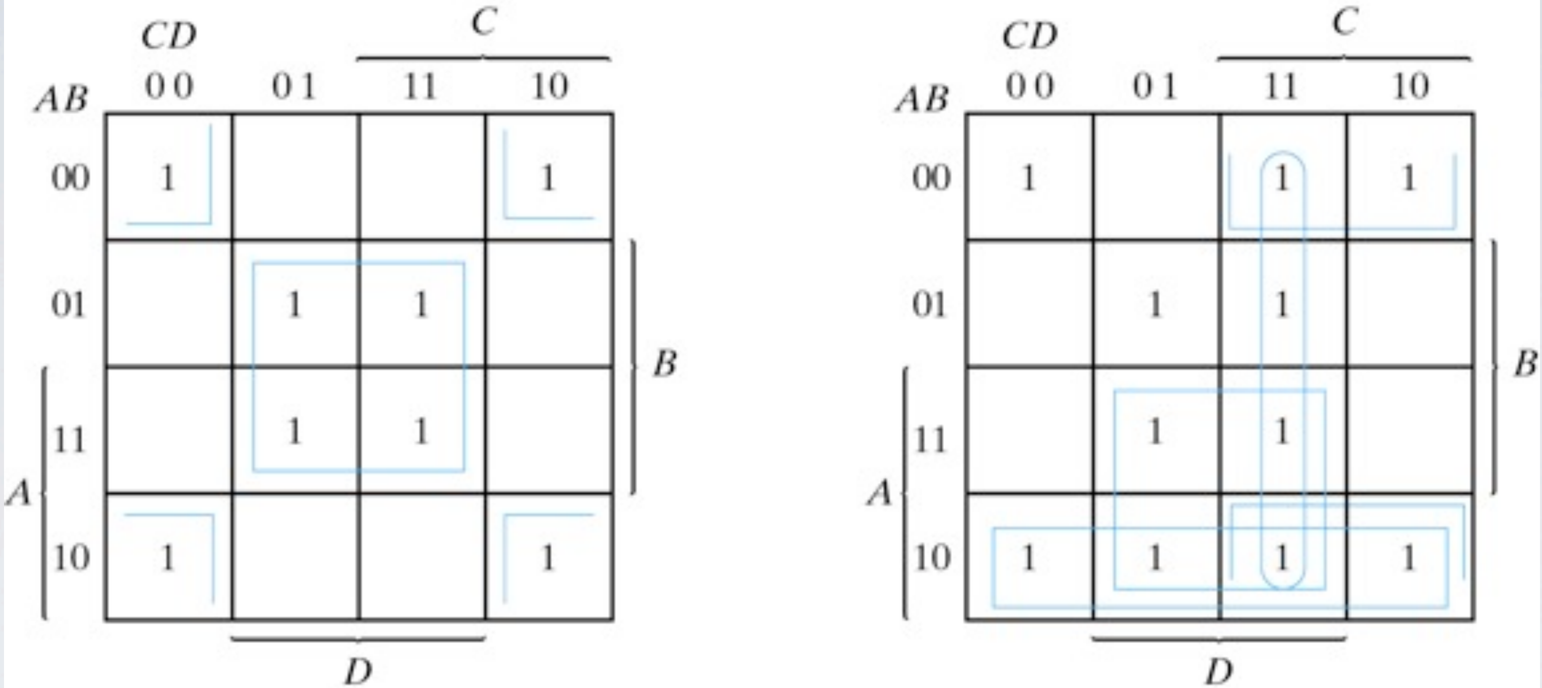


Fig.3-10 Map for Example 3-6; $A'B'C + B'CD' + A'BCD' + AB'C' = B'D' + B'C' + A'CD'$

PRIME IMPLICANTS

- In choosing adjacent squares in a map, we must ensure that
  - all the minterm of the function are covered when we combine the squares
  - the number of terms in the expression is minimized
  - there are no redundant terms (i.e. minterms covered by other terms)
- *Prime implicant (PI):* product term obtained by combining the maximum possible number of adjacent squares.
- If a minterm in a square is covered by only one PI then the PI is *essential*.
- To avoid redundant terms, do (1) essential prime implicants, (2) prime implicants, (3) other terms



(a) Essential prime implicants
BD and B′D′

(b) Prime implicants CD, B′C
AD, and AB′

Fig. 3-11 Simplification Using Prime Implicants

Map in (b): do the 1's in (a) first, then CD and AB'

