Lecture Notes for INEL 6055: Chapter 2

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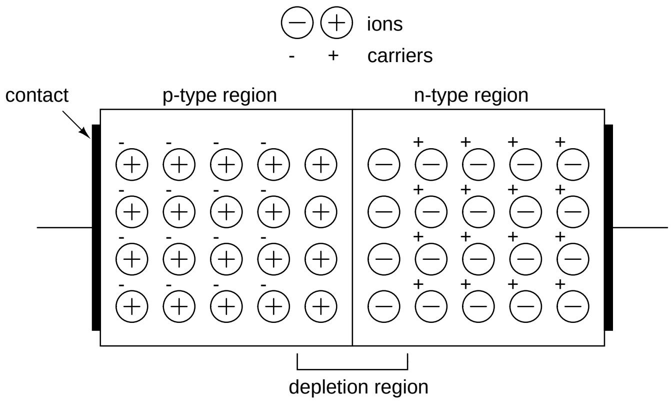
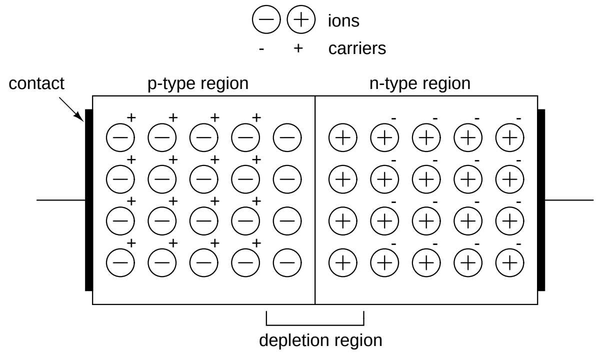
1 Junction Diode

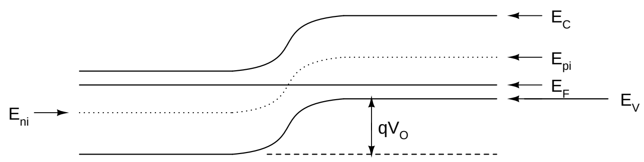
- Build by forming adjacent p- and n-type regions. Interface between regions is called the *pn junction*
- There are concentration gradients for free electrons (n-type side) and (p-type side). This gives place to diffusion currents across the junction.
- The holes that diffuse across the junction become excess minority carriers in the n-type region. Similarly, diffused electrons are excess minority carriers in the p-type region.
- Near the junction, device is depleted of majority carriers - *depletion region*.
- fixed ions remain in the depletion region. These ions generate an electric potential called the *build-in voltage*.
- Diffusion across the junction continues until the build-in voltage is large enough to prevent.



1.1 Build-in voltage

- As carriers diffuse across the junction, fixed ions remain. The electric charge represented by these ions give place to an electric field that cause the energy bands to bent.
- The Fermi level is constant throughout a system in thermal equilibrium.





- The voltage drop across the depletion region is said to form a *barrier*.This internal voltage gives place to a drift current I_S across the junction.
- An equilibrium is reached when the diffusion current due to the concentration gradient equals the drift current due to the build-in voltage. At equilibrium

$$I_D = I_S$$

- Since the Fermi level is the same at both sides of the junction, the bands will bent by qV_0. To find the build-in potential V_0, observe that

$$qV_0 = E_{pi} - E_{ni}$$

where E_{pi} and E_{ni} correspond to the middle of the forbidden band in the p- and n-type regions, respectively.

From chapter 1,

$$p = N_V e^{-\frac{E_F - E_V}{kT}}$$

For intrinsic material, $p = p_i$ and $E_F = E_{pi}$. Thus

$$\begin{aligned} p_i &= N_V e^{-\frac{E_{pi} - E_V}{kT}} \\ N_A &= N_V e^{-\frac{E_F - E_V}{kT}} \end{aligned}$$

$$\begin{aligned}
\frac{N_A}{p_i} &= \frac{e^{-\frac{E_F - E_V}{kT}}}{e^{-\frac{E_{pi} - E_V}{kT}}} \\
&= e^{\frac{E_{pi} - E_F}{kT}} \\
E_{pi} &= E_F + kTln\left(\frac{N_A}{p_i}\right)
\end{aligned}$$

