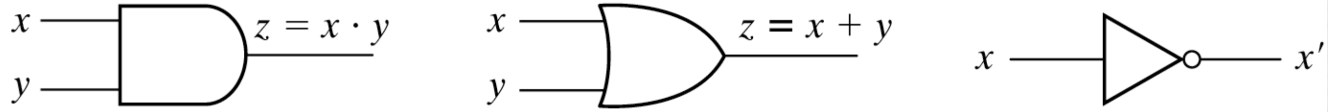
BOOLEAN ALGEBRA

INEL4205 - SPRING 2008

OPERACIONES BÁSICAS

- AND - SALIDA ES "1" SI TODAS LAS ENTRADAS SON "1"
- OR - SALIDA ES "1" SI ALGUNA ENTRADA ES "1"
- NOT (COMPLEMENTO) - SALIDA ES "1" SI LA ENTRADA ES "0", "0" SI LA ENTRADA ES "1"



(a) Two-input AND gate (b) Two-input OR gate (c) NOT gate or inverter

Fig. 1-4 Symbols for digital logic circuits

TABLAS DE VERDAD

- LA TALA DE VERDAD PRESENTA LA SALIDA PARA TODAS LAS COMBINACIONES POSIBLES DE LAS ENTRADAS
- SI HAY "N" ENTRADAS, EL TOTAL DE COMBINACIONES UNICAS ES 2^N

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

AND

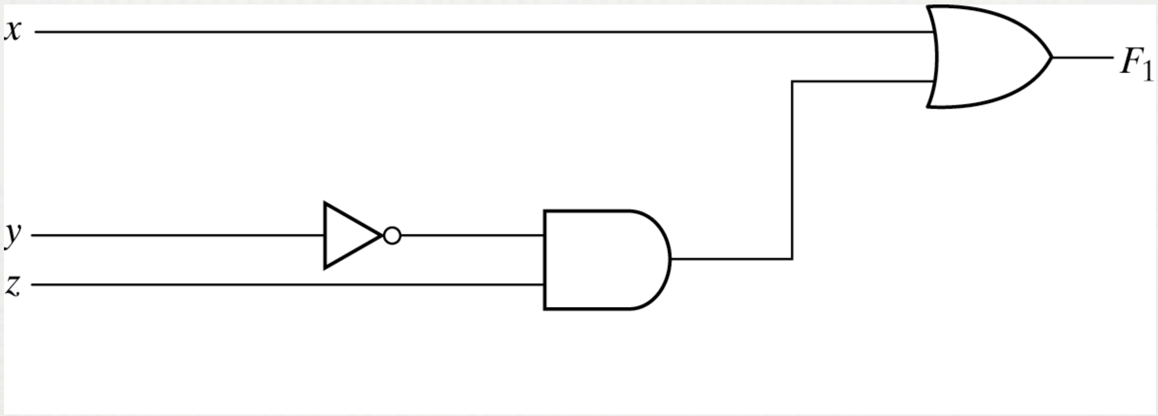
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

OR

A	Y
0	1
1	0

NOT

EXPRESIÓN BOOLEANA



ESCRIBA LA TABLA DE VERDAD Y
LA EXPRESIÓN BOOLEANA QUE REPRESENTA
LA SALIDA "F_1" EN TERMINOS DE LAS ENTRADAS
"x", "y" Y "z"

