

# Lecture Notes for INEL 6055: Chapter 3

Manuel Toledo Quiñones  
Assistant Professor  
Electrical and Computer Engineering Department  
University of Puerto Rico  
Mayaguez, Puerto Rico  
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## 1 Diode Equation

- See slides 5 and 6.
- Forward voltage injects excess minority carriers at the depletion region boundaries.
- Excess minority carriers diffuse due to the concentration gradient.
- Electrons drift through the depletion region, then diffuse through the neutral regions.
- Assumptions:
  - No recombination in the depletion region.
  - Negligible voltage drop in the neutral regions.
- Diffusion equation for electrons: linear approximation.

$$J_n = qD_n \frac{\partial n_p(x)}{\partial x} \approx \frac{n_{pe} - n_p(w_p)}{L_n}$$

where

- $n_{pe}$  is equilibrium concentration of electrons in p-type side.
- $L_n$  is the *diffusion length* of electrons.
- Fermi-Dirac Distribution:
$$f = \frac{1}{1 - e^{(E-E_F)/kT}} \approx e^{-\frac{E-E_F}{kT}}$$
- Energy barrier when voltage  $V$  is applied:  $q(V_0 - V_D)$ .
- $V_0$  is the build-in voltage.

- Excess electron concentration at  $w_p$

$$n_p(w_p) \propto \int_{q(V_0 - V_D)}^{\infty} e^{-\frac{E - E_F}{kT}} dE$$

After integration:  $n_p(w_p) = C e^{-\frac{E - E_F}{kT}} = C e^{\frac{qV_D}{kT}}$

- Since for  $v_D = 0$ ,  $n_p(w_p) = C = n_{pe}$ ,

$$n_p(w_p) = n_{pe} e^{\frac{qV_D}{kT}}$$

- The electron current density becomes

$$\begin{aligned} J_n &= qD_n \frac{n_{pe} - n_p(w_p)}{L_n} \\ &= qD_n \frac{n_{pe}(1 - e^{\frac{qV_D}{kT}})}{L_n} \\ &= \frac{qD_n n_{pe}}{L_n} (1 - e^{\frac{qV_D}{kT}}) \end{aligned}$$

- Using  $n_{pe} = \frac{n_i^2}{N_A}$ , replacing  $L_n$  by  $w_{anode}$  because  $L_n \gg w_{anode}$ , and multiplying by the junction area  $A$ , we get the current due to electrons:

$$i_n = -\frac{qAD_n n_i^2}{N_A w_{anode}} (e^{\frac{qV_D}{kT}} - 1)$$

where the fact that conventional current is opposite to electron flow was used.

- The junction current due to both holes and electrons is

$$i_D = I_S (e^{\frac{qV_D}{kT}} - 1)$$

where

$$I_S = \frac{qAD_n n_i^2}{N_A w_{anode}} + \frac{qAD_p n_i^2}{N_D w_{cathode}}$$

## 1.1 Second-order Corrections

- There is recombination in the depletion region.
- Carrier with energy below  $q(V_0 - V_D)$  can penetrate a fraction of the depletion region, and can recombine there.
- Therefore, it is possible for an electron and a hole, each with energy  $q\frac{V_0 - V_D}{2}$  to reach the middle of the d.r. and recombine, producing a current proportional to  $\frac{V_0 - V_D}{2}$ .

- To take into account for recombination in the depletion-region, the diode equation is modified to

$$i_D = I_S \left( e^{\frac{qV_D}{\eta kT}} - 1 \right)$$

where  $\eta$  is a parameter that take values between 1 and 2.

- See slide 7.
- The resistance due to the metal-semiconductor contacts and neutral regions are taken into account by adding a resistor  $r_S$  to the diode's model.
- See slide 8.
- Measurement of diode static parameters: see slide 10.

## 1.2 Stored-charge capacitance

- For reverse-biased junctions, the depletion-layer capacitance dominates the dynamic response.
- For forward-bias voltages large than about  $\frac{V_0}{2}$  the *stored-charge capacitance*  $C_S$  is more important.
- $C_S$  is due to excess minority-carrier accumulation at the edge of the depletion region, which represent a stored charge  $Q_S$ .
- See slide 11.
- If a forward-bias is suddenly turned off, removing  $Q_S$  takes time. Current  $i_D$  does not drop to zero until  $Q_S$  is removed.
- See slide 12.
- This process of  $C_S$  discharge determines the time-response and high-frequency behavior of the diode.
- SPICE model incorporates this effect by using a parameter called the *transit time*  $\tau_t$ .  $C_S = \tau_t \frac{di_D}{dV_{D0}}$ .