Similarly, for the n-type side,

$$n = N_C e^{-\frac{E_C - E_F}{kT}}$$

For intrinsic material, $n = n_i$ and $E_F = E_{pi}$. Thus

$$\begin{aligned}
n_i &= N_C e^{-\frac{E_C - E_{ni}}{kT}} \\
N_D &= N_C e^{-\frac{E_C - E_F}{kT}} \\
\frac{N_D}{n_i} &= \frac{e^{-\frac{E_C - E_F}{kT}}}{e^{-\frac{E_C - E_{ni}}{kT}}} \\
&= e^{\frac{E_F - E_{ni}}{kT}} \\
E_{ni} &= E_F - kTln\left(\frac{N_D}{n_i}\right)
\end{aligned}$$

- From these, after subtracting, dividing by q and equating $\frac{kT}{q} = V_T$, the build-in voltage is found to be

$$V_O = V_T ln\left(\frac{N_A N_D}{n_i^2}\right)$$

1.2 Width of the Depletion Region

- For charge equality $qx_pN_A = qx_nN_D$, where x_p and x_n represent the depletion region width in the p- and n-type regions, respectively. This can be expressed as

$$\frac{x_p}{x_n} = \frac{N_D}{N_A}$$

The total width of the depletion region, given by $x_p + x_n$, can be shown to be given by

$$W_{dep} = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right) V_O}$$

1.3 Biased Diodes

- An external voltage can force carriers to cross the junction. For that the positive terminal must be connected to the p-type region and the negative terminal to the n-type region. The diode is said to be *forward biased*. Because of this, the n- and p-type regions are called *cathode* and *anode*, respectively.

- If the external voltage makes the cathode more positive than the anode, then the diode is *reversed biased*.

1.4 Reverse Bias Junction Capacitance

1.4.1 Abrupt Profile

- Charge accumulated in each side of the depletion region is

$$q_p = q_n = qN_D x_n A$$

- Poisson's Equation:

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\epsilon_s}$$

  Must be integrated twice to find φ.

$$\frac{d^2\varphi}{dx^2} = \begin{cases} 0 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S} & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S} & \text{if } x \geq x_p \\ 0 & \text{if } x \geq -x_p \end{cases}$$

- Solving problem for equilibrium conditions.
- First integration:

$$\frac{d\varphi}{dx} = \begin{cases} C_1 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}x + C_2 & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S}x + C_3 & \text{if } x \geq x_p \\ C_4 & \text{if } x \geq -x_p \end{cases}$$

- Boundary conditions for first integration:
  - E must be continuous
  - $E(-x_n) = -\frac{d\varphi}{dx} = 0$
  - $E(-x_p) = -\frac{d\varphi}{dx} = 0$
- First integration after using b.c.

$$\frac{d\varphi}{dx} = \begin{cases} 0 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}(x + x_n) & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S}(x - x_p) & \text{if } x \geq x_p \\ 0 & \text{if } x \geq -x_p \end{cases}$$

- Second integration:

$$\varphi = \begin{cases} C_5 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}(\frac{x^2}{2} + x_n x) + C_6 & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S}(\frac{x^2}{2} - x_p x) + C_7 & \text{if } x \geq x_p \\ C_8 & \text{if } x \geq -x_p \end{cases}$$

- Boundary conditions for second integration:
  - φ is constant for $x \leq -x_n$ and $x \geq x_p$
  - $\varphi(-x_n) = V_0$
  - $\varphi(x_p) = 0$ (defined as ground)
- Second integration after using b.c.:

$$\varphi = \begin{cases} V_0 & \text{if } x \leq -x_n \\ V_0 - \frac{qN_D}{2\epsilon_S}(x+x_n)^2 & \text{if } -x_n \leq x \leq 0 \\ +\frac{qN_A}{2\epsilon_S}(x-x_p)^2 & \text{if } 0 \leq x \leq x_p \\ 0 & \text{if } x \geq x_p \end{cases}$$

