Lecture Notes for INEL 6055: Chapter 6

Manuel Toledo Quiñones
Assistant Professor
Electrical and Computer Engineering Department
University of Puerto Rico
Mayaguez, Puerto Rico

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1 Introduction

- A reverse junction as a current source. See slide 2.
- Minority electrons and holes can freely fall down or bubble up.
- We can control the current by adjusting the quantity of minority carriers.This can be done by increasing the temperature or by exposure to light.
- A *forward-biased* pn junction build next to the reverse-biased junction can provide the additional carriers.
- The resulting two-junction device is a BJT. The voltage across the forward-biased junction controls the supply of carriers and thus reverse-biased junction current.
- Consider an NPN BJT operating in the active region:
  - See slide 3.
  - Forward-biased base-emitter junction and reverse-biased base-collector junction.
  - Electrons flow from the emitter into the base, and holes from the base into the emitter, producing currents I_{nE} and I_{pE}, respectively.
  - See slide 4.
  - The total emitter current is $I_E = I_{nE} + I_{pE}$.
  - Electrons entering the emitter can flow into the collector.
  - Holes flowing from base to emitter can not do "useful" work.
  - The *emitter efficiency* is defined by

$$\gamma_E = \frac{I_{nE}}{I_E} = \frac{1}{1 + \frac{I_{pE}}{i_{nE}}}$$

  - I_{nE} and I_{pE} depend on V_{BE} and on the majority carrier concentrations. To increase efficiency, dope the emitter as much as possible and the base as little as possible.
  - The ratio of successfully-collected to emitted electrons is the *transport factor*:

$$\alpha_T = \frac{I_{nC}}{I_{nE}}$$

  - Electrons can recombine at the base; thus to maximize α_T, make the base region as thin as possible.
  - A reverse current I_{CB0} flows through the collector-base junction. This leakage current can be neglected in active operation, but is important in cutoff.
  - I_E depends exponentially on V_{BE}. For operation as a voltage-controlled current source, the *transconductance* g_m that relates I_C to V_{BE} is the device gain. The exponential relationship makes g_m high.
  - Both base and emitter are used as amplifier inputs (CE and CB configurations). The corresponding current gains are $\beta = \frac{I_C}{I_B}$ and $\alpha = \frac{I_C}{I_E}$:

$$\alpha \approx \frac{I_{nC}}{I_E} = \frac{I_{nC}}{I_{nE}}\frac{I_{nE}}{I_E} = \alpha_T \gamma_E$$

  - α and β are related:

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\alpha = \frac{\beta}{\beta + 1}$$

- Slide 5 shows the four modes of operating a BJT.

2 Bipolar IC Technologies

- Structure of an IC BJT. See slide 7.
- The N+ buried layer provides a low-resistance path for electrons to flow out of the collector.
- Bipolar technology heavily relies on the quality of the N-epi layer.
- Layer deposition techniques:
  - See
    http://kottan-labs.bgsu.edu/teaching/workshop2001/chapter5.htm
    http://www.uoregon.edu/ hutchlab/semilab/thinfilm.html
    http://www.filebox.vt.edu/users/sprice/semiconductorprocess.html
    http://www.postech.ac.kr/mse/semicon/facilities.htm
  - evaporation
  - sputtering
  - chemical vapor deposition (CVD)
  - molecular beam epitaxy (MBE)

3 BJT Modeling

3.1 Ebers-Moll Model

3.1.1 Injection Version

- See slide 14b.
- Forward active mode

$$I_F = I_{ES}\left(e^{V_{BE}/V_T} - 1\right)$$

- Reverse active mode

$$I_R = I_{CS}\left(e^{V_{BC}/V_T} - 1\right)$$

- The terminal currents are:

$$\begin{aligned}
I_C &= \alpha_F I_F - I_R \\
&= \alpha_F I_{ES}\left(e^{V_{BE}/V_T} - 1\right) - I_{CS}\left(e^{V_{BC}/V_T} - 1\right) \\
I_E &= -I_F + \alpha_R I_R \\
&= -I_{ES}\left(e^{V_{BE}/V_T} - 1\right) + \alpha_R I_{CS}\left(e^{V_{BC}/V_T} - 1\right)
\end{aligned}$$

- The base current is the difference between the emitter and collector currents;

$$I_B = -I_E - I_C$$

3.1.2 Transport Version

- See slide 14c.
- For the two models to be equivalent:

$$I_{EC} = \alpha_R I_R = I_S\left(e^{V_{BC}/V_T} - 1\right)$$

$$I_{CC} = \alpha_F I_F = I_S\left(e^{V_{BE}/V_T} - 1\right)$$

where, for normal active mode,

$$I_S = \alpha_F I_{ES}$$

and for inverse active mode

$$I_S = \alpha_R I_{CS}$$

- The terminal currents are:

$$\begin{aligned}
I_C &= I_{CC} - I_{CE}/\alpha_R \\
&= I_S\left(e^{V_{BE}/V_T} - 1\right) - I_S/\alpha_R\left(e^{V_{BC}/V_T} - 1\right) \\
I_E &= -I_{CC}/\alpha_F + I_{CE} \\
&= -I_S/alpha_F\left(e^{V_{BE}/V_T} - 1\right) + I_S\left(e^{V_{BC}/V_T} - 1\right) \\
I_B &= -I_E - I_C
\end{aligned}$$

