INEL 6055 Homework No. 1

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For the following problems refer to the semiconductor resistor designed in the example on section 1.1.2 of the lecture notes. It is acceptable to substitute finite differences for partial derivatives or to calculate partial derivatives graphically.

1. Determine the surface donor concentration if the conductivity profile shown in figure shown on figure 2 correspond to an N-type device. (10 points)

2. Estimate the temperature coefficient of the $100k\Omega$ resistor. The temperature coefficient is defined as

$$\alpha = \frac{1}{R_0}\frac{\partial R}{\partial T}$$

   Assume room temperature (300K) and that the linear thermal expansion coefficient of Silicon is 3×10^{-6} parts per degree Kelvin. (20 points)

3. Find the voltage that, when applied to the resistor, will increase the room-temperature incremental resistance to twice its nominal value. (20 points)

① $\sigma_0 = 10\ (\Omega\text{-cm})^{-1} = q N_{D0} \mu_n$

Using $\mu_n = 1450\ \text{cm}^2/\text{V-s} \Rightarrow N_{D0} = \dfrac{10\ (\Omega\cdot\text{cm})^{-1}}{(1.6\times10^{-19}\,\text{C})(1450\ \frac{\text{cm}^2}{\text{V-s}})}$

$$\boxed{N_{D0} \cong 4.3 \times 10^{16}\ \text{cm}^{-3}}$$

② $R = \dfrac{L}{\sigma x_j W} \Rightarrow \alpha = \dfrac{1}{R_0}\dfrac{\partial R}{\partial T} = \dfrac{1}{R_0}\left[\dfrac{1}{\sigma_0}\dfrac{\partial}{\partial T}\left(\dfrac{L}{x_j W}\right) + \dfrac{L}{x_j W}\dfrac{\partial}{\partial T}\left(\dfrac{1}{\sigma}\right)\right]$

Let $R_0 = \dfrac{L}{\sigma_0 x_j W}$

$$\alpha = \underbrace{\frac{\cancel{\frac{1}{\sigma_0}}\frac{\partial}{\partial T}\left(\frac{L}{x_j W}\right)}{\cancel{\frac{1}{\sigma_0}}\frac{L}{x_j W}}}_{3\times10^{-6}\ K^{-1}} + \frac{\cancel{\frac{L}{x_j W}}\frac{\partial}{\partial T}\frac{1}{\sigma}}{\frac{\cancel{L}}{\sigma\, \cancel{x_j W}}}$$

$$\therefore \quad \alpha = 3\times10^{-6} + \frac{\frac{\partial}{\partial T}\frac{1}{\sigma}}{1/\sigma}$$

$$\sigma \simeq q\mu_n N_D$$

Since q is a constant and all donors are ionized at room temperature,

$$\sigma(T) = qN_D\left[\mu_n(T)\right]$$

$$\frac{\partial}{\partial T}\frac{1}{\sigma} = \frac{\partial}{\partial\sigma}\left(\frac{1}{\sigma}\right)\frac{\partial\sigma}{\partial T} = -\frac{1}{\sigma^2}\frac{\partial\sigma}{\partial T}$$

$$\alpha = 3\times10^{-6}\,K^{-1} - \cancel{\sigma}\left(\frac{1}{\sigma^{\cancel{2}}}\right)\frac{\partial\sigma}{\partial T}$$

$$= 3\times10^{-6}\,K^{-1} - \frac{1}{\cancel{qN_D}\mu_n}\,\cancel{qN_D}\frac{\partial\mu_n}{\partial T}$$

$$= 3\times10^{-6}\,K^{-1} - \frac{1}{\mu_N}\frac{\partial\mu_N}{\partial T}$$

Use

$$\mu_n = 88\,T_n^{-0.57} + \frac{1250\,T_n^{-2.33}}{1+\left(\frac{N}{1.26\times10^{17}\,T_n^{2.4}}\right)0.88\,T_n^{-0.146}}$$

N = average donor conc.

Since $\bar{\sigma} = 4.18\,(\Omega\text{-cm})^{-1}$ (from example; see page 3 Notes)

$$\bar{\sigma} = q\mu_n\bar{N}_D \Rightarrow \bar{N}_D = \frac{4.18\,(\Omega\text{-cm})^{-1}}{(1.6\times10^{-19}\,C)(1450\,cm^2/V\text{-}s)}$$

$$= 1.8\times10^{16}\,cm^{-3}$$

$$\mu_n = 88\,T_n^{-0.57} + \frac{1250\,T_n^{-2.33}}{1+\left(\frac{0.143}{T_n^{2.4}}\right)0.88\,T_n^{-0.146}}$$

Use $\frac{\partial\mu_n}{\partial T} \simeq \frac{\Delta\mu_n}{\Delta T}$

for $T_n = 1$ & 1.01 ($300^\circ K$ & $303^\circ K$)

$$\frac{\partial\mu_n}{\partial T} \simeq -7.63$$

$$\alpha \simeq 3\times10^{-6}\,K^{-1} - \frac{1}{1450}(-7.63) = 3\times10^{-6}\,K^{-1} + 5.3\times10^{-3}\,K^{-1}$$

$$\boxed{\alpha \simeq 5.3\times10^{-3}\,K^{-1}}$$

IXYS → Si Chip resistors: $\alpha \sim 1.02\%\,/^\circ C$ $\xrightarrow{\text{ok}}$

(3) $R = \dfrac{L}{\sigma x_j W} = \dfrac{V}{I} = \dfrac{E/L}{j(x_j W)} = \dfrac{1}{x_j L W} \dfrac{E}{j}$

$j = +qn v_d$

$\therefore \quad R = \dfrac{1}{x_j L W}\left(\dfrac{1}{q N_D}\right)\dfrac{E}{v_d} = A \dfrac{E}{v_d}$

The incremental resistance can be defined as

$R' = \dfrac{\Delta V}{\Delta I}$ (slope around some operating point)

$= A \dfrac{\Delta E}{\Delta v_d}$ → for the resistance to double the slope of $\Delta v_d / \Delta E$ must be half

$$v_d = v_\ell \frac{E}{E_c}\left(\frac{1}{1+(E/E_c)^{1.11}}\right)^{1/1.11}$$

For $E \ll E_c$, $v_d \simeq v_\ell \dfrac{E}{E_c}$ ⇐ nominal value (low field)

$\therefore \quad \dfrac{\Delta v_d}{\Delta E} = \dfrac{v_\ell}{E_c} = \dfrac{1.07 \times 10^7 \text{ cm/s}}{6910 \text{ V/cm}} = 1548.5 \dfrac{\text{V}\cdot\text{cm}^2}{\text{s}}$ ↳ slope at low field

To numerically find the slope at a field E_1

(1) determine v_{d1} at E_1

(2) " v_{d2} at $E_1 + \Delta E_1$ (ΔE_1 can be 1% of E_1)

(3) find slope $= \dfrac{v_{d2} - v_{d1}}{(E_1 + \Delta E_1) - E_1} = \dfrac{v_{d2} - v_{d1}}{\Delta E_1}$

We want to find E_1 at which slope $= \frac{1}{2}(1548.5) \simeq 774.25$

I get about $E_1 = 3281$ V/cm

The voltage is $V = (3281 \frac{\text{V}}{\text{cm}})(125 \mu\text{m})(10^{-4} \frac{\text{cm}}{\mu\text{m}})$

$\boxed{V \simeq 41 \text{ V}}$