# Programmable Logic and Memories 

INEL 4205 - Logic Circuits - Spring 2008

(a) Conventional symbol

(b) Array logic symbol

Fig. 7-1 Conventional and Array Logic Diagrams for OR Gate


Fig. 7-2 Block Diagram of a Memory Unit

Memory address

| Binary | decimal | Memory contest |
| :---: | :---: | :---: |
| 0000000000 | 0 | 1011010101011101 |
| 0000000001 | 1 | 1010101110001001 |
| 0000000010 | 2 | 0000110101000110 |
|  | $:$ | : |
| 1111111101 | 1021 | 1001110100010100 |
| 1111111110 | 1022 | 0000110100011110 |
| 1111111111 | 1023 | 1101111000100101 |

Fig. 7-3 Content of a $1024 \times 16$ Memory

## Steps to Write into RAM

- Apply address to address lines
- Apply data to data input lines
- Activate write input \& enable chip

For reads: do I and 3 using read input

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Fig. 7-4 Memory Cycle Timing Waveforms

(a) Logic diagram


Read/Write
(b) Block diagram

Fig. 7-5 Memory Cell


Fig. 7-6 Diagram of a $4 \times 4$ RAM

## Row/Column decoding

- IK-word memory requires 10 address bits and a $10 \times 1024$ decoder
- The decoding can also be done with two $5 \times 32$ decoder, one for the row and one for the column. The cell connected to the rowcolumn intersection is selected.


Fig. 7-7 Two-Dimensional Decoding Structure for a 1K-Word Memory


Fig. 7-8 Address Multiplexing for a 64 K DRAM


Fig. 7-9 ROM Block Diagram


Fig. 7-10 Internal Logic of a $32 \times 8$ ROM
Used as programable logic, a PROM stores the truth table for N functions of M inputs. $\mathrm{N}=$ number of bits in each cell.
$M=$ number of address bits; there are $2^{M}$ memory locations in the PROM.
" $x$ " indicates a connection, and a " $l$ " in the truth table


Fig. 7-11 Programming the ROM According to Table 7-3
Address 00000: cell contents is 10110110

Design a combinatorial circuit using a ROM.The circuit accepts a 3-bit input number and outputs a binary number equal to the square of the input.

Table 7-4
Truth Table for Circuit of Example 7-1

| Inputs |  |  | Outputs |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{A}_{\mathbf{2}}$ | $\boldsymbol{A}_{\mathbf{1}}$ | $\boldsymbol{A}_{\mathbf{0}}$ |  | $\boldsymbol{B}_{\mathbf{5}}$ | $\boldsymbol{B}_{\mathbf{4}}$ | $\boldsymbol{B}_{\mathbf{3}}$ | $\boldsymbol{B}_{\mathbf{2}}$ | $\boldsymbol{B}_{\mathbf{1}}$ | $\boldsymbol{B}_{\mathbf{0}}$ |


(a) Block diagram

| $A_{2}$ | $A_{1}$ | $A_{0}$ | $B_{5}$ | $B_{4}$ | $B_{3}$ | $B_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 |

(b) ROM truth table

Fig. 7-12 ROM Implementation of Example 7-1

(a) Programmable read-only memory (PROM)

(b) Programmable array logic (PAL)

(c) Programmable logic array (PLA)

Fig. 7-13 Basic Configuration of Three PLDs


Fig. 7-14 PLA with 3 Inputs, 4 Product Terms, and 2 Outputs

PLA has limited \# of ANDs: designer an use F or F' to minimize the \# of distinct product terms, the complement $F$ ' if necessary.

Example: Implement the following boolean functions in a PLA:

$$
\begin{aligned}
& F_{1}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\sum(0,1,2,4) \\
& \mathrm{F}_{2}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\sum(0,5,6,7)
\end{aligned}
$$



$$
\begin{aligned}
& F_{1}=A^{\prime} B^{\prime}+A^{\prime} C^{\prime}+B^{\prime} C^{\prime} \\
& F_{1}=(A B+A C+B C)^{\prime}
\end{aligned}
$$



PLA programming table
Outputs

|  | Product <br> term | Inputs |  | (C) | (T) |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1 | 1 | - | 1 | 1 |
| $A B$ | 2 | 1 | - | 1 | 1 | 1 |
| $A C$ | 3 | - | 1 | 1 | 1 | - |
| $B C$ | 4 | 0 | 0 | 0 | - | 1 |

Fig. 7-15 Solution to Example 7-2

7-21 Derive the PLA programming table for the combinational circuit that squares a 3-bit number. Minimize the number of product terms. (See Fig. 7-12 for the equivalent ROM implementation.)

(a) Block diagram

| $A_{2}$ | $A_{1}$ | $A_{0}$ | $B_{5}$ | $B_{4}$ | $B_{3}$ | $B_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 |

(b) ROM truth table

Fig. 7-12 ROM Implementation of Example 7-1

3. ( 25 pts ) Determine the PLA programming table needed to implement the following two boolean functions. Minimize the number of product terms. Show all your work, including the Karnaugh maps used in the minimization.

$$
\begin{aligned}
& \mathrm{F}_{1}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\sum(1,3,4,5,7,13,15) \\
& \mathrm{F}_{2}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\sum(0,2,3,6,7,8,10,11,12,14)
\end{aligned}
$$

Write your result in the following table.


Note: not all rows need to be used

AND gates inputs

PAL


Fig. 7-16 PAL with Four Inputs, Four Outputs, and Three-Wide AND-OR Structure PAL: Only inputs to AND gates can be programmed but one term $\left(\mathrm{F}_{\mathrm{I}}\right)$ can be re-used in other functions

$$
\begin{aligned}
w(A, B, C, D) & =\sum(2,12,13) \\
x(A, B, C, D) & =\sum(7,8,9,10,11,12,13,14,15) \\
y(A, B, C, D) & =\sum(0,2,3,4,5,6,7,8,10,11,15) \\
z(A, B, C, D) & =\sum(1,2,8,12,13)
\end{aligned}
$$

Manipulate expressions so that a common term is identified. Assign common term to $\mathrm{F}_{1}$.

$$
\begin{aligned}
w & =A B C^{\prime}+A^{\prime} B^{\prime} C D^{\prime} \\
x & =A+B C D \\
y & =A^{\prime} B+C D+B^{\prime} D^{\prime} \\
z & =A B C^{\prime}+A^{\prime} B^{\prime} C D^{\prime}+A C^{\prime} D^{\prime}+A^{\prime} B^{\prime} C^{\prime} D \\
& =w+A C^{\prime} D^{\prime}+A^{\prime} B^{\prime} C^{\prime} D
\end{aligned}
$$

Table 7-6
PAL Programming Table

|  | AND Inputs |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Product Term | $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\boldsymbol{C}$ | $\boldsymbol{D}$ | $\boldsymbol{W}$ | Outputs |
| 1 | 1 | 1 | 0 | - | - | $w=A B C^{\prime}$ |
| 2 | 0 | 0 | 1 | 0 | - | $+A^{\prime} B^{\prime} C D^{\prime}$ |
| 3 | - | - | - | - | - |  |
| 4 | 1 | - | - | - | - | $x=A$ |
| 5 | - | 1 | 1 | 1 | - | $+B C D$ |
| 6 | - | - | - | - | - |  |
| 7 | 0 | 1 | - | - | - | $y=A^{\prime} B$ |
| 8 | - | - | 1 | 1 | - | $+C D$ |
| 9 | - | 0 | - | 0 | - | $+B^{\prime} D^{\prime}$ |
| 10 | - | - | - | - | 1 | $z=w$ |
| 11 | 1 | - | 0 | 0 | - |  |
| 12 | 0 | 0 | 0 | 1 | - | $+A C^{\prime} D^{\prime}$ |
|  |  |  |  |  | $+A^{\prime} B^{\prime} C^{\prime} D$ |  |

AND gates inputs


Fig. 7-17 Fuse Map for PAL as Specified in Table 7-6

- PAL practice: problem 7-24


Fig. 7-18 Sequential Programmable Logic Device


Fig. 7-19 Basic Macrocell Logic


Fig. 7-20 General CPLD Configuration



Figure 1: Simplified Block Diagram of XC4000 Series CLB (RAM and Carry Logic functions not shown)

## Hamming code: Error Detection and Correction

$$
\begin{array}{lcccccccccccc}
\text { Bit position: } & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
& P_{1} & P_{2} & 1 & P_{4} & 1 & 0 & 0 & P_{8} & 0 & 1 & 0 & 0 \\
P_{1} & =\operatorname{XOR} \text { of bits }(3,5,7,9,11)=1 \oplus 1 \oplus 0 \oplus 0 \oplus 0=0 \\
P_{2} & =\operatorname{XOR} \text { of bits }(3,6,7,10,11)=1 \oplus 0 \oplus 0 \oplus 1 \oplus 0=0 \\
P_{4} & =\operatorname{XOR} \text { of bits }(5,6,7,12)=1 \oplus 0 \oplus 0 \oplus 0=1 \\
P_{8} & =\text { XOR of bits }(9,10,11,12)=0 \oplus 1 \oplus 0 \oplus 0=1 \\
\text { Bit position: } & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12
\end{array}
$$

- Compute correction bits $C_{1}=\operatorname{XOR}$ of bits $(1,3,5,7,9,11)$

$$
\begin{aligned}
& C_{2}=\text { XOR of bits }(2,3,6,7,10,11) \\
& C_{4}=\text { XOR of bits }(4,5,6,7,12) \\
& C_{8}=\text { XOR of bits }(8,9,10,11,12)
\end{aligned}
$$

For no error:
$\begin{array}{llll}C_{8} & C_{4} & C_{2} & C_{1}\end{array}$

With error in bit 1 :
$0 \quad 0$
$0 \quad 0 \quad 0$

With error in bit 5:
0
$0 \quad 0 \quad 1$
$\begin{array}{llll}0 & 1 & 0 & 1\end{array}$

Table 7-2
Range of Data Bits for $k$ Check Bits

| Number of <br> Check Bits, $\boldsymbol{k}$ | Range of <br> Data Bits, $\boldsymbol{n}$ |
| :---: | :---: |
| 3 | $2-4$ |
| 4 | $5-11$ |
| 5 | $12-26$ |
| 6 | $27-57$ |
| 7 | $58-120$ |

$$
2^{k}-1-k \geq n
$$

## Single-error correction, double-error

- To detect a double-error, add an aditional parity bit $P$
$=$ XOR (all other bits)
- I2-bit example: $\mathrm{P}_{13}=X O R(1 . . \mid 2)$
- If
- $C=0 \& P=0$ : no error
- $C \neq 0 \& P=I$ : single error at bit indicated by $C$
- $C \neq 0 \& P=0$ : double error detected
- $C=0 \& P=1$ : error in $P_{13}$

7-10 Given the 8-bit data word 01011011 , generate the 13-bit composite word for the Hamming code that corrects single errors and detects double errors.

7-11 Obtain the 15-bit Hamming code word for the 11-bit data word 11001001010.
7-12 A 12-bit Hamming code word containing 8 bits of data and 4 parity bits is read from memory. What was the original 8-bit data word that was written into memory if the 12 -bit word read out is as follows:
(a) 000011101010
(b) 101110000110
(c) 101111110100


Fig. P7-17