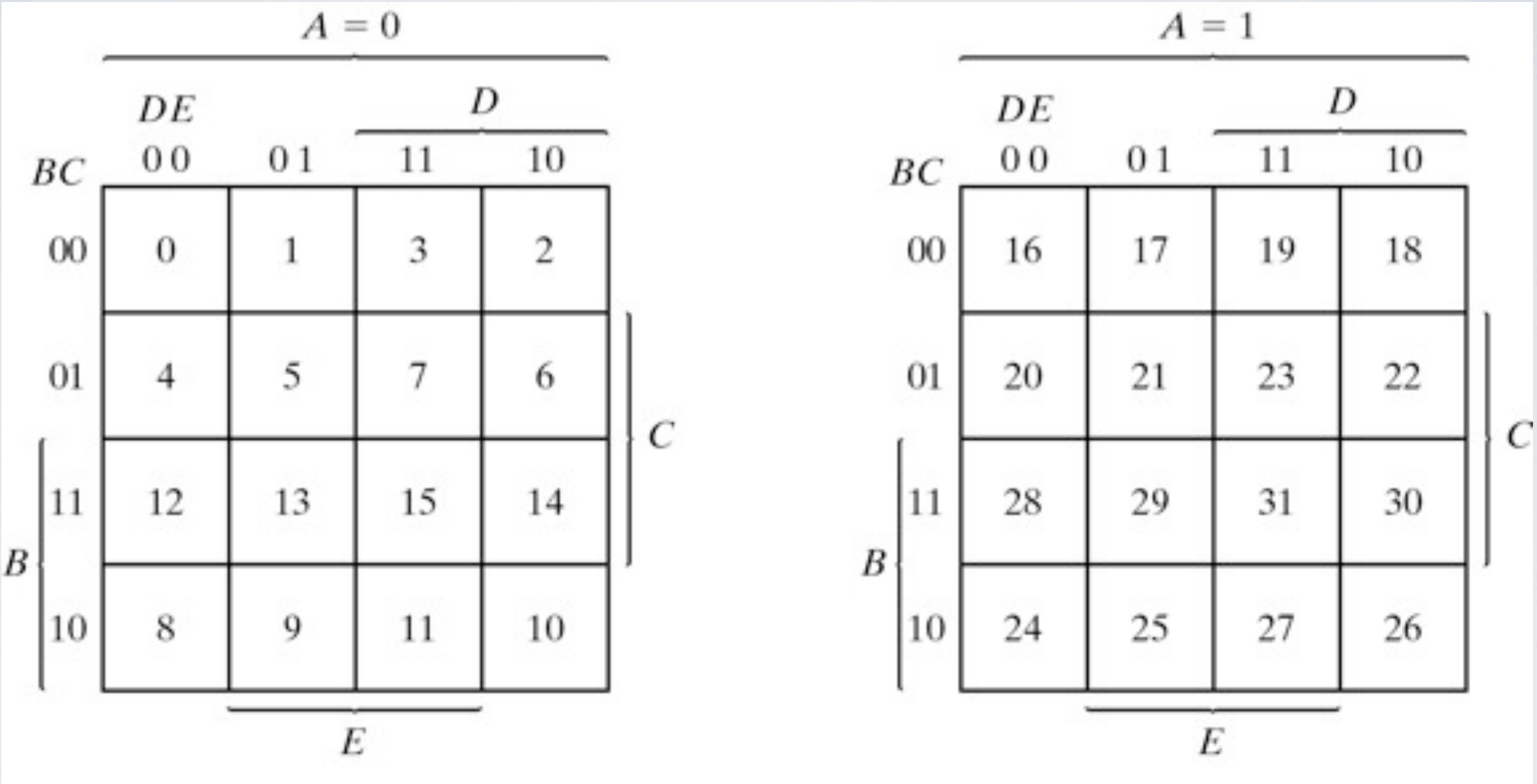


Fig. 3-12 Five-variable Map

Example: $F(A,B,C,D,E) = A'B'E' + BD'E + ACE$

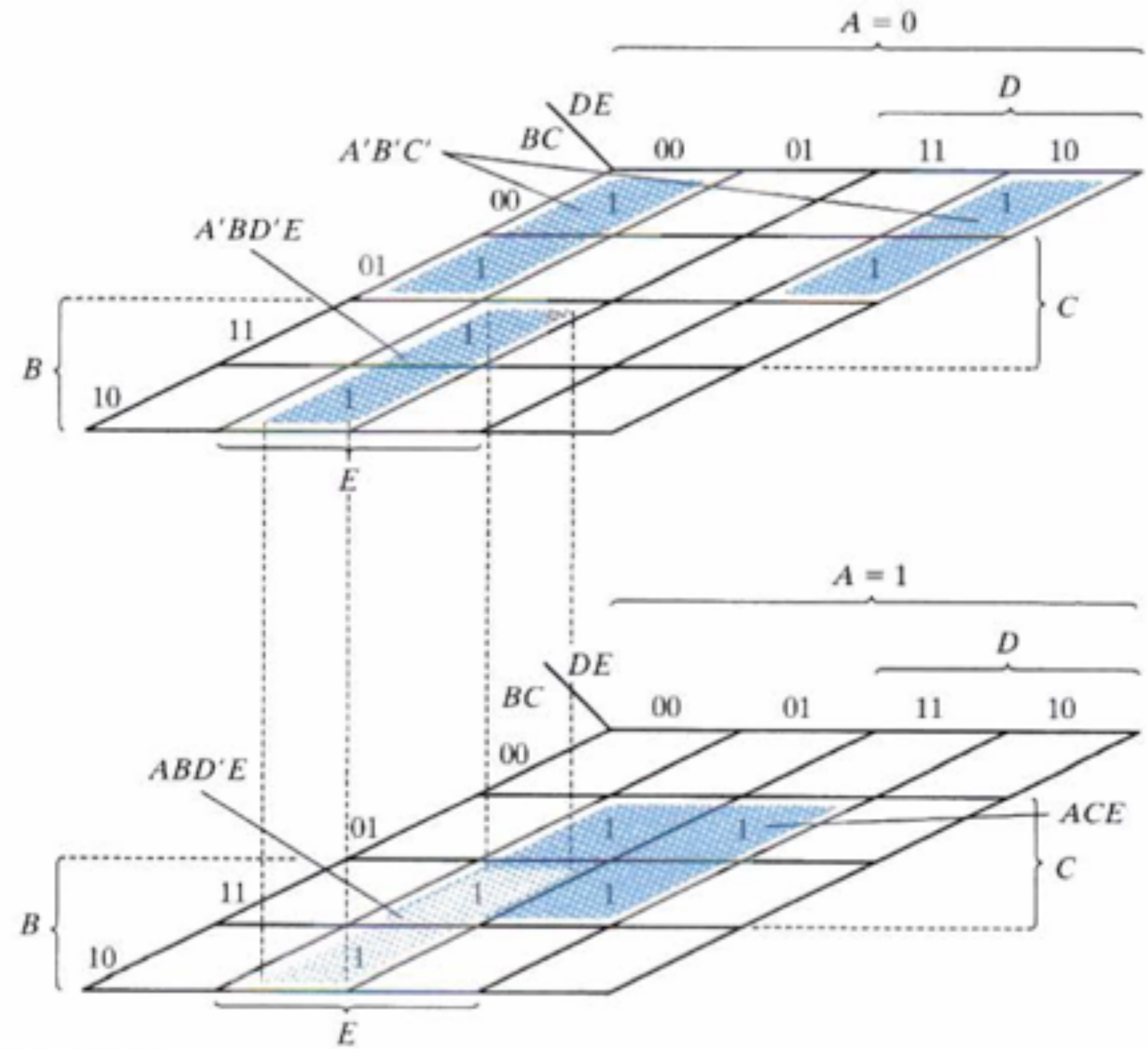


FIGURE 3.13
Map for Example 3.7, $F = A'B'E' + BD'E + ACE$

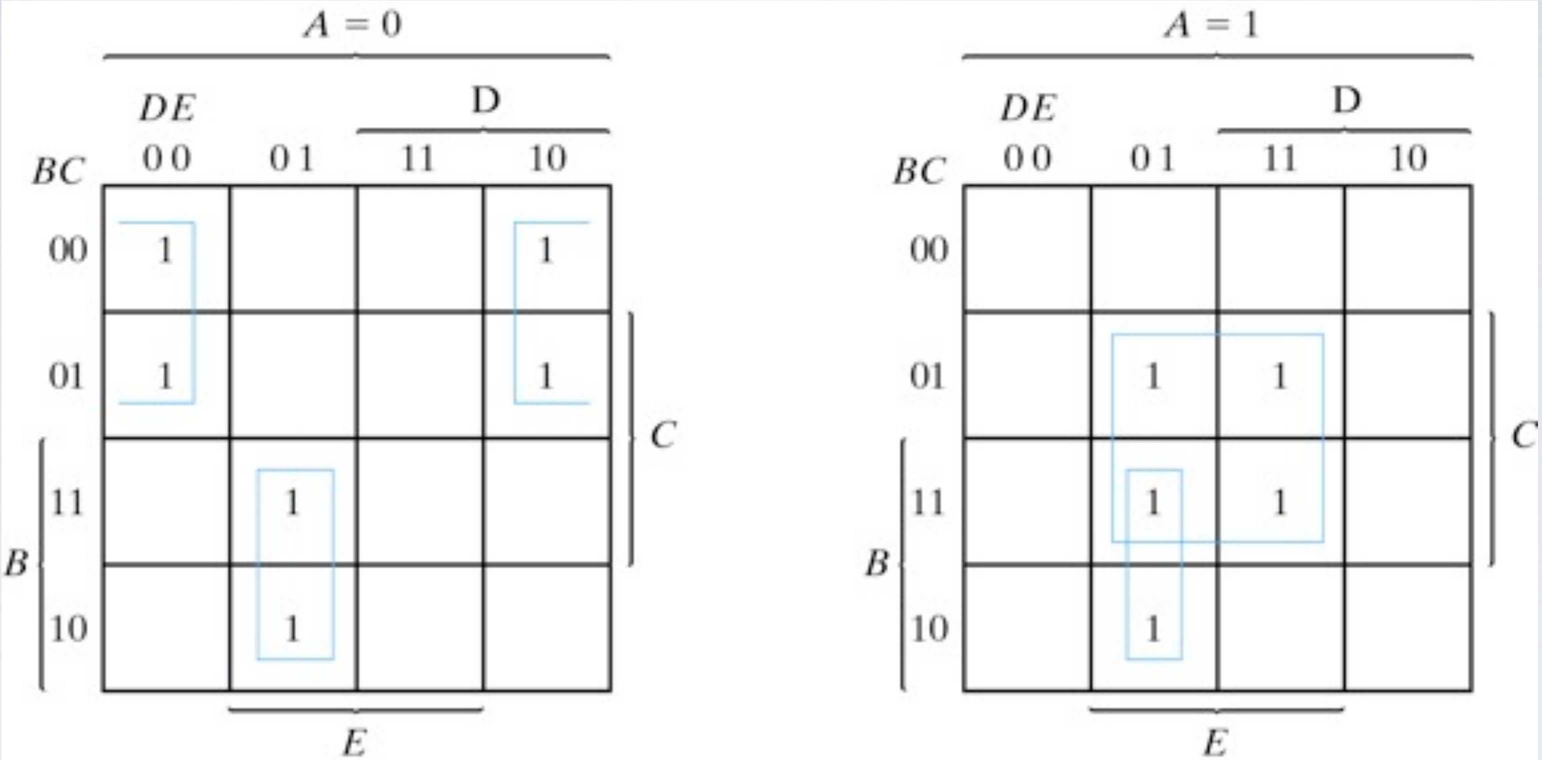


Fig. 3-13 Map for Example 3-7; $F = A'B'E' + BD'E + ACE$

$$\sum (0,1,2,5,8,9,10)$$

- Example 3-8: Simplify to a minimal expression using the:
  - 1's to produce a sum of products (AND-OR)
  - 0's to produce a complemented sum of products (AND-NOR)
  - 0's to produce a product of sums (OR-AND)
  - 1' to produce a complemented product of sums (OR-NAND)

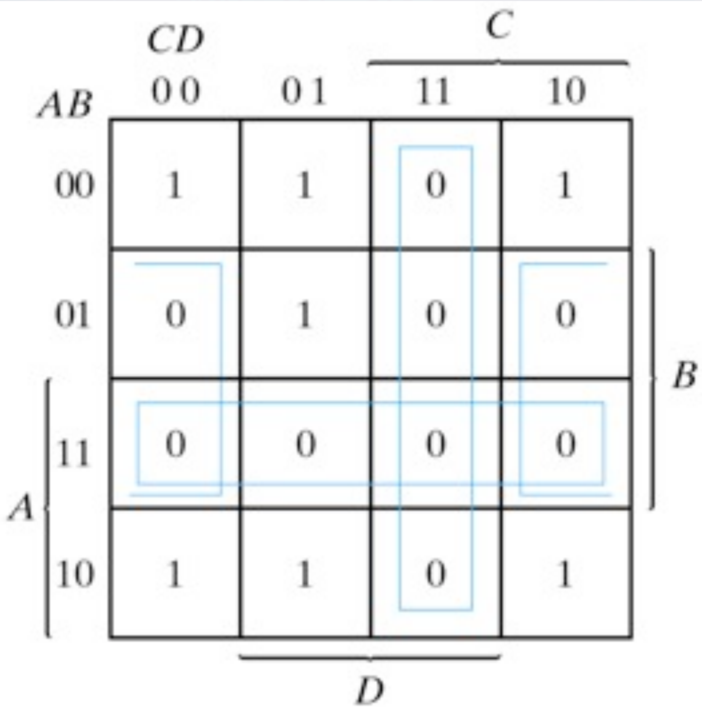
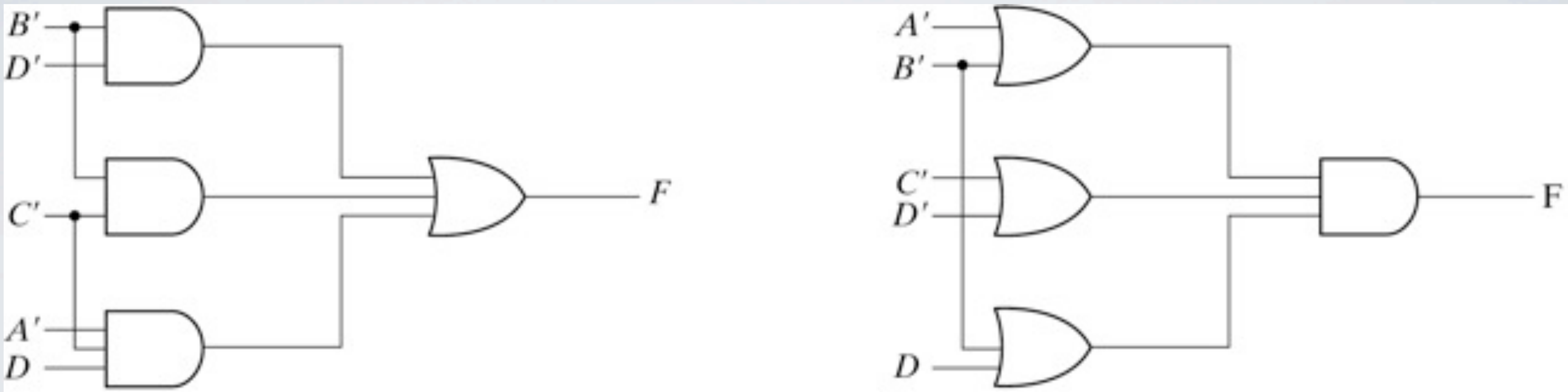