- Continuity at $x = 0$

$$V_0 - \frac{qN_D}{2\epsilon_S}x_n^2 = +\frac{qN_A}{2\epsilon_S}x_p^2$$

- Expressing $x_p = \frac{N_D}{N_P}x_n$ and

$$W_{dep} = x_n + x_p = (1 + \frac{N_D}{N_A})x_n = \frac{N_A + N_D}{N_A}x_n$$

$$\begin{aligned} V_0 &= \frac{q}{2\epsilon_S}\left(N_D x_n^2 + N_A x_p^2\right) \\ &= \frac{qN_D}{2\epsilon_S}\left(x_n^2 + \frac{N_A}{N_D}x_p^2\right) \\ &= \frac{qN_D}{2\epsilon_S}x_n^2\left(\frac{N_A + N_D}{N_A}\right) \end{aligned}$$

$$x_n = \sqrt{\frac{2\epsilon_S}{qN_D}\frac{N_A}{N_D + N_A}V_0}$$

- The charge is given by

$$\begin{aligned} q_j &= N_D q A x_n \\ &= qA\sqrt{\frac{2\epsilon_S}{q}\frac{N_A N_D}{N_D + N_A}V_0} \end{aligned}$$

- For a reversed biased junction, replace V_0 with $V_0 + V_{bias}$

$$q_j = qA\sqrt{\frac{2\epsilon_S}{q}\frac{N_A N_D}{N_D + N_A}(V_0 + V_{bias})}$$

- Apply the definition of capacitance,

$$C = \frac{dq}{dV}$$

to the junction charge using the applied voltage, V_{bias} as the voltage. Thus,

$$C_j = \frac{dq_j}{dV_{bias}} |_{V_{bias}=V_R}$$

where V_R is the applied bias voltage.

$$C_j = \frac{qA}{2}\sqrt{\frac{2\epsilon_S}{q}\frac{N_A N_D}{N_D + N_A}\frac{1}{V_0 + V_R}}$$

- in terms of

$$C_{j0} = A\sqrt{\frac{q\epsilon_s}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_0}}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

- From our previous expression for C_j,

$$C_j^2 = \frac{qA^2\epsilon_S}{2}\frac{N_A N_D}{N_D + N_A}\frac{1}{V_0 + V_R}$$

$$V_R = \frac{qA^2\epsilon_S}{2}\frac{N_A N_D}{N_D + N_A}\frac{1}{C_j^2} - V_0$$

If we measure C_j as we change V_R and plot V_R versus $1/C_j^2$, we should get a straight line with intercept $-V_0$ and slope

$$\frac{qA^{2\epsilon_S}}{2}\frac{N_A N_D}{N_D + N_A}$$

This only works if the junction is abrupt.

1.5 Linear Junction

- Charge density: $\rho = -ax$

$$\frac{d^2\varphi}{dx^2} = \frac{a}{\epsilon_S}x$$

- First integration:

$$\frac{d\varphi}{dx} = \frac{a}{\epsilon_S}\frac{1}{2}x^2 + C_1$$

B.C.: $E = \frac{d\varphi}{dx}$ continuous.
At $x = x_p = +\frac{w}{2}$, $E = 0$. Thus

$$C_1 = -\frac{a}{\epsilon_S}\frac{1}{2}x_p^2$$

and

$$\frac{d\varphi}{dx} = \frac{a}{2\epsilon_S}(x^2 - x_p^2)$$

- Second integration:

$$\varphi = \frac{a}{2\epsilon_S}(\frac{1}{3}(x^3 - x_p^2 x) + C_2$$

B.C.: $\varphi = 0$ at x_p

$$\begin{aligned} C_2 &= -\frac{a}{2\epsilon_S}(\frac{1}{3}(x_p^3 - x_p^2 x_p) \\ &= \frac{a}{3\epsilon_S}x_p \end{aligned}$$

and

$$\varphi = \frac{a}{2\epsilon_S}(\frac{1}{3}(x^3 - x_p^2 x + x_p)$$

- At $x = -x_n = -\frac{w}{2}$, $\varphi = V_0 + V_{bias}$,

$$\begin{aligned} V_0 + V_{bias} &= \frac{a}{3\epsilon_S}(x_n^3 + x_p^3) \\ &= \frac{a}{12\epsilon_S}w^3 \end{aligned}$$