3.1.3 SPICE Version

- See slide 15.
- Terminal currents:

$$
\begin{aligned}
I_C &= I_{CC} - I_{CE}/\alpha_R \\
&= -I_{?1} + I_{CT} \\
&= -I_{?1} + I_{CC} - I_{CE} \\
I_{?1} &= I_{CC} - I_{CE} - I_{CC} + I_{CE}/\alpha_R \\
&= I_{CE}\left(\frac{1-\alpha_R}{\alpha_R}\right) \\
&= I_{CE}/\beta_R \\
I_E &= -I_{CC}/\alpha_F + I_{CE} = -I_{?2} - I_{CC} + I_{CE} \\
I_{?2} &= I_{CC}/\alpha_F - I_{CE} - I_{CC} + I_{CE} \\
&= I_{CC}\left(\frac{1-\alpha_F}{\alpha_F}\right) \\
&= I_{CC}/\beta_F
\end{aligned}
$$

- In terms of I_S,

$$
\begin{aligned}
I_C &= I_S\left(e^{V_{BE}/V_T} - 1\right) - \left(1 + \frac{1}{\beta_R}\right) I_S\left(e^{V_{BC}/V_T} - 1\right) \\
I_E &= -\left(1 + \frac{1}{\beta_F}\right) I_S\left(e^{V_{BE}/V_T} - 1\right) + I_S\left(e^{V_{BC}/V_T} - 1\right) \\
I_B &= -I_E - I_C \\
&= \frac{1}{\beta_F} I_S\left(e^{V_{BE}/V_T} - 1\right) + \frac{1}{\beta_R} I_S\left(e^{V_{BC}/V_T} - 1\right)
\end{aligned}
$$

- For the normal active mode,

$$
\begin{aligned}
I_C &= I_S e^{V_{BE}/V_T} \\
I_E &= -\left(1 + \frac{1}{\beta_F}\right) I_S e^{V_{BE}/V_T} \\
I_B &= \frac{1}{\beta_F} I_S e^{V_{BE}/V_T} = \frac{I_C}{\beta_F}
\end{aligned}
$$

3.2 Second Order Effects

3.2.1 Early Effect

- See slide 16.

- Also called the *base modulation effect*.
- Base-collector depletion layer changes width, causing an increase in I_S (because more hole-electron pairs are generated in the depletion region) and therefore in I_C.
- Represented in terms of

$$I_S = I_{S0}\left(1 + \frac{|V_{BC}|}{|V_A|}\right)$$

3.2.2 Parasitic resistances r_E, r_B and r_C.

3.2.3 Parasitic Capacitances

- See slides 18 and 19.
- For normal active mode,

$$C_\mu = C_{dC}$$

$$C_\pi = C_{dE} + C_{sE}$$

3.2.4 Gummel-Poon Effect

- β's are not constant with current; see slide 20.
- Explained in terms of charge accumulation in the base at high voltage, in the capacitances C_{dE} and C_{sE}.
- Majority carrier concentration in the base: doping + depletion layer charge + stored charge;

$$\begin{aligned} Q_{BT} &= Q_{B0} \\ &+C_{dE}V_{BE} + C_{dC}V_{BC}\frac{A_E}{A_C} \\ &+\frac{Q_{B0}}{Q_{BT}}\tau_F I_S\left(e^{V_{BE}/V_T}-1\right) + \frac{Q_{B0}}{Q_{BT}}\tau_R I_S\left(e^{V_{BC}/V_T}-1\right) \end{aligned}$$

  where $Q_{B0} = qN_AA_E$.

- Because I_S is inversely proportional to the carrier density, multiplying I_S by Q_{B0}/Q_{BT} effectively replaces the doping level by the total charge; in the normal active mode,

$$I_C \approx \frac{I_{S0}}{q_b}e^{V_{BE}}V_T$$

where

$$
\begin{aligned}
q_b &= \frac{Q_{BT}}{Q_{B0}} \\
&= 1 + \frac{C_{dE}V_{BE}}{Q_{B0}} + \frac{C_{dC}V_{BC}}{Q_{B0}}\frac{A_E}{A_C} \\
&\quad + \frac{1}{q_b Q_{B0}}\tau_F I_S\left(e^{V_{BE}/V_T} - 1\right)
\end{aligned}
$$

$$
+\frac{1}{q_b Q_{B0}}\tau_R I_S\left(e^{V_{BC}/V_T} - 1\right)
$$

$$
\begin{aligned}
q_b^2 &= q_b\left(1 + \frac{C_{dE}V_{BE}}{Q_{B0}} + \frac{C_{dC}V_{BC}}{Q_{B0}}\frac{A_E}{A_C}\right) \\
&\quad + \frac{1}{Q_{B0}}\tau_F I_S\left(e^{V_{BE}/V_T} - 1\right) + \frac{1}{Q_{B0}}\tau_R I_S\left(e^{V_{BC}/V_T} - 1\right)
\end{aligned}
$$