## 1.3 Temperature Effects

- Exponent in diode equation shows temperature dependence.
- $I_S$  is proportional to  $n_i^2$  which have a strong temperature dependence.
- Slide 13 shows the temperature dependence of a Silicon diode. This dependence can be approximated by a temperature coefficient of  $-2mV/^\circ C$ .
- See table 3.6 in textbook for temperature dependence of  $C_d$ .

## 2 Breakdown

### 2.1 Avalanche Breakdown

- See Slides 14, 15 and 16.
- Electrons move through the depletion region under reverse bias,
- Kinetic energy gained between collisions:  $E_{kin} = -qEx$ 
  - $E$ : applied field
  - $x$ : distance between collisions
- Average distance between collisions = scattering length =  $l_{sc}$
- Because  $l_{sc} \ll w_d$ , the width of the depletion region, many collisions happen before the electron crosses the depletion region.
- *Avalanche breakdown* happens when  $E_{kin} = -qEl_{sc} > E_{ionization}$ , the *ionization energy* needed to create an electron-hole pair.
- Temperature increases phonon scattering, reducing  $l_{sc}$  and thus increasing the field necessary for avalanche breakdown. The breakdown voltage exhibits a positive temperature coefficient.
- Avalanche breakdown voltage depends on doping. Higher doping increases built-in voltage and reduces breakdown voltage.
- Avalanche occurs at voltages larger than 6V.

### 2.2 Tunneling Breakdown

- Electron wave-function penetrates across the depletion region.
- Reverse bias produces band bending, which reduces depletion region width. See slide 16 right-hand side figure (a) and (b).
- As bending increase, conduction band in the n-type side falls below valence band in the p-type side.
- When this happens a tunneling current is established.
- Tunneling is effective due to the large concentration of electrons in the valence band of the p-type side of the junction. Even a very small probability can cause a significant tunneling current.
- Very narrow depletion regions are required for tunneling to be effective. This requires high doping levels.
- For tunneling diodes the t.c. is negative.
- Tunneling occurs at lower voltages than avalanche, typically.  $\approx 6V$ .

### 3 Metal-Semiconductor Contact

- Used to construct *Schottky diodes*.
- See slide 17.
- Energy-band diagrams are drawn so that the vacuum level match.
- Electrons in solid have negative energy with respect to that of an electron in vacuum.
- The energy needed to remove an electron from the metal is the *work function*,  $q\phi_m$ , is the difference between the vacuum energy and the Fermi level.
- See slide 18.
- The Fermi level of an n-type semiconductor is higher than the metal's. Conduction band electrons are at a higher energy in the semiconductor than in the metal.
- When the metal-semiconductor is formed, electrons from the semiconductor flow into the metal leaving behind ions and forming a depletion region.
- Thermal equilibrium is reached when the Fermi levels are equal at the interface.
- See slide 19.
- A potential barrier equal to the difference between the Fermi levels is formed.

$$qV_0 = q\phi_m - q\phi_s$$

- The barrier in the metal is

$$q\phi_B = q\phi_m - q\xi_s$$

where  $\xi_s$  is the *electron affinity* (difference in energy between bottom of the conduction band and vacuum) of the semiconductor.

- The reverse bias current in a Schottky diode is

$$I_S = I_{S0} e^{-\frac{q\phi_B}{kT}}$$

where  $I_{S0}$  is a constant that depends on temperature. This quantity plays the same role as the saturation current in the junction diode.

- The forward-bias current in the Schottky diode can be modeled with the same equation used for the junction diode:

$$I_D = I_S e^{\frac{V_D}{kT}}$$

- the *built-in voltage*  $V_0$  is smaller for Schottky diodes than for junction diodes. See slide 20 and 21..

- In Schottky diodes all carriers are electrons. Thus there is no charge storage and consequently Schottky diodes are faster than junction diodes. The cost is a larger  $I_S$ .
- In the case of heavily-doped n-type semiconductor, a very narrow energy barrier is formed. This barrier is not high enough to prevent electrons from crossing. Also the depletion region is very narrow and tunneling takes place. Since the barrier is not effective to stop electrons from crossing across the interface, an *Ohmic contact* is formed.
- See slides 22 and 23.