or

$$w^3 = \frac{12\epsilon_S}{a}(V_0 + V_{bias})$$

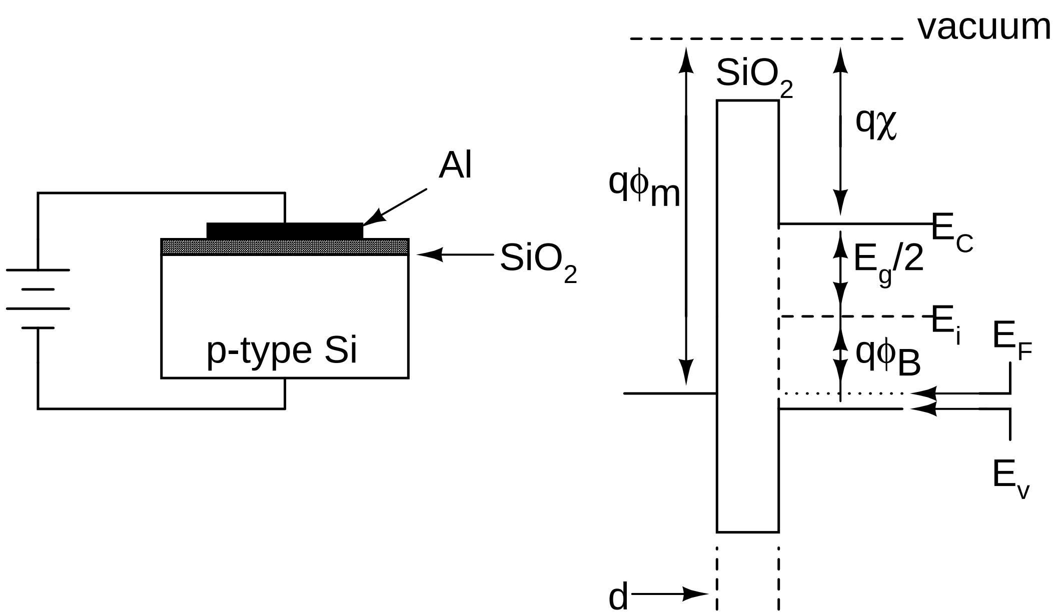
- The junction capacitance is given by

$$\begin{aligned} C_j &= \frac{A\epsilon_S}{w} \\ &= A\epsilon_S\sqrt[3]{\frac{a}{12\epsilon_S}}\frac{1}{\sqrt[3]{V_0 + V_{bias}}} \\ &= A\sqrt[3]{\frac{a\epsilon_S^2}{12V_0}}\frac{1}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}} \\ &= \frac{C_{j0}}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}} \end{aligned}$$

where

$$C_{j0} = A\sqrt[3]{\frac{a\epsilon_S^2}{12V_0}}$$

2 Ideal MOS Capacitor



Work function: difference between Fermi and vacuum levels
$q\phi_S$, $q\phi_m$: semiconductor and metal work functions, respectively.
Electron affinity $q\chi$: difference between the conduction band edge and the vacuum level.
Fermi potential $q\phi_F$: difference between the mid-gap and the Fermi level. Proportional to doping type and level.

- At zero bias, $\phi_m = \phi_S$. Work function difference is zero:

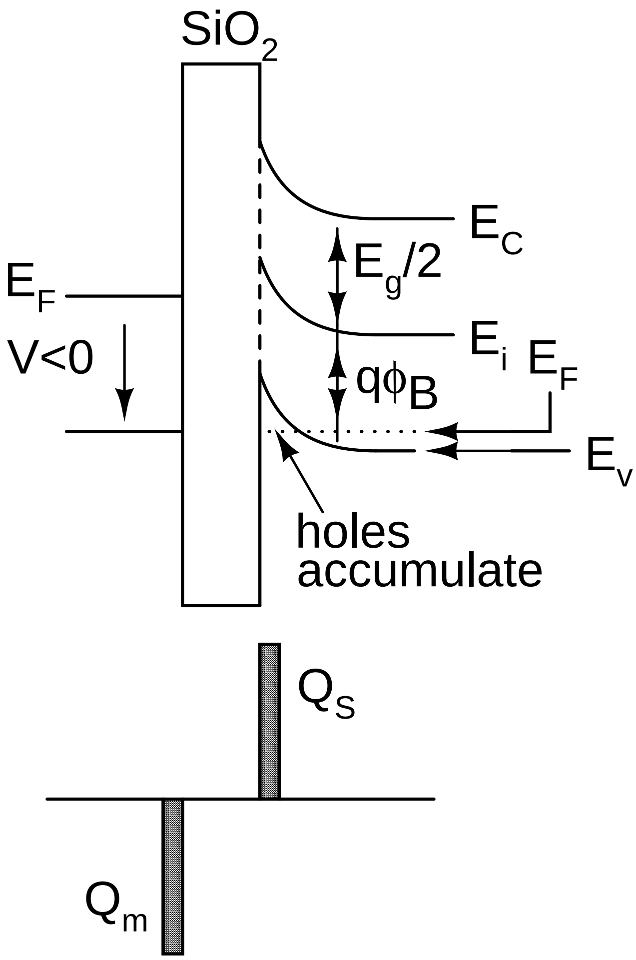
$$q\phi_m - \left[q\chi + \frac{E_G}{2} + q\phi_B\right] = 0$$