Fig. 3-14 Map for Example 3-8; $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10)$
$= B'D' + B'C' + A'C'D = (A' + B')(C' + D')(B' + D)$



(a) $F = B'D' + B'C' + A'C'D$

(b) $F = (A' + B')(C' + D')(B' + D)$

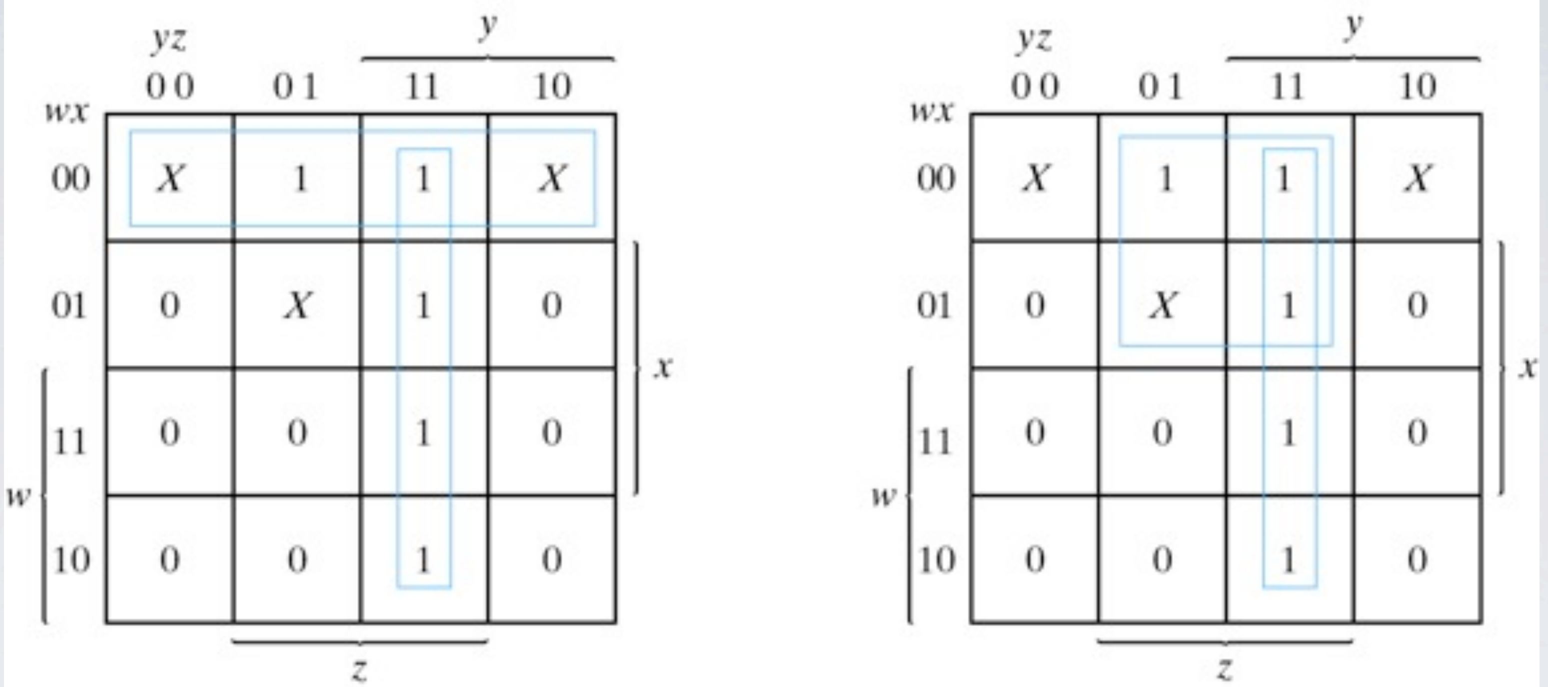
Fig. 3-15 Gate Implementation of the Function of Example 3-8

Table 3.2
Truth Table of Function F

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

x \ yz	00	01	11	10
0	0	1	1	0
1	1	0	0	1

Fig. 3-16 Map for the Function of Table 3-2



(a) $F = yz + w'x'$

(a) $F = yz + w'z$

Fig. 3-17 Example with don't-care Conditions

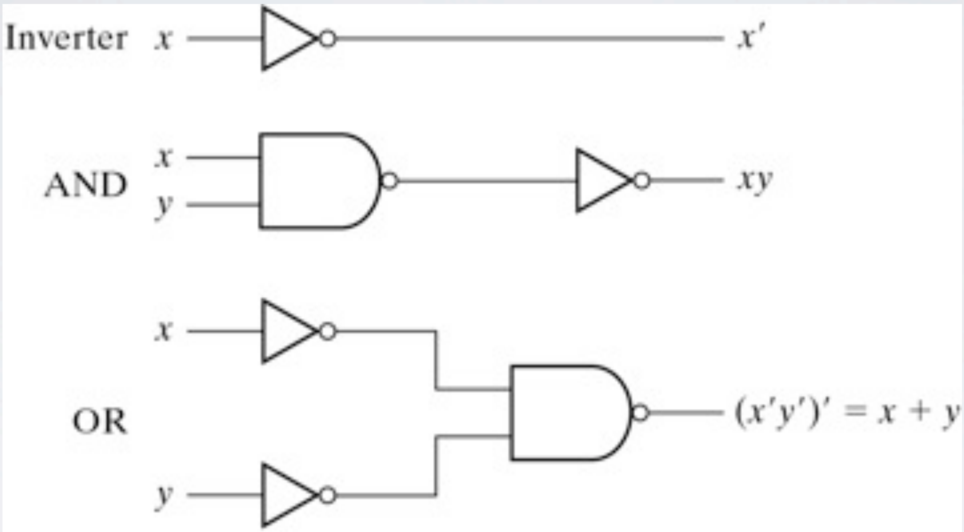
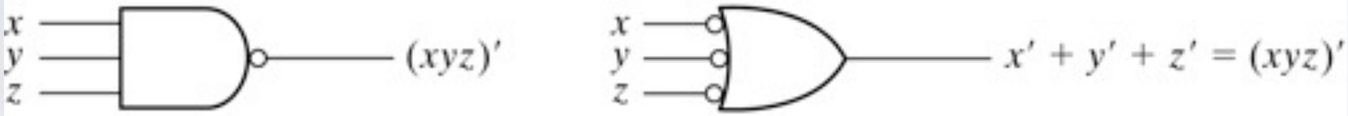


