## ARITHMETIC CIRCUITS <br> INEL 4205 - Ch. 4 - Set I <br> Spring 2012



Fig. 4-1 Block Diagram of Combinational Circuit


Fig. 4-2 Logic Diagram for Analysis Example
You can use intermediate variables to construct the boolean Functions from a complex logic diagram.

Table 4-2
Truth Table for Code-Conversion Example

| Input BCD |  |  |  |  | Output Excess-3 Code |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | $B$ | $C$ | $D$ |  | $w$ | $x$ | $y$ |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |  |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |  |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |  |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |  |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |  |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |  |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |  |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |  |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |  |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |  |

## Table 4-2 BCD to Excess-3 Code




Fig. 4-3 Maps for BCD to Excess-3 Code Converter


Fig. 4-4 Logic Diagram for BCD to Excess-3 Code Converter

Table 4-3
Half Adder

| $x$ | $y$ | $C$ | $S$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |$S$




$$
\text { (a) } \begin{aligned}
S & =x y^{\prime}+x^{\prime} y \\
C & =x y
\end{aligned}
$$

(b) $S=x \oplus y$ $C=x y$

Fig. 4-5 Implementation of Half-Adder

Table 4-4
Full Adder

| $\boldsymbol{x}$ | $\boldsymbol{y}$ | $\boldsymbol{z}$ | C | $\boldsymbol{S}$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |


$S=x^{\prime} y^{\prime} z+x^{\prime} y z^{\prime}+x y^{\prime} z^{\prime}+x y z$


$$
\begin{aligned}
S & =x y+x z+y z \\
& =x y+x y^{\prime} z+x^{\prime} y z
\end{aligned}
$$

Fig. 4-6 Maps for Full Adder


Fig. 4-7 Implementation of Full Adder in Sum of Products


Fig. 4-8 Implementation of Full Adder with Two Half Adders and an OR Gate


Fig. 4-9 4-Bit Adder


Fig. 4-10 Full Adder with P and G Shown


Fig. 4-10 Full Adder with P and G Shown

$$
\begin{aligned}
P_{i} & =A_{i} \oplus B_{i} \\
G_{i} & =A_{i} B_{i}
\end{aligned}
$$

$$
\begin{aligned}
S_{i} & =P_{i} \oplus C_{i} \\
C_{i+1} & =G_{i}+P_{i} C_{i}
\end{aligned}
$$



Fig. 4-10 Full Adder with P and G Shown

$$
\begin{aligned}
P_{i} & =A_{i} \oplus B_{i} \\
G_{i} & =A_{i} B_{i}
\end{aligned}
$$

$$
C_{0}=\text { input carry }
$$

$$
C_{1}=G_{0}+P_{0} C_{0}
$$

$$
C_{2}=G_{1}+P_{1} C_{1}=G_{1}+P_{1}\left(G_{0}+P_{0} C_{0}\right)=G_{1}+P_{1} G_{0}+P_{1} P_{0} C_{0}
$$

$$
C_{3}=G_{2}+P_{2} C_{2}=G_{2}+P_{2} G_{1}+P_{2} P_{1} G_{0}+P_{2} P_{1} P_{0} C_{0}
$$



Fig. 4-11 Logic Diagram of Carry Lookahead Generator


Fig. 4-12 4-Bit Adder with Carry Lookahead


Fig. 4-13 4-Bit Adder Subtractor
$M=I$ complements the $B$-bits and adds $I$, thus forming the 2's complement of B
carries: $0 \quad 1$

| +70 | 0 | 1000110 | -70 | 1 | 0111010 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| +80 | 0 | 1010000 | -80 | 1 | 0110000 |
| +150 | 1 | 0010110 | -150 | 1101010 |  |

## overflow occurs if:

For unsigned numbers: carry out of msb
for Signed numbers: if carry into sign bit is different than carry out of sign bit


Fig. 4-14 Block Diagram of a BCD Adder


Fig. 4-15 2-Bit by 2-Bit Binary Multiplier


Fig. 4-16 4-Bit by 3-Bit Binary Multiplier

## MAGNITUDE COMPARATOR

$$
\begin{gathered}
A=A_{3} A_{2} A_{1} A_{0} \\
B=B_{3} B_{2} B_{1} B_{0} \\
x_{i}=A_{i} B_{i}+A_{i}^{\prime} B_{i}^{\prime} \quad \text { for } i=0,1,2,3 \\
(A=B)=x_{3} x_{2} x_{1} x_{0}
\end{gathered}
$$

$$
(A>B)=A_{3} B_{3}^{\prime}+x_{3} A_{2} B_{2}^{\prime}+x_{3} x_{2} A_{1} B_{1}^{\prime}+x_{3} x_{2} x_{1} A_{0} B_{0}^{\prime}
$$

$$
(A<B)=A_{3}^{\prime} B_{3}+x_{3} A_{2}^{\prime} B_{2}+x_{3} x_{2} A_{1}^{\prime} B_{1}+x_{3} x_{2} x_{1} A_{0}^{\prime} B_{0}
$$



Fig. 4-17 4-Bit Magnitude Comparator