Table 2-1
Postulates and Theorems of Boolean Algebra

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

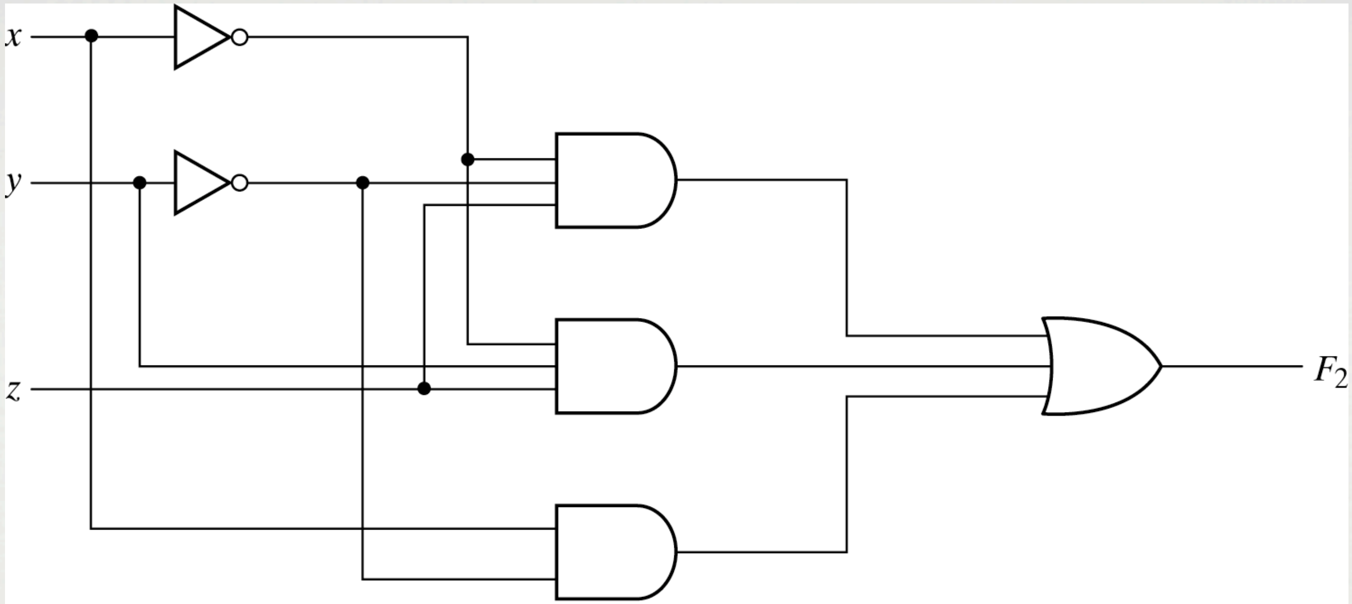
POSTULADO 4B FORMA 2 $X + X'Y = X + Y$

TEOREMA DEL CONSENSO $XY+X'Z+YZ = XY+X'Z$

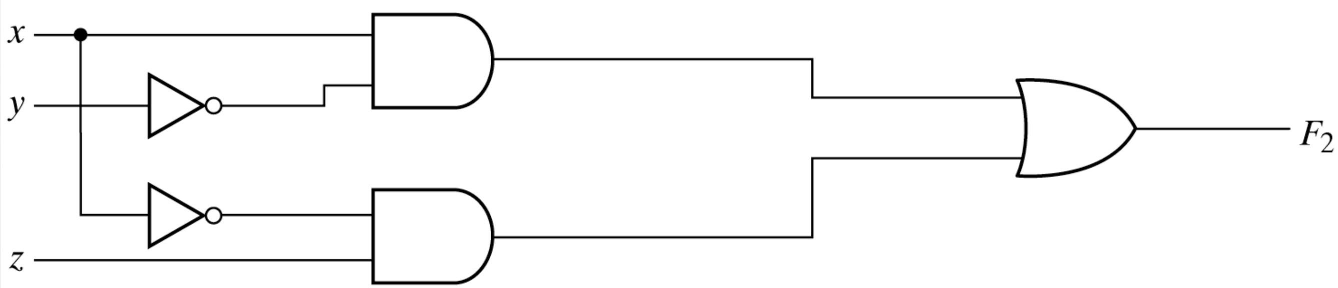
SIMPLIFICACIÓN

- SIMPLIFIQUE LA SIGUIENTE EXPRESION BOOLEANA

$$F_2 = X'Y'Z + X'YZ + XY'$$



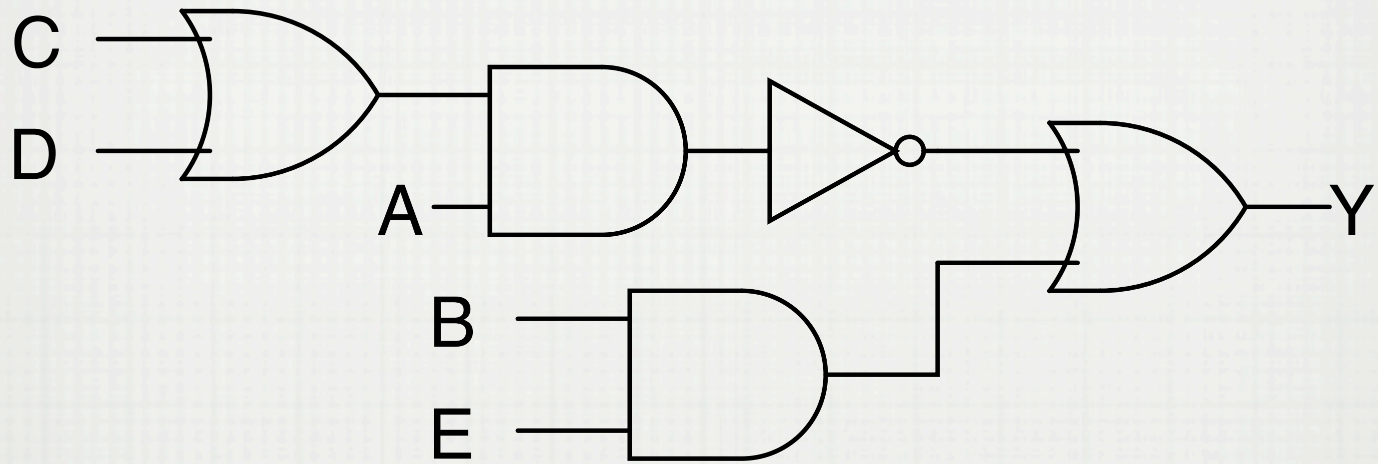
(a) $F_2 = x'y'z + x'yz + xy'$



(b) $F_2 = xy' + x'z$

Fig. 2-2 Implementation of Boolean function F_2 with gates

EJEMPLO



- ESCRIBA LA EXPRESIÓN BOOLEANA Y LA TABLA DE VERDAD PARA LA "Y" DEL DIAGRAMA DE ARRIBA

- DIBUJE EL DIAGRAMA PARA UN CIRCUITO QUE PRODUZCA LA SIGUIENTE SALIDA

$$ACF+DC'F'$$

EJEMPLOS

1. DETERMINE EL COMPLEMENTO DE LA SIGUIENTE EXPRESION BOOLEANA:

$$(A'+B)C'$$

2. DEMUESTRE QUE:

- $[(AB'+C)D'+E]' = [(A'+B)C'+D]E'$
- $X+XY=X$

EJEMPLOS

SIMPLIFIQUE:

- $X(X'+Y)$
- $X+X'Y$
- $(X+Y)(X+Y')$
- $XY+X'Z+YZ$
- $(X+Y)(X'+Z)(Y+Z)$

PRACTICA: PROBS. 2.2, 2.3 Y 2.4

TEOREMAS ADICIONALES

APLICANDO EL POSTULADO 4B

$$X + X'Y = X + Y$$

LLAMEMOSLO POSTULADO 4B FORMA 2

TEOREMA DEL CONSENSO

$$XY+X'Z+YZ = XY+X'Z+YZ(X+X')$$

$$XY+X'Z+YZ = XY(1+Z)+X'Z(1+Y)$$

$$\boxed{XY+X'Z+YZ = XY+X'Z}$$

SIMPLIFIQUE

$$ABCD' + A'B'CD + CD'$$

$$ABCD' + A'B'CD + CD'$$

$$(AB+1)CD' + A'B'CD$$

$$C(D' + A'B'D)$$

POR EL POSTULADO 4B FORMA 2

$$C(D' + A'B')$$

SIMPLIFIQUE

$AB'C' + CD' + BC'D'$

$$AB'C + CD' + BC'D'$$

$$AB'C + (C + BC')D'$$

POSTULADO 4B FORMA 2

$$AB'C + CD' + BD'$$

SIMPLIFIQUE

$$(A + B')(A' + B' + D)(B' + C + D')$$

$$(A + B')(A' + B' + D)(B' + C + D')$$

DUALIDAD

$$AB' + A'B'D + B'CD'$$

$$(A + A'D + CD')B'$$

$$(A + D + CD')B'$$

$$(A + D + C)B'$$

DUALIDAD

$$ACD + B'$$

$$XY + X'YZ' + YZ$$

$$XY + X'YZ' + YZ$$
$$XY + X'YZ' + YZ(X + X')$$
$$XY(1 + Z) + X'Y(Z' + Z)$$
$$XY + X'Y$$
$$Y$$

$$(XY' + Z)(X + Y')Z$$

$$(XY' + Z)(X + Y')Z$$
$$(XY' + ZX + ZY')Z$$
$$XY'Z + XZ + Y'Z$$
$$(X + 1)Y'Z + XZ$$
$$(X + Y')Z$$

$$XY' + Z + (X' + Y)Z'$$

$$XY' + Z + (X' + Y)Z'$$

POSTULADO 4B FORMA 2

$$XY' + X' + Y + Z$$

POSTULADO 4B FORMA 2

$$X' + Y' + Y + Z$$

$$\boxed{1}$$

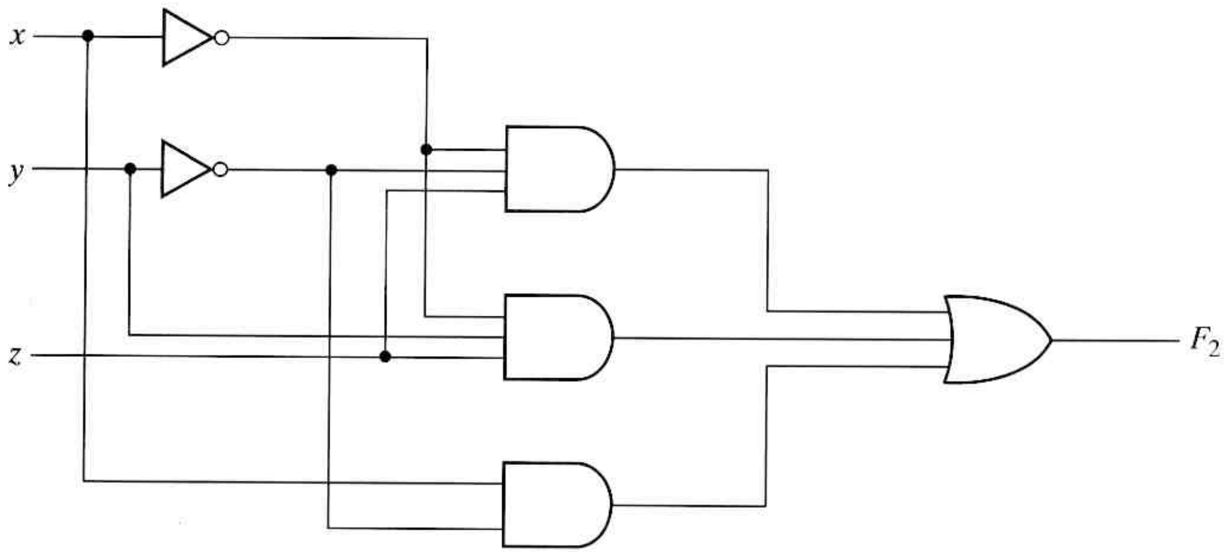
OTRA FORMA:

$$(XY' + Z)' = (X' + Y)Z'$$

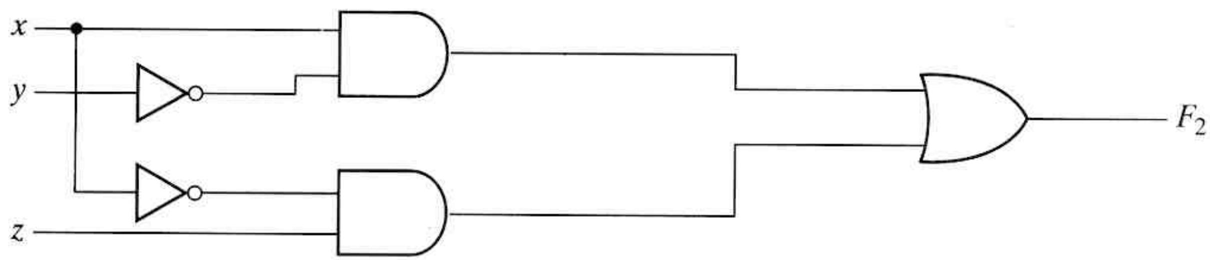
ASI QUE LA EXPRESIÓN TIENE LA FORMA

$$A + A'$$

Y POR LO TANTO SIEMPRE ES 1



(a) $F_2 = x'y'z + x'yz + xy'$ ← 3 TERMS; 8 LITERALS



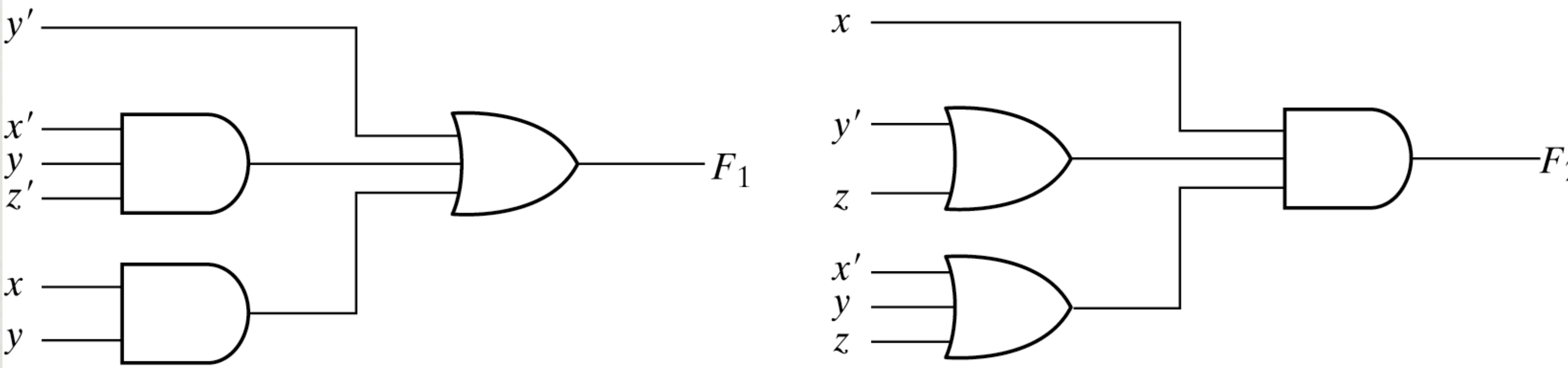
(b) $F_2 = xy' + x'z$ ← 2 TERMS; 4 LITERALS