Fig. 3-18 Logic Operations with NAND Gates



(a) AND–invert

(b) Invert–OR

Fig. 3-19 Two Graphic Symbols for NAND Gate

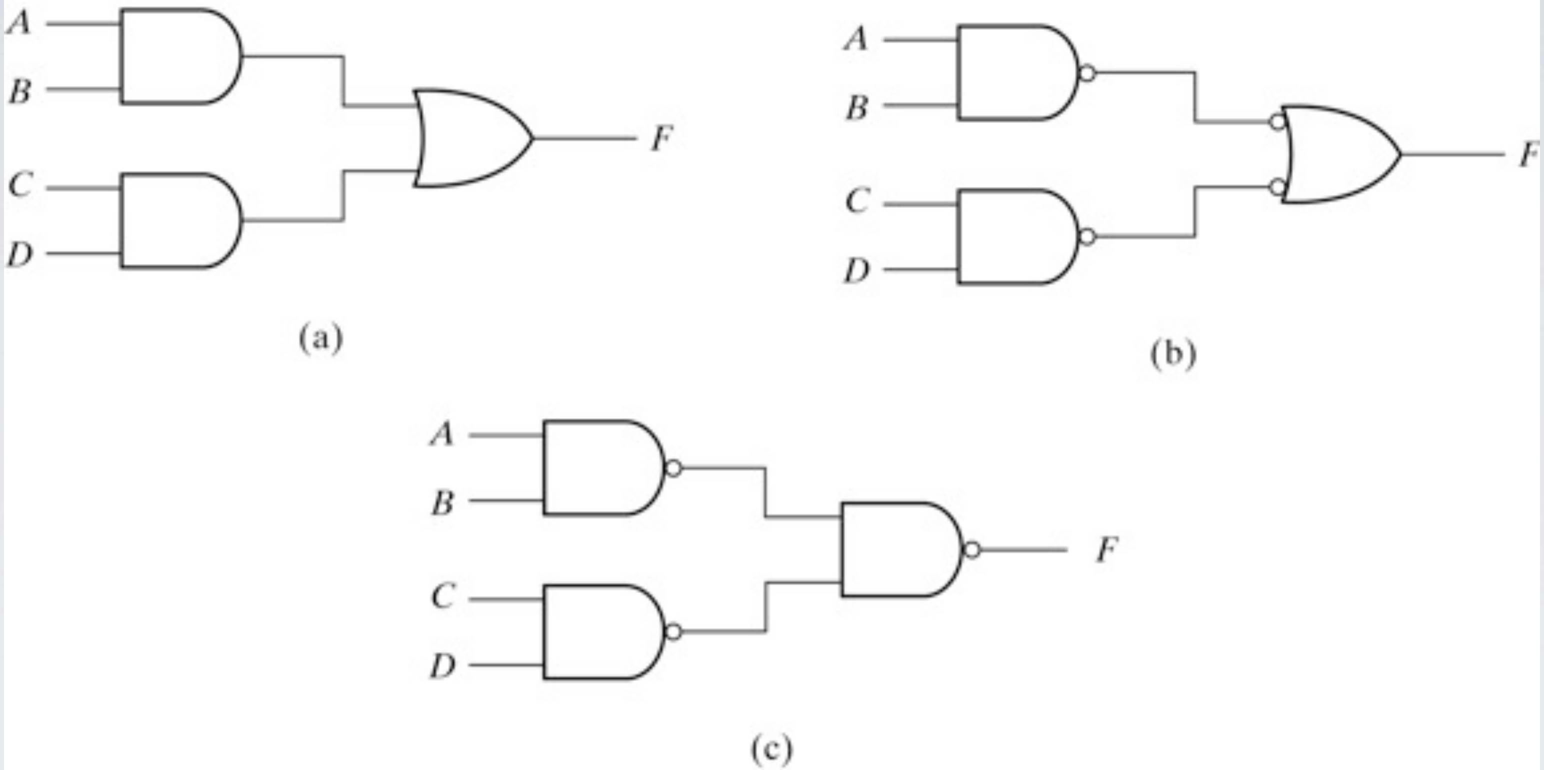


Fig. 3-20 Three Ways to Implement $F = AB + CD$

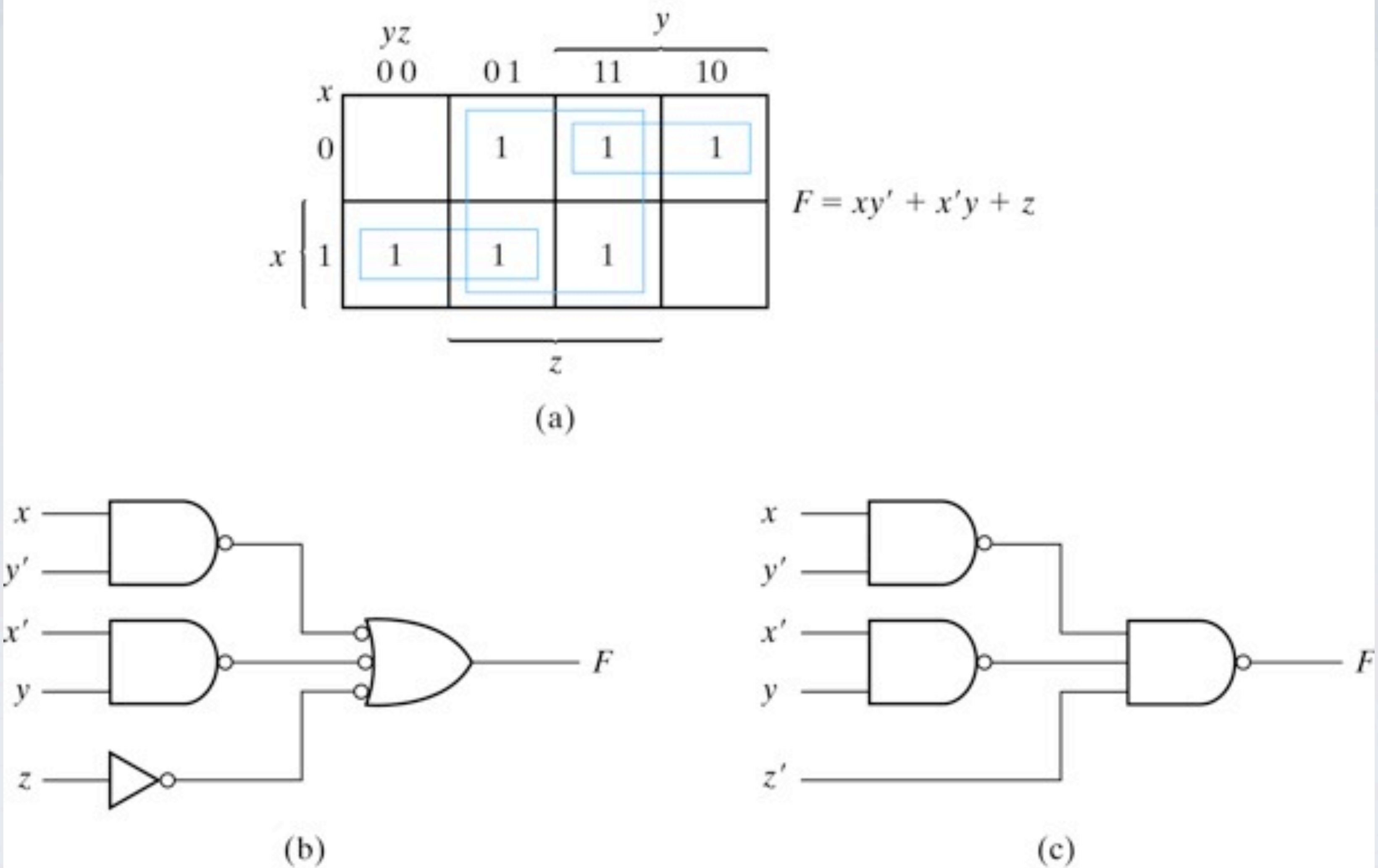
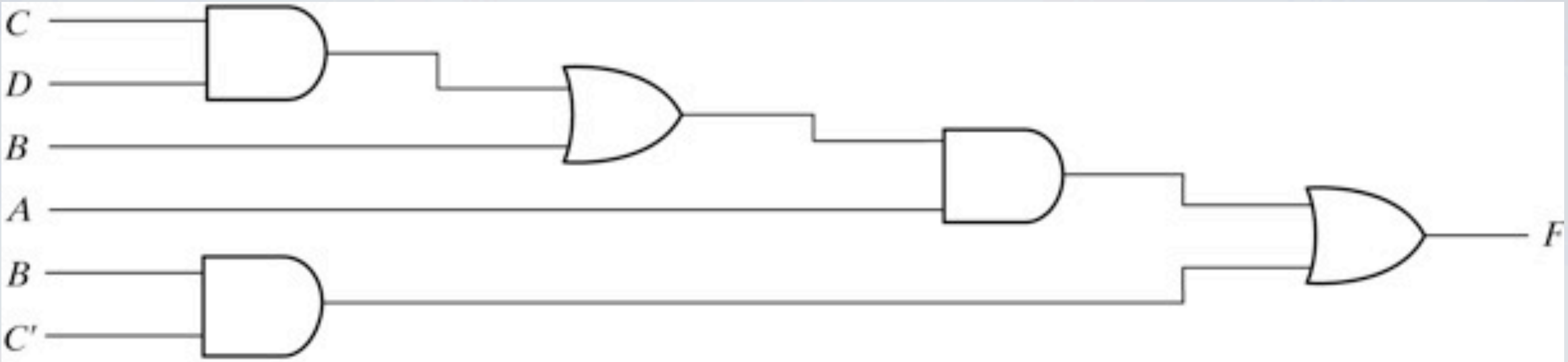
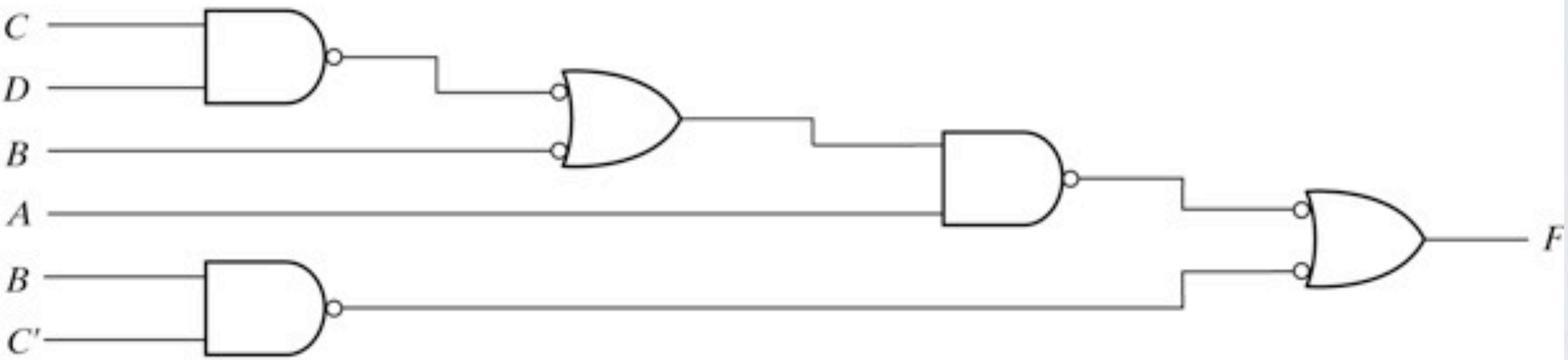