- charges accumulate in the $Si - SiO_2$ interface and in the metal.
- no carrier transport in the oxide under dc-bias conditions.

2.0.1 MOS Biasing

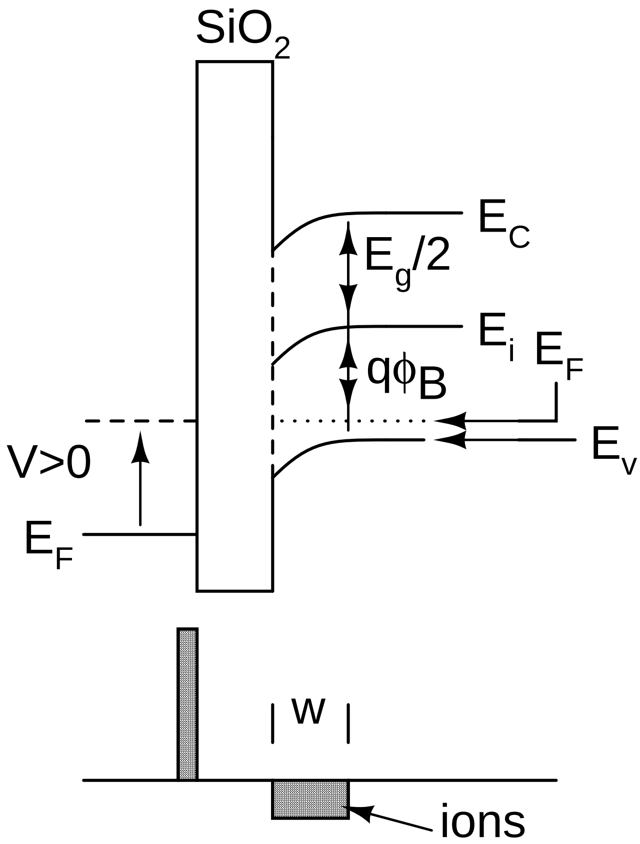
p-type Si

- Accumulation mode: If a negative voltage is applied to the metal



- majority carriers (holes) will be attracted to the $Si - SiO_2$ interface
- bands will bent near the surface.
- $p_p = n_i e^{\frac{E_i - E_F}{kT}}$

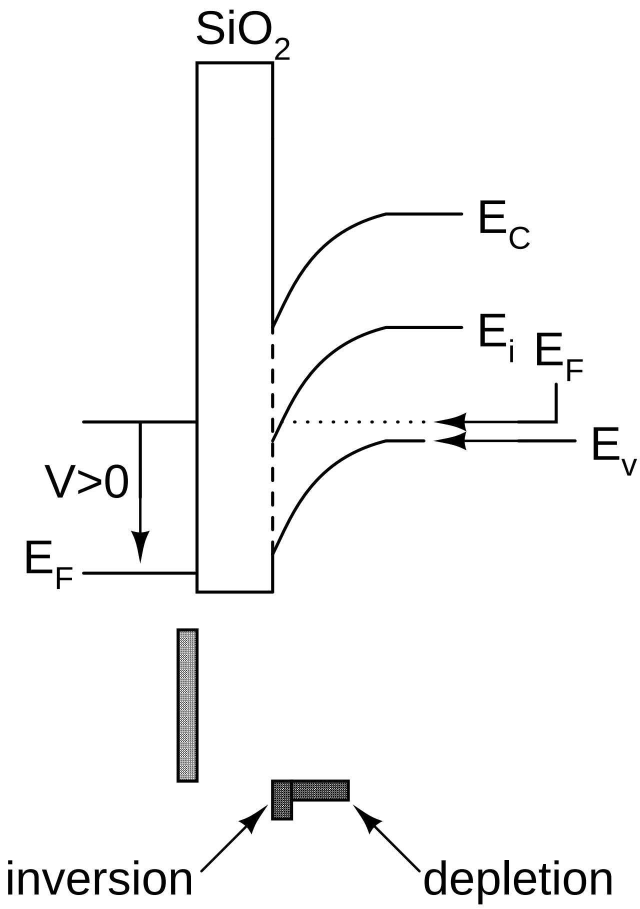
- Depletion mode: If a small positive voltage is applied to the metal,