FIGURE 2-2

Implementation of Boolean function F_2 with gates

Table 2-3
Minterms and Maxterms for Three Binary Variables

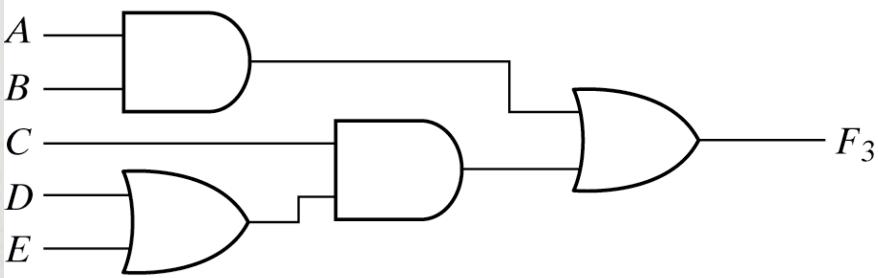
			Minterms		Maxterms	
x	**y**	**z**	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7



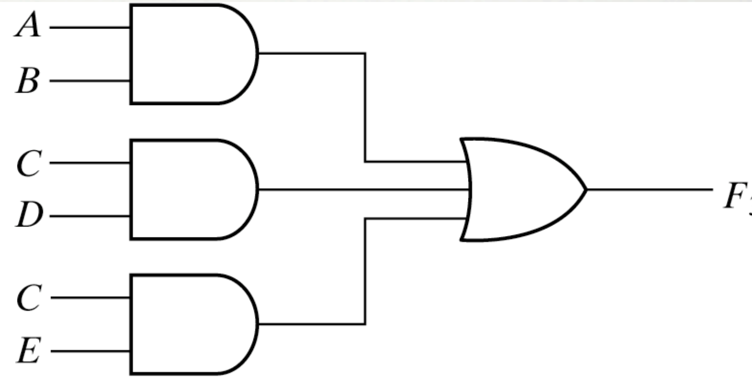
(a) Sum of Products

(b) Product of Sums

Fig. 2-3 Two-level implementation



(a) $AB + C(D + E)$



(b) $AB + CD + CE$

Fig. 2-4 Three- and Two-Level implementation

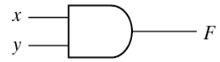
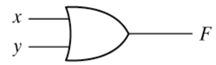
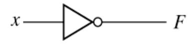
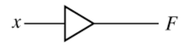
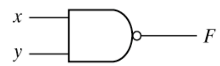
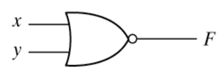
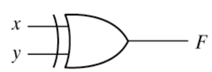
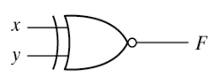
Name	Graphic symbol	Algebraic function	Truth table
AND		$F = xy$	x y | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = x + y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
Inverter		$F = x'$	x | F 0 | 1 1 | 0
Buffer		$F = x$	x | F 0 | 0 1 | 1
NAND		$F = (xy)'$	x y | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = (x + y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1