Fig. 3-21 Solution to Example 3-10

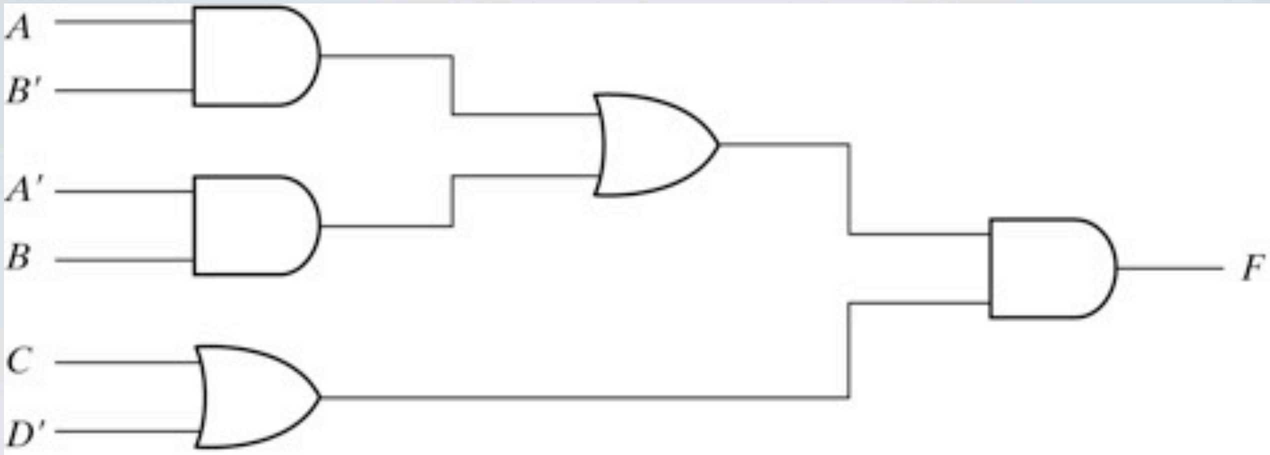


(a) AND-OR gates

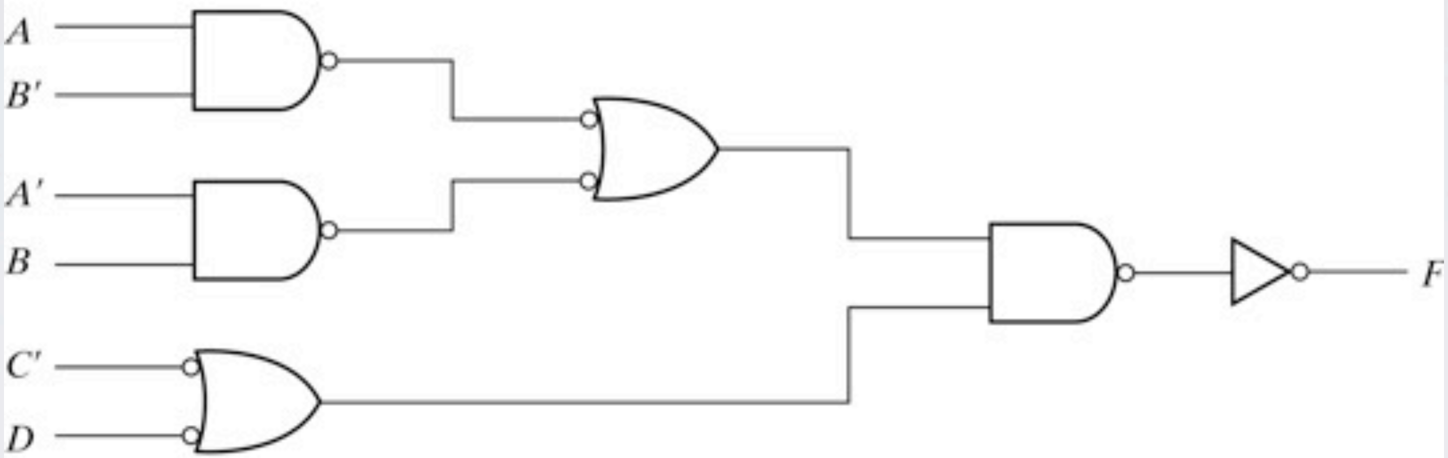


(a) NAND gates

Fig. 3-22 Implementing $F = A(CD + B) + BC$



(a) AND-OR gates



(b) NAND gates

Fig. 3-23 Implementing $F = (AB' + A'B)(C + D')$

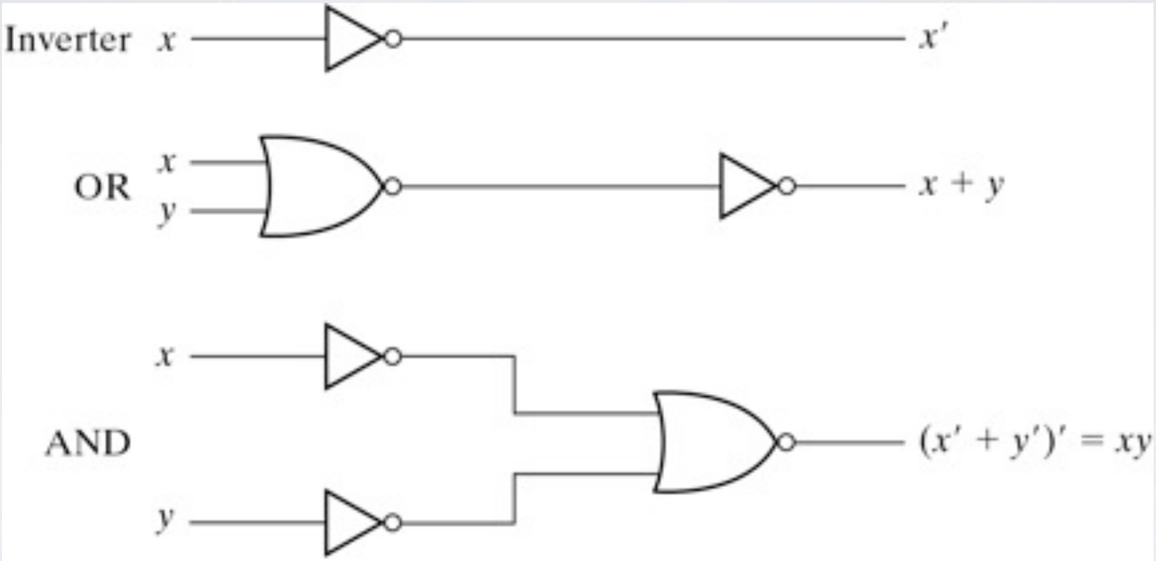
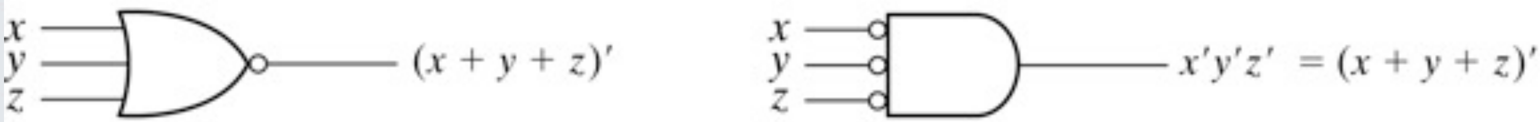