- bands bent downward near the surface.
- majority carriers are driven away from interface.
- a depletion region is formed.

$$Q_{SC} = -qN_A w$$

- Weak inversion mode: A larger positive voltage is applied to the metal.



- bands bent downward near the surface.
- intrinsic level reach the Fermi level.
- depletion region still exists.
- minority carriers accumulate under metal
- the electron concentration under the metal is

$$n_p = n_i e^{\frac{E_F - E_i}{kT}}$$

- electron concentration is small

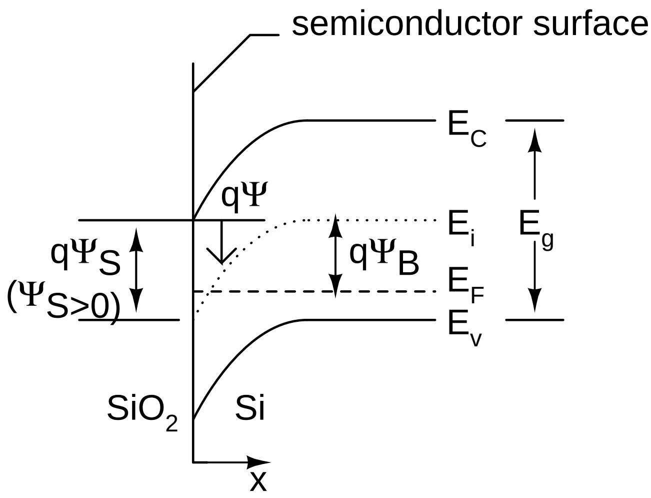
- Strong inversion mode: A still larger positive voltage is applied to the metal.
  - band bent further; conduction band edge comes close to the Fermi level.
  - for strong inversion

$$n_p \approx N_A$$

  - In the strong inversion mode, small increases in band bending yield large increases in the electron charge in the inversion layer; the surface depletion region reaches a maximum width w_m

$$Q_S = Q_n + Q_{SC} = Q_n - qN_A w_m$$

2.1 Detailed Analysis



- $\Psi = 0$ in the semiconductor bulk, by definition.
- Ψ_S is called the surface potential.
- electron and hole concentration as a function of Ψ

$$n_p = n_i e^{\frac{q(\Psi - \Psi_B)}{kT}}$$

$$p_p = n_i e^{\frac{q(\Psi_B - \Psi)}{kT}}$$

  where Ψ is positive when the bands bent downward.
- At the surface,

$$n_S = n_i e^{\frac{q(\Psi_S - \Psi_B)}{kT}}$$

$$p_S = n_i e^{\frac{q(\Psi_B - \Psi_S)}{kT}}$$

- $\Psi_S < 0$: bands bent upward; accumulation of holes
- $\Psi_S = 0$: flat-band condition
- $\Psi_B > \Psi_S > 0$: bands bent downward; depletion of holes
- $\Psi_B = \Psi_S$: midgap with $n_s = n_p = n_i$ (intrinsic concentration)
- $\Psi_S > \Psi_B$:inversion
- $\Psi(x)$ can be obtained from Poisson's equation

$$\frac{d^2\Psi}{dx^2} = -\frac{\rho_s(x)}{\epsilon_s}$$

- If the semiconductor is depleted to width w, the charge density in the depletion region is

$$\rho_s = -qN_A$$

– Setting $d\Psi/dx = 0$ and $\Psi = 0$ in the bulk, integration of Poisson's equation yields

$$\Psi(x) = \Psi_S \left(1 - \frac{x}{w}\right)^2$$

where the surface potential is

$$\Psi_S = \frac{qN_A w^2}{2\epsilon_S}$$

– For strong inversion, we can use the condition $n_S = N_A$. From

$$p_p = n_i e^{\frac{q(\Psi_B - \Psi)}{kT}}$$

evaluated at the bulk where $\Psi = 0$ and $p \approx N_A$,

$$N_A = n_i e^{\frac{q\Psi_B}{kT}} = n_S = n_i e^{\frac{q(\Psi_S - \Psi_B)}{kT}}$$