This can be rewritten as the quadratic:

$$
q_b^2 - q_1 q_b - q_2 = 0
$$

where

$$
\begin{aligned}
q_1 &= 1 + \frac{C_{dE}V_{BE}}{Q_{B0}} + \frac{C_{dC}V_{BC}}{Q_{B0}}\frac{A_E}{A_C} \\
&= 1 + \frac{V_{BE}}{|V_B|} + \frac{V_{BC}}{|V_A|} \\
V_B &= \frac{Q_{B0}}{C_{dE}} \\
V_A &= \frac{Q_{B0}}{C_{dC}}\frac{A_C}{A_E} \\
q_2 &= \frac{1}{Q_{B0}}\tau_F I_S\left(e^{V_{BE}/V_T} - 1\right) + \frac{1}{Q_{B0}}\tau_R I_S\left(e^{V_{BC}/V_T} - 1\right) \\
&= \frac{I_{S0}}{I_{KF}}\left(e^{V_{BE}/V_T} - 1\right) + \frac{I_{S0}}{I_{KR}}\left(e^{V_{BC}/V_T} - 1\right) \\
I_{KF} &= \frac{Q_{B0}}{\tau_F} \\
I_{KR} &= \frac{Q_{B0}}{\tau_R}
\end{aligned}
$$

The solution is:

$$
q_b = \frac{q_1}{2} + \frac{\sqrt{q_1 + 4q_2}}{2}
$$

which in the SPICE model is approximated as

$$
q_b \approx \frac{q_1}{2}\left(1 + \sqrt{1 + 4q_2}\right)
$$

- The complete BJT equations at the Gummel-Poon level in SPICE are:

$$
\begin{aligned}
I_C &= \frac{I_{S0}}{q_b}\left(e^{V_{BE}/V_T} - e^{V_{BC}/V_T}\right) \\
&\quad -\frac{I_{S0}}{\beta_{RM}}\left(e^{V_{BC}/V_T} - 1\right) - C_4 I_{S0}\left(e^{V_{BC}/(n_{CL}V_T)} - 1\right) \\
I_E &= -\frac{I_{S0}}{q_b}\left(e^{V_{BE}/V_T} - e^{V_{BC}/V_T}\right) \\
&\quad -\frac{I_{S0}}{\beta_{FM}}\left(e^{V_{BE}/V_T} - 1\right) - C_2 I_{S0}\left(e^{V_{BE}/(n_{EL}V_T)} - 1\right) \\
I_B &= -I_E - I_C \\
&= +\frac{I_{S0}}{\beta_{FM}}\left(e^{V_{BE}/V_T} - 1\right) + C_2 I_{S0}\left(e^{V_{BE}/(n_{EL}V_T)} - 1\right) \\
&\quad +\frac{I_{S0}}{\beta_{RM}}\left(e^{V_{BC}/V_T} - 1\right) + C_4 I_{S0}\left(e^{V_{BC}/(n_{CL}V_T)} - 1\right)
\end{aligned}
$$

- Terms associated with coefficients C_2 and C_4 in the above equations are added to account for the fact that, at low biasing levels, the recombination of carriers in the bulk and surface depletion layer, as well as other surface leakage mechanisms, lead to an increase in the base current and a corresponding decrease in β. See slide 20.

- The Ebers-Moll equations can be obtained from the Gummen-Poon level equations by
  - letting $I_{KF} = I_{KR} \to \infty$ so that $q_2 \to 0$
  - letting $|V_A| = |V_B| \to \infty$ so that $q_1 \to 1$
  - let C_2 and C_4 remain at their default value of 0

4 Measurement of SPICE Parameters

4.1 I_S and β

In the normal active mode,

$$I_C \simeq I_S e^{V_{BE}/V_T}$$

and

$$I_B = I_C/\beta_F$$

so that

$$ln I_C = ln I_S + \frac{V_{BE}}{V_T}$$

and

$$ln I_B = ln I_S + \frac{V_{BE}}{V_T} - ln\beta_F$$

See slide 21.

4.2 V_A

See slide 22.

4.3 r_E and I_{KF}

See slide 23.

To estimate r_E, observe that

$$lnI_B == lnI_S + \frac{V_{BE}}{V_T} - ln\beta_F$$

fits the data well for small I_B. The fitting improves for intermediate values of V_{BE} if $r_E \neq 0$ is selected.

For still larger values of V_{BE}, there will be a departure from linearity even if $r_E \neq 0$ is selected because of the Gummen-Poon effect. In this region of the graph, $q_b \gg 1$ and

$$I_C \approx \sqrt{I_{S0} I_{KF}} e^{V_{BE}/(2V_t)}$$

Thus,

$$lnI_C \approx ln\sqrt{I_{S0} I_{KF}} + \frac{1}{2V_t} V_{BE}$$

The parameter C_2 could be estimated if the graph departs from a line with slope $1/V_t$ for small values of V_{BE}. The change in slope of the line and its value at $V_{BE} = 0$ can then be used to estimate n_{EL} and $C_2 I_{S0}$.