Fig. 3-24 Logic Operations with NOR Gates



(a) OR–invert

(a) Invert–AND

Fig. 3-25 Two Graphic Symbols for NOR Gate

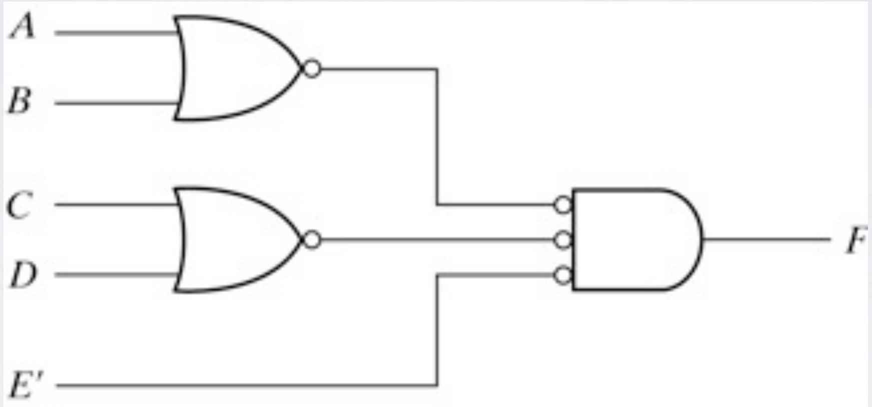


Fig. 3-26 Implementing $F = (A + B)(C + D)E$

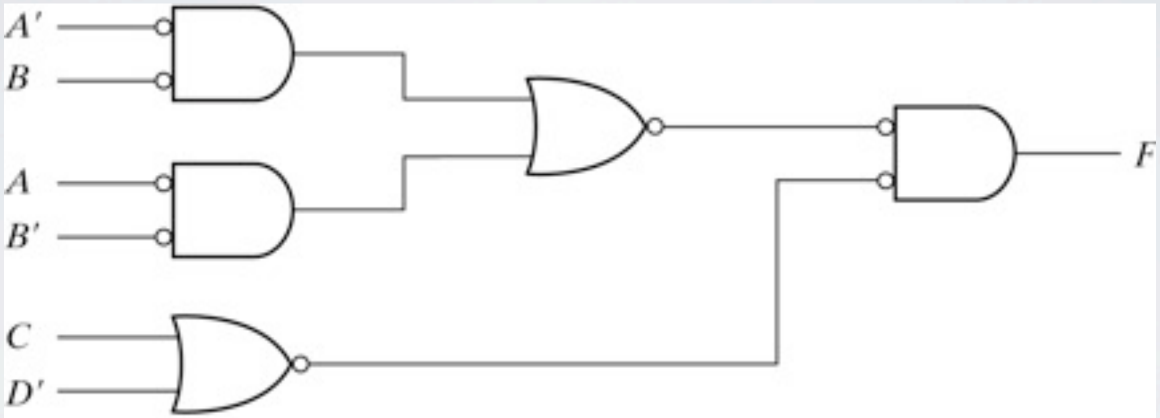
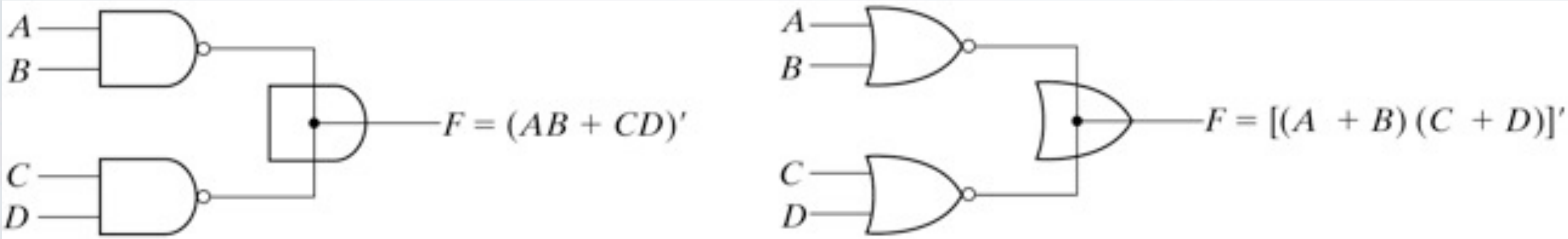


Fig. 3-27 Implementing $F = (AB' + A'B)(C + D')$ with NOR Gates



(a) Wired-AND in open-collector TTL NAND gates.

(AND-OR-INVERT)

(b) Wired-OR in ECL gates

(OR-AND-INVERT)

Fig. 3-28 Wired Logic

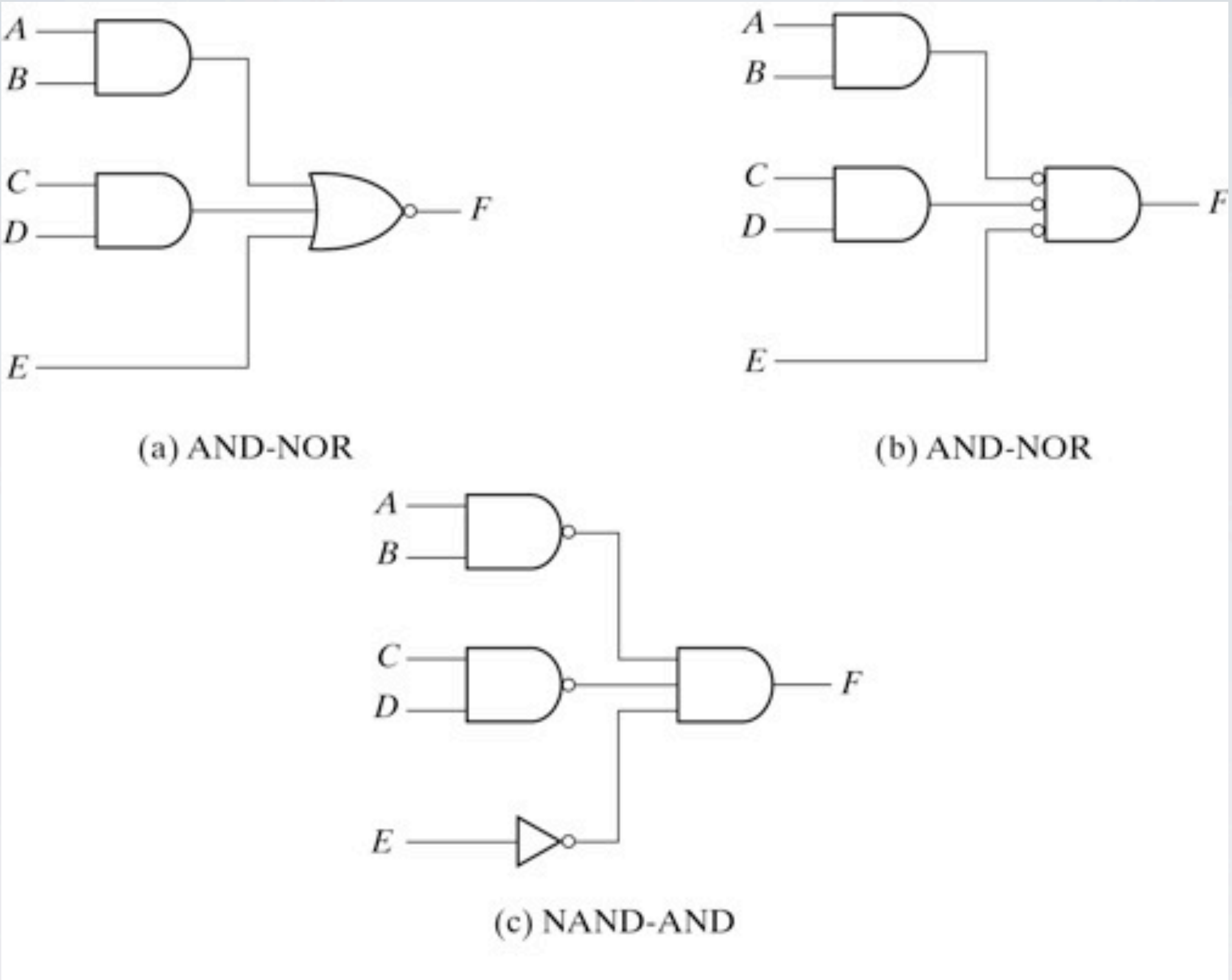


Fig. 3-29 AND-OR-INVERT Circuits; $F = (AB + CD + E)'$

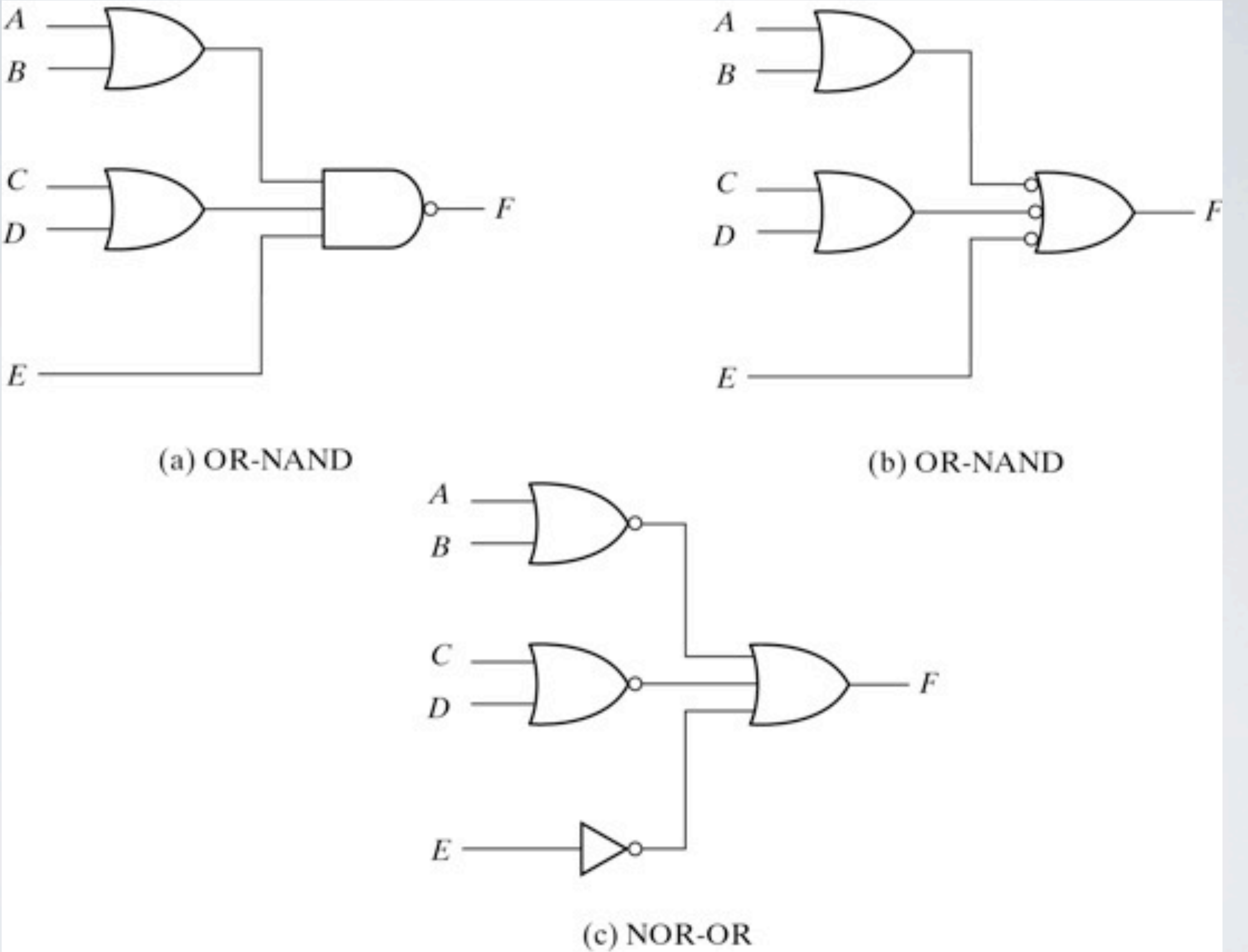
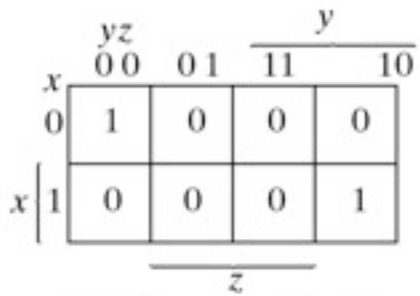
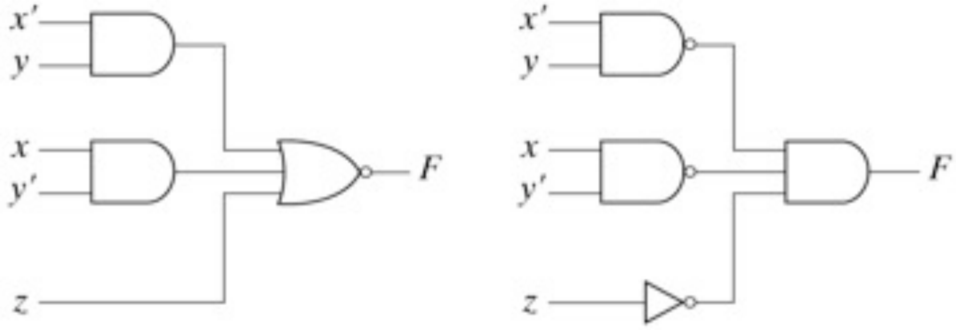


Fig. 3-30 OR-AND-INVERT Circuits; $F = [(A + B)(C + D)E]'$



$F = x'y'z' + xyz'$
$F' = x'y + xy' + z$

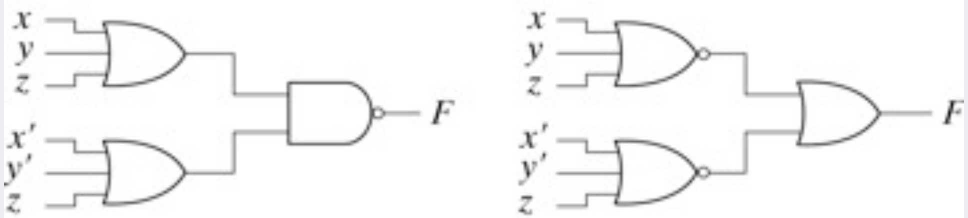
(a) Map simplification in sum of products.



AND-NOR

NAND-AND

(b) $F = (x'y + xy' + z)'$

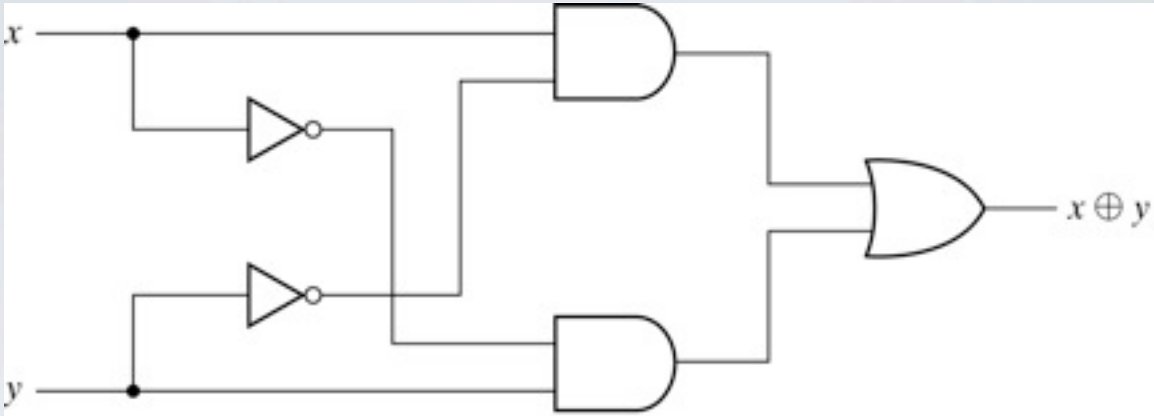


OR-NAND

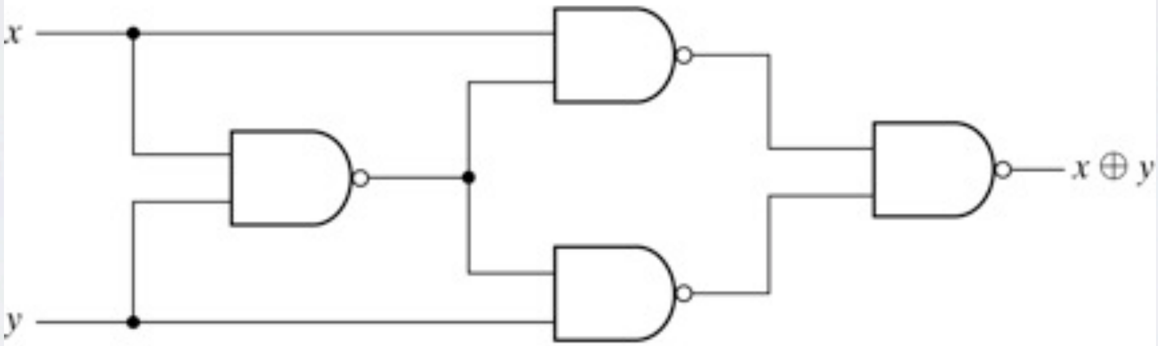
NOR-OR

(c) $F = [(x + y + z)(x' + y' + z)]'$

Fig. 3-31 Other Two-level Implementations

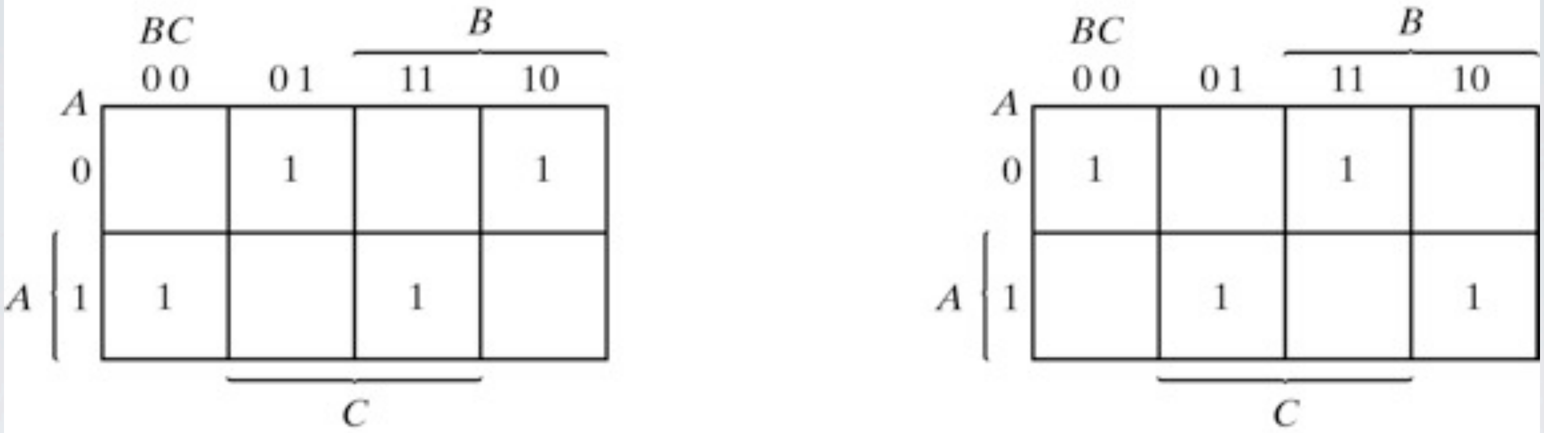


(a) With AND-OR-NOT gates



(b) With NAND gates

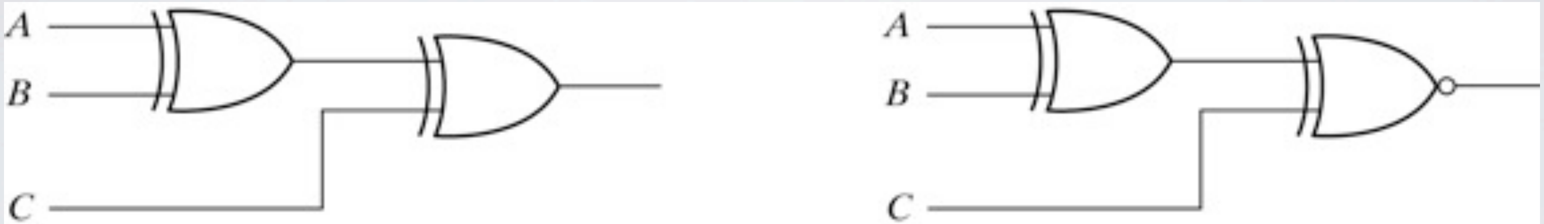
Fig. 3-32 Exclusive-OR Implementations



(a) Odd function
$F = A \oplus B \oplus C$

(a) Even function
$F = (A \oplus B \oplus C)'$