we get $\Psi_B = \Psi_S - \Psi_B$. For strong inversion w reaches a maximum w_m and

$$\begin{aligned}\Psi_{S,inv} &= 2\Psi_B \\ &\approx \frac{2kT}{q} ln\left(\frac{N_A}{n_i}\right) \\ &= \frac{qN_A w_m^2}{2\epsilon_S}\end{aligned}$$

and

$$w_m = 2\sqrt{\frac{\epsilon_S kT ln\left(\frac{N_A}{n_i}\right)}{q^2 N_A}}$$

The charge in the depletion region is

$$Q_{SC} = -qN_A w_m$$

2.2 Capacitance

– In the absence of any work function differences, when a voltage V is applied across the MOS structure it appears partly across the oxide and partly across the semiconductor

$$\begin{aligned}V &= V_o + \Psi_S \\ V_o &= \mathcal{E} d \\ &= \frac{|Q_S| d}{\epsilon_{ox}} \\ &= \frac{|Q_S|}{C_o}\end{aligned}$$

– C can be found by considering two capacitors in series

$$C = \frac{C_o C_j}{C_o + C_j} F/cm^2$$

where $C_j = \epsilon_s / w$.

– From the above equations

$$\frac{C}{C_o} = \frac{1}{\sqrt{1 + \frac{2\epsilon_{ox}^2 V}{qN_A \epsilon_s d^2}}}$$

which predicts that the capacitance will decrease with applied voltage.

– Whe a negative voltage is applied, no depletion region is formed and $C = C_o = \epsilon_{ox}/d$.

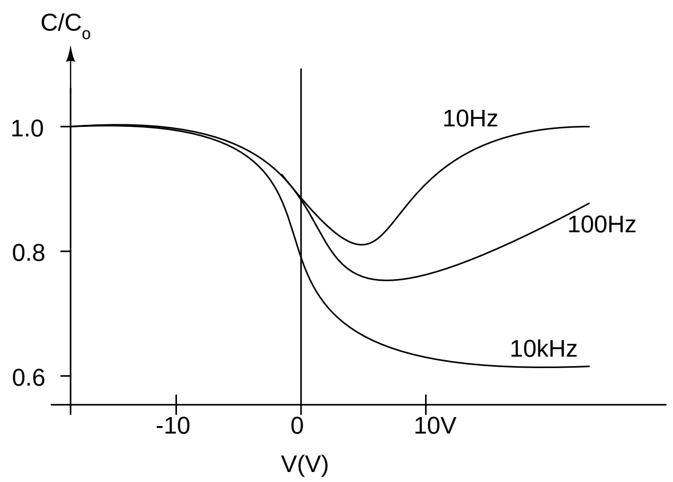
– When strong inversion occurs, $\Psi_S = \Psi_{S,inv}$ and $Q_S = qN_A w_m$. The voltage at which this happens is called the *threshold voltage*,

$$\begin{aligned} V_T &= \frac{qN_A w_m}{C_o} + 2\Psi_B \\ &= \frac{\sqrt{2\epsilon_s q N_A (2\Psi_B)}}{C_o} + 2\Psi_B \end{aligned}$$

– Once strong depletion takes place, capacitance remains at a minimum value that can be obtained by letting $C_j = \epsilon_S / w_m$,

$$\begin{aligned} C &= \frac{C_o C_j}{C_o + C_j} \\ &= \frac{(\epsilon_{ox}/d) \times (\epsilon_S / w_m)}{\epsilon_{ox}/d + \epsilon_S / w_m} \\ &= \frac{\epsilon_{ox}}{\frac{\epsilon_{ox}}{epsilon_S} w_m + d} \end{aligned}$$

– The above equation is correct at high frquencies. It assumes that when the metal voltage changes, all incremental charge appears at the edge of the depletion region. At frequencies below 100 Hz, however, the depletion region generation-recombination mechanism is faster that the voltage variations. This leads to charge exchange with the inversion layer in step with the metal voltage variations. As a result the capacitance in strong inversion will be that of the inversion layer alone.


C/C_o
1.0
0.8
0.6
10Hz
100Hz
10kHz
-10
0
10V
V(V)