Fig. 2-5 Digital logic gates

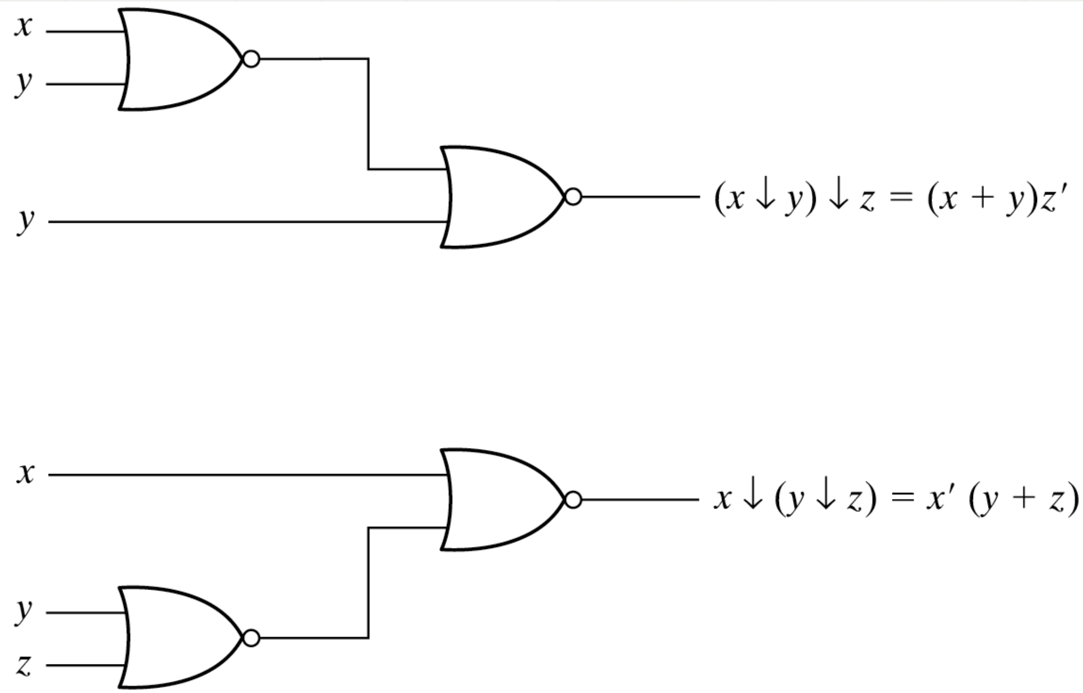
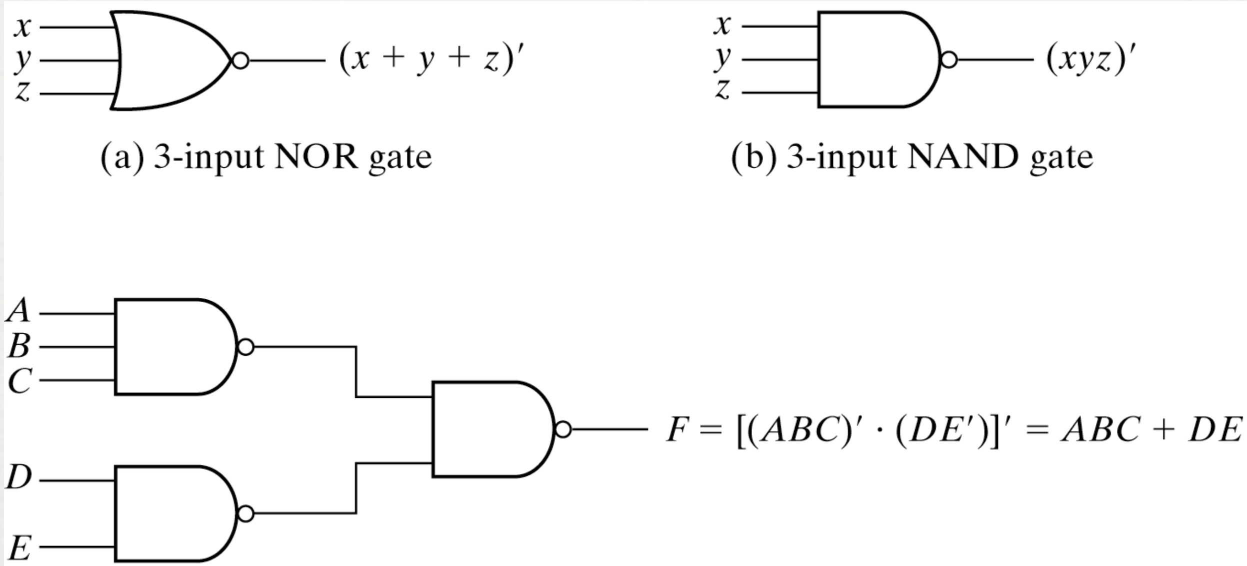


Fig. 2-6 Demonstrating the nonassociativity of the NOR operator; $(x \downarrow y) \downarrow z \neq x \downarrow (y \downarrow z)$

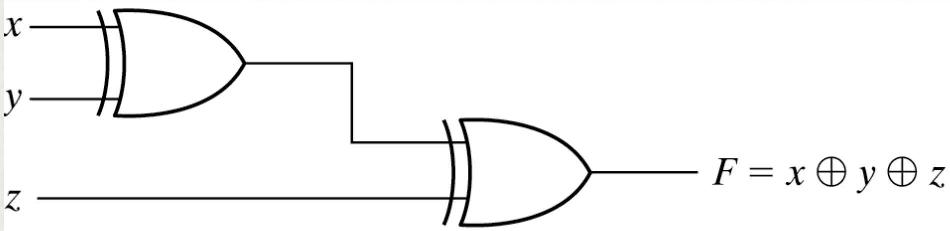


(a) 3-input NOR gate

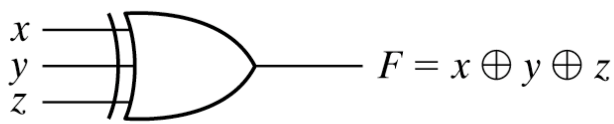
(b) 3-input NAND gate

(c) Cascaded NAND gates

Fig. 2-7 Multiple-input and cascated NOR and NAND gates



(a) Using 2-input gates



(b) 3-input gate

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(c) Truth table

Fig. 2-8 3-input exclusive-OR gate

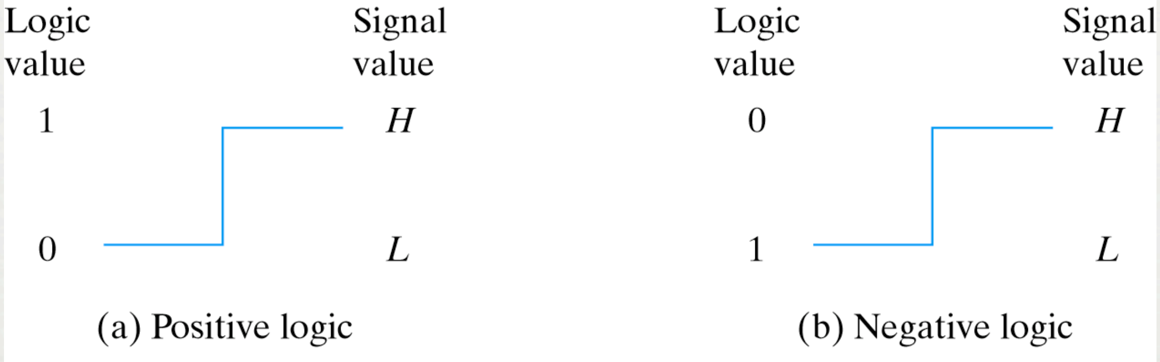
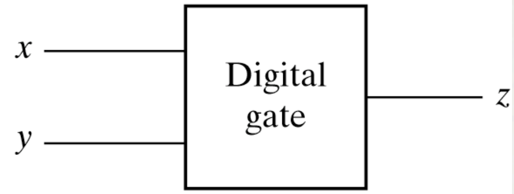


Fig. 2-9 signal assignment and logic polarity

x	y	F
L	L	L
L	H	L
H	L	L
H	H	H

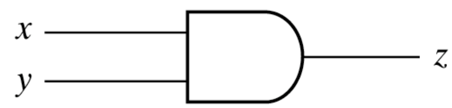
(a) Truth table with H and L



(b) Gate block diagram

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

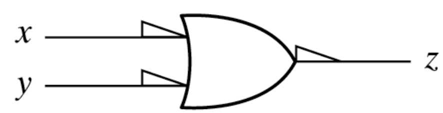
(c) Truth table for positive logic



(d) Positive logic AND gate

x	y	z
1	1	1
1	0	1
0	1	1
0	0	0

(e) Truth table for negative logic



(f) Negative logic OR gate

Fig. 2-10 Demonstration of positive and negative logic