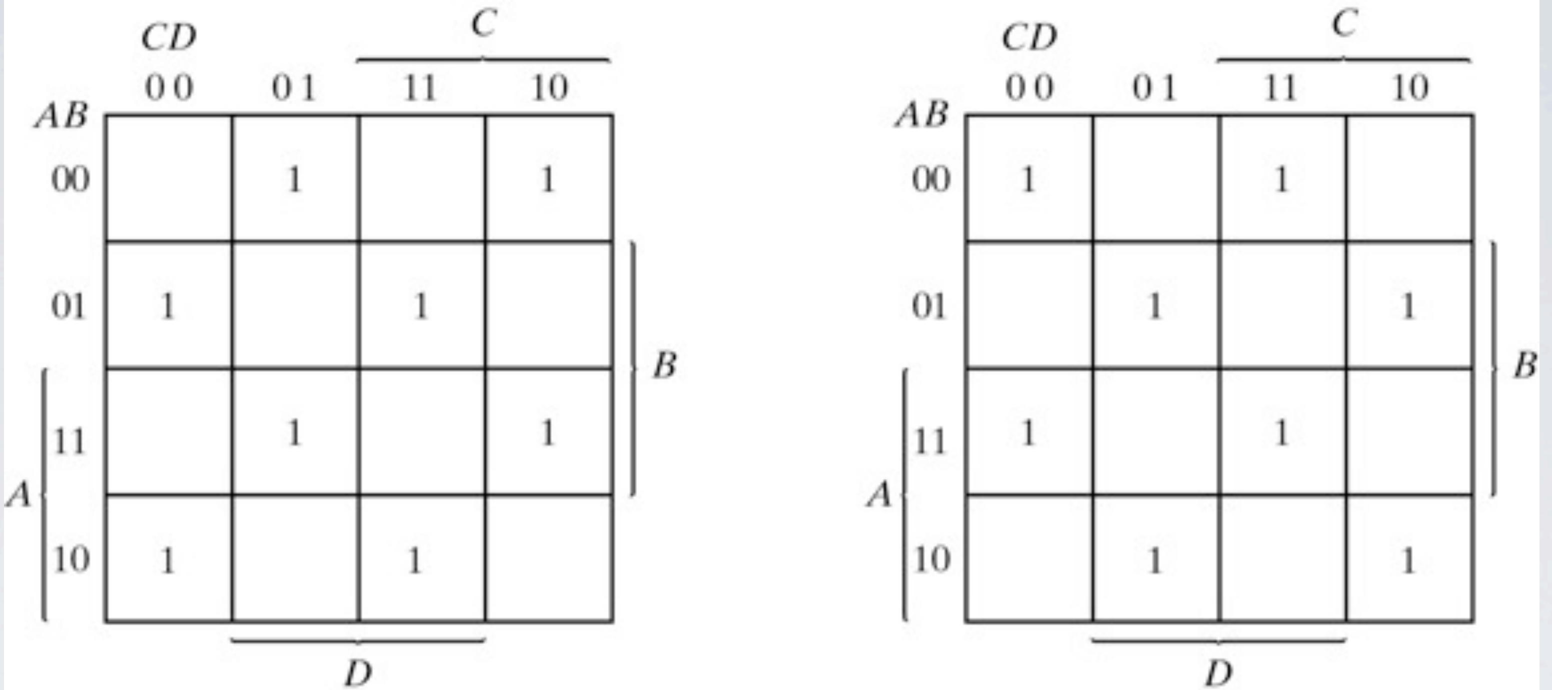
Fig. 3-33 Map for a Three-variable Exclusive-OR Function



(a) 3-input odd function

(b) 3-input even function

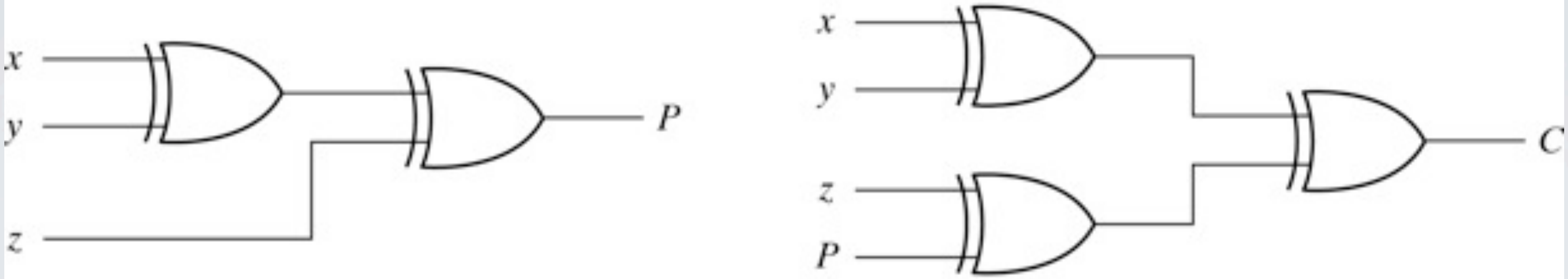
Fig. 3-34 Logic Diagram of Odd and Even Functions



(a) Odd function
$F = A \oplus B \oplus C \oplus D$

(b) Even function
$F = (A \oplus B \oplus C \oplus D)'$

Fig. 3-35 Map for a Four-variable Exclusive-OR Function



(a) 3-bit even parity generator

(a) 4-bit even parity checker

Fig. 3-36 Logic Diagram of a Parity Generator and Checker

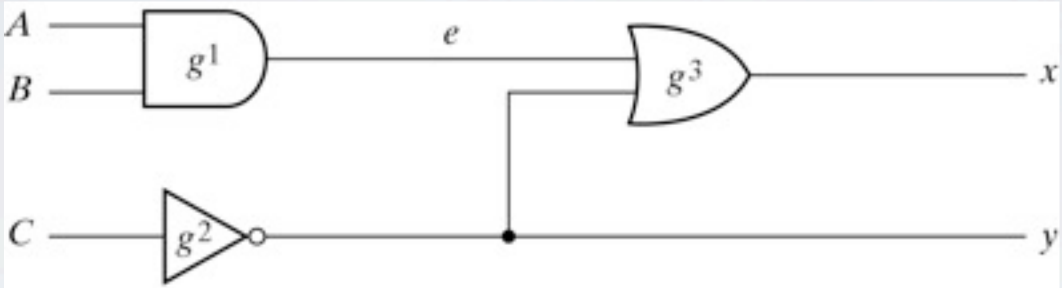


Fig. 3-37 Circuit to Demonstrate HDL

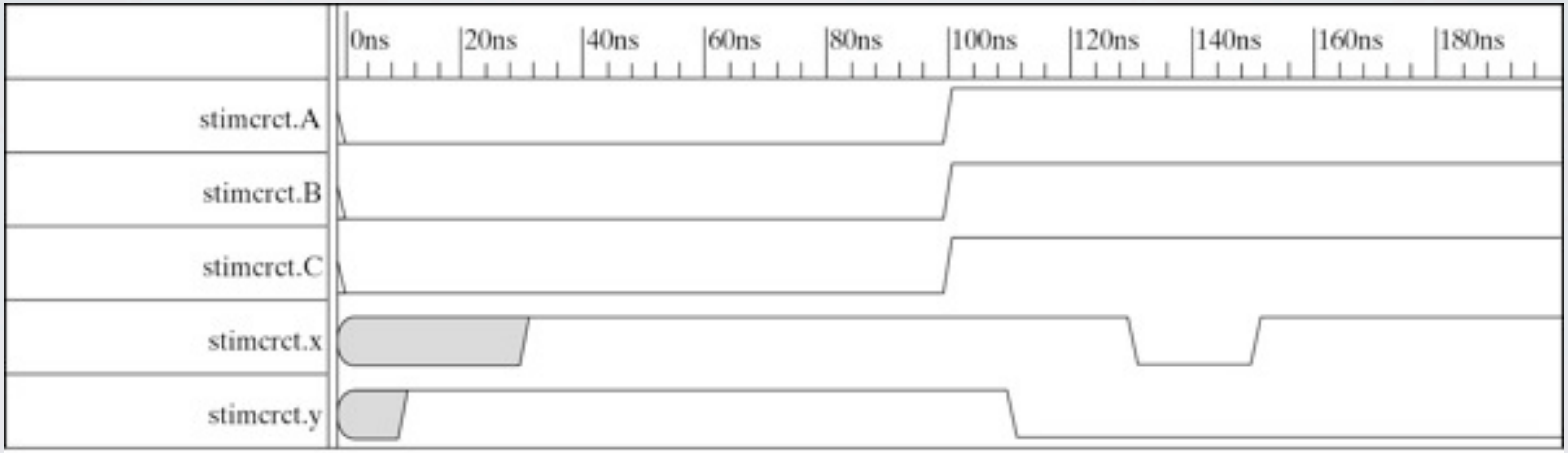


Fig. 3-38 Simulation Output of HDL Example